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$rg\alpha$-Homeomorphisms in Topological Spaces

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Abstract

A bijection $f : (X, \tau) \to (Y, \sigma)$ is called regular generalized α-homeomorphism if f and f^{-1} are $rg\alpha$-continuous. Also we introduce new class of maps, namely $rg\alpha c$-homeomorphisms which form a subclass of $rg\alpha$-homeomorphisms. This class of maps is closed under composition of maps. We prove that the set of all $rg\alpha c$-homeomorphisms forms a group under the operation composition of maps.

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1 Introduction

Generalized closed mappings were introduced and studied by Malghan [11]. Generalized open maps, rg-closed maps, wg-closed maps and wg-open maps, w-open maps and w-closed maps, g^*-open maps and g^*-closed maps, pre-semi-open maps, almost open maps, α-open maps and gpr-closed maps have been introduced and studied by Sundaram [21], Arockiarani [2], Nagaveni [14], Sheik Jhon [17], Pushpalatha [16], Crossely and Hildebrand [6], Singal and Singal [19], Mashhour [13] and Gnanambal [7] respectively. We give the definitions of some of them which are used in our present study

In this paper, we introduce the concept of $rg\alpha$-homeomorphism and study the relationship between homeomorphisms, w-homeomorphisms, g-homeomorphisms and rwg-homeomorphisms. Also we introduce new class of maps $rg\alpha$-homeomorphisms which form a subclass of $rg\alpha$-homeomorphisms. This class of maps is closed under composition of maps. We prove that the set of all homeomorphisms $rg\alpha c$-forms a group under the operation composition of maps.

Let us recall the following definition which we shall require later.

Definition 1.1. A subset A of a space $(X,\ \tau)$ is called
1) a **preopen set** [12] if $A \subseteq intcl\,(A)$ and a **preclosed set** if $clint\,(A) \subseteq A$.
2) a **semiopen set** [8] if $A \subseteq clint\,(A)$ and a **semiclosed set** if $intcl\,(A) \subseteq A$.
3) a α**-open set** [15] if $A \subseteq intclint\,(A)$ and a α**-closed set** if $clintcl\,(A) \subseteq A$.
4) a **semi-preopen set** [1] if $A \subseteq clintcl\,(A)$ and a **semi-preclosed set** if $intclint\,(A) \subseteq A$.
5) a **regular open set** [20] if $A = intcl\,(A)$ and a **regular closed set** if $A = clint\,(A)$.

The intersection of all semiclosed (resp. semiopen) subsets of (X, τ) containing A is called the semi-closure (resp. semi-kernal) of$A$ and is denoted by scl(A)(resp. sker(A)).

Definition 1.2. A subset A of a space $(X,\ \tau)$ is called
1) **generalized closed set** (briefly, g-closed)[9] if $cl\,(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
2) **weakely closed set** (briefly, w-closed)[18] if $cl\,(A) \subseteq U$ whenever $A \subseteq U$ and U is semiopen in X.
3) **regular weakly generalized closed set** (briefly, rwg-closed)[14] if $clint\,(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
4) **regular semiopen set** [5] if there is a regular open set U such that $U \subset A \subset cl(U)$.
5) **regular** α**-open set** (briefly, $r\alpha$-open) [22] if there is a regular open set U such that $U \subset A \subset \alpha cl(U)$.
6) **regular** w**-closed set** (briefly, rw-closed) [25] if $cl\,(A) \subset U$ whenever $A \subset U$ and U is regular semiopen in X.
7) **regular generalized** α**-closed set** (briefly, $rg\alpha$-closed) [22] if $\alpha cl\,(A) \subset U$ whenever $A \subset U$ and U is regular α-open in X.

The complements of the above mentioned closed sets are their respective open sets.

Definition 1.3. A map $f : (X,\ \tau) \to (Y,\ \sigma)$ is called
(i) g**-continuous** [3] if $f^{-1}(V)$ is g-closed set of $(X,\ \tau)$ for every closed set V of $(Y,\ \sigma)$,
(ii) w**-continuous** [17] if $f^{-1}(V)$ is w-closed set of $(X,\ \tau)$ for every closed set V of $(Y,\ \sigma)$,
(iii) rwg**-continuous** [14] if $f^{-1}(V)$ is rwg-closed set of $(X,\ \tau)$ for every closed set V of $(Y,\ \sigma)$,
(iv) $rg\alpha$**-continuous** [23] if $f^{-1}(V)$ is $rg\alpha$-closed set of $(X,\ \tau)$ for every closed set V of $(Y,\ \sigma)$.

Definition 1.4. A map $f : (X,\ \tau) \to (Y,\ \sigma)$ is said to be
(i) **irresolute** [6] if $f^{-1}(V)$ is semiopen in $(X,\ \tau)$ for each semiopen set V of

$(Y,\ \sigma)$,
(ii) w**-irresolute** [17] if $f^{-1}(V)$ is w-closed in $(X,\ \tau)$ for each w-closed set V of $(Y,\ \sigma)$,
(iii) $rg\alpha$**-irresolute** [23] if $f^{-1}(V)$ is $rg\alpha$-closed in $(X,\ \tau)$ for each $rg\alpha$-closed set V of $(Y,\ \sigma)$.

Definition 1.5. A map $f : (X,\ \tau) \to (Y,\ \sigma)$ is said to be
(i) $rg\alpha$**-closed** [24] if $f(F)$ is $rg\alpha$-closed in $(Y,\ \sigma)$ for every closed set F of $(X,\ \tau)$.

Definition 1.6. A map $f : (X,\ \tau) \to (Y,\ \sigma)$ is said to be
(i) $rg\alpha$**-open** [24] if $f(F)$ is $rg\alpha$-open in $(Y,\ \sigma)$ for every open set F of $(X,\ \tau)$.

Definition 1.7. A bijective function $f : (X,\ \tau) \to (Y,\ \sigma)$ is called
(i) **generalized homeomorphism** (g-homeomorphism) [10] if both f and f^{-1} are g-continuous,
(ii) gc**-homeomorphism** [10] if both f and f^{-1} are gc-irresolute,
(iii) rwg**-homeomorphism** [14] if both f and f^{-1} are rwg-continuous,
(iv) w^***-homeomorphism** [17] if both f and f^{-1} are w-irresolute,
(v) w**-homeomorphism** [17] if both f and f^{-1} are w-contiuous.

2 $rg\alpha$-homeomorphisms in Topological Spaces

We introduce the following definition
Definition 2.1. A bijection $f : (X,\ \tau) \to (Y,\ \sigma)$ is called **regular generalised α-homeomorphism** (briefly, $rg\alpha$-homeomorphism) if f and f^{-1} are $rg\alpha$-continuous.

We denote the family of all $rg\alpha$-homeomorphisms of a topological space $(X,\ \tau)$ onto itself by $rg\alpha$-$h(X,\ \tau)$.

Example 2.1 *Consider $X = Y = \{a,\ b,\ c,\ d\}$ with topologies $\tau = \{X,\ \phi,\ \{a\},\ \{b\},\ \{a,b\},\ \{a,\ b,\ c\}\}$ and $\sigma = \{Y,\ \phi,\ \{a\},\ \{b\},\ \{a,\ b\}\}$. Let $f : (X,\tau) \to (Y,\ \sigma)$ be the identity map. Then f is bijective, $rg\alpha$-continuous and f^{-1} is $rg\alpha$-continuous. Therefore f is $rg\alpha$-homeomorphism.*

Theorem 2.1. *Every homeomorphism is an $rg\alpha$-homeomorphism, but not conversely.*

Proof. Let $f : (X,\ \tau) \to (Y,\ \sigma)$ be a homeomorphism. Then f and f^{-1} are continuous and f is bijection. As every continuous function is $rg\alpha$-continuous, we have f and f^{-1} are $rg\alpha$-continuous. Therefore f is $rg\alpha$-homeomorphism. ∎

The converse of the above Theorem need not be true, as seen from the following example.

Example 2.2 *Consider* $X = Y = \{a,\ b,\ c,\ d\}$ *with topologies* $\tau = \{X,\ \phi,\ \{a\},\ \{b\},\ \{a,\ b\},\ \{a,\ b,\ c\}\}$ *and* $\sigma = \{Y,\ \phi,\ \{a\},\ \{b\},\ \{a,\ b\}\}$. *Define a map* $f : (X,\ \tau) \to (Y,\ \sigma)$ *be the identity map. Then* f *is* $rg\alpha$*-homepmorphism but it is not homeomorphism, since the inverse image of open set* $\{a,\ b,\ c\}$ *in* X *is* $\{a,\ b,\ c\}$ *which is not open set in* Y.

Theorem 2.2. *Every* w*-homeomorphism is an* $rg\alpha$*-homeomorphism but not conversely.*

Proof. Let $f : (X,\ \tau) \to (Y,\ \sigma)$ be a w-homeomorphism. Then f and f^{-1} are w-continuous and f is bijection. As every w-continuous function is $rg\alpha$-continuous, we have f and f^{-1} are $rg\alpha$-continuous. Therefore f is $rg\alpha$-homeomorphism. ∎

The converse of the above Theorem is not true in general as seen from the following example.

Example 2.3 *Consider* $X = Y = \{a,\ b,\ c,\ d\}$ *with topologies* $\tau = \{X,\ \phi,\ \{a\},\ \{b\},\ \{a,\ b\},\ \{a,\ b,\ c\}\}$ *and* $\sigma = \{Y,\ \phi,\ \{a\},\ \{b\},\ \{a,\ b\}\}$. *Let* $f : (X,\ \tau) \to (Y,\ \sigma)$ *be the identity map. Then this function is* $rg\alpha$*-homeomorphism but it is not* w*-homeomorphism, since the inverse image of the open set* $\{a,\ b,\ c\}$ *in* X *is* $\{a,\ b,\ c\}$ *which is not* w*-open set in* Y.

Theorem 2.3. *Every* rw*-homeomorphism is an* $rg\alpha$*-homeomorphism but not conversely.*

Proof. Let $f : (X,\ \tau) \to (Y,\ \sigma)$ be a rw-homeomorphism. Then f and f^{-1} are rw-continuous and f is bijection. As every rw-continuous function is $rg\alpha$-continuous, we have f and f^{-1} are $rg\alpha$-continuous. Therefore f is $rg\alpha$-homeomorphism. ∎

The converse of the above theorem is not true in general as seen from the following example.

Example 2.4 *Consider* $X = Y = \{a,\ b,\ c,\ d\}$ *with topologies* $\tau = \{X,\ \phi,\ \{a\},\ \{b\},\ \{a,\ b\},\ \{a,\ b,\ c\}\}$ *and* $\sigma = \{Y,\ \phi,\ \{a\},\ \{b\},\ \{a,\ b\}\}$. *Let* $f : (X,\ \tau) \to (Y,\ \sigma)$ *be the identity map. Then this function is* $rg\alpha$*-homeomorphism but it is not* rw*-homeomorphism, since the inverse image of the open set* $\{a,\ b,\ c\}$ *in* X *is* $\{a,\ b,\ c\}$ *which is not* rw*-open set in* Y.

Corollary 2.1. *Every* w^**-homeomorphism is an* $rg\alpha$*-homeomorphism but not conversely.*

Proof. From Sheik John [18], it follows that every w^*-homeomorphism is a w-homeomorphism but not conversely. By Theorem 2.3., every w-homeomorphism is a $rg\alpha$-homeomorphism but not conversely and hence w^*-homeomorphism is a $rg\alpha$-homeomorphism but not conversely. ∎

Theorem 2.4. *Every $rg\alpha$-homeomorphism is an rwg-homeomorphism but not conversely.*

Proof. Let $f : (X,\ \tau) \to (Y,\ \sigma)$ be $rg\alpha$-homeomorphism. Then f and f^{-1} are $rg\alpha$-continuous and f is bijection. Since every $rg\alpha$-continuous function is rwg-continuous, we have f and f^{-1} are rwg-continuous. Therefore f is rwg-homeomorphism. ■

The converse of the above Theorem is not true in general as seen from the following example.

Example 2.5 *Consider $X = Y = \{a,\ b,\ c,\ d\}$ with topologies $\tau = \{X,\ \phi,\ \{a\},\ \{a,\ b\},\ \{a,\ b,\ c\}\}$ and $\sigma = \{Y,\ \phi,\ \{a\},\ \{b\},\ \{a,\ b\},\ \{a,\ b,\ c\}\}$. Define a map $f : (X,\ \tau) \to (Y,\ \sigma)$ by $f(a) = c$, $f(b) = b$, $f(c) = a$ and $f(d) = d$. Then f is rwg-homeomorphism but it is not $rg\alpha$-homeomorphism, since the inverse image of the open set $\{a,\ b\}$ in X is $\{b,\ c\}$ which is not $rg\alpha$-open set in Y.*

Remark 2.1. *$rg\alpha$-homeomorphism and g-homeomorphism are independent as seen from the following example.*

Example 2.6 *Let $X = Y = \{a,\ b,\ c\}$ with topologies $\tau = \{X,\ \phi,\ \{a\}\}$ and $\sigma = \{Y,\ \phi,\ \{a,\ b\}\}$. Define a map $f : (X,\ \tau) \to (Y,\ \sigma)$ by $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then f is $rg\alpha$-homeomorphism but it is not g-homeomorphism, since the inverse image of the open set $\{a\}$ in X is $\{c\}$ which is not g-open set in Y.*

Example 2.7 *Let $X = Y = \{a,\ b,\ c,\ d\}$ with topologies $\tau = \sigma = \{X,\ \phi,\ \{a\},\ \{b\},\ \{a,\ b\}\}$. Define a map $f : (X,\ \tau) \to (Y,\ \sigma)$ by $f(a) = c$, $f(b) = b$, $f(c) = a$ and $f(d) = d$. Then f is g-homeomorphism but it is not $rg\alpha$-homeomorphism, since the inverse image of the open set $\{a,\ b\}$ in X is $\{b,\ c\}$ which is not $rg\alpha$-open set in Y.*

Theorem 2.5. *Let $f : (X,\ \tau) \to (Y,\ \sigma)$ be a bijective $rg\alpha$-continuous map. Then the following are equivalent.*
(i) f is a $rg\alpha$-open map, (ii) f is $rg\alpha$-homeomorphism,
(iii) f is a $rg\alpha$-closed map.

Proof. Proof follows from Theorem 2.17 in [24]. ■

Remark 2.2. *The composition of two $rg\alpha$-homeomorphism need not be a $rg\alpha$-homeomorphism in general as seen from the following example.*

Example 2.8 *Consider* $X = Y = Z = \{a,\ b,\ c\}$, $\tau = \{X,\ \phi,\ \{a\},\ \{a,\ c\}\}$, $\sigma = \{Y,\ \phi,\ \{a,\ b\}\}$ *and* $\eta = \{Z,\ \phi,\ \{a\},\ \{b\},\ \{a,\ b\}\}$. *Let* $f : (X,\ \tau) \to (Y,\ \sigma)$ *and* $g : (Y,\ \sigma) \to (Z,\ \eta)$ *be the identity maps. Then both* f *and* g *are* $rg\alpha$*-homeomorphisms, but their composition* $g \circ f : (X,\ \tau) \to (Z,\ \eta)$ *is not* $rg\alpha$*-homeomorphism, because for the open set* $\{a,\ c\}$ *of* $(X,\ \tau)$, $g \circ f(\{a,\ c\}) = g(f(\{a,\ c\}) = g(\{a,\ c\}) = \{a,\ c\}$, *which is not* $rg\alpha$*-open in* $(z,\ \eta)$. *Therefore* $g \circ f$ *is not* $rg\alpha$*-open and so* $g \circ f$ *is not* $rg\alpha$*-homeomorphism.*

Definition 2.2. *A bijection* $f : (X,\ \tau) \to (Y,\ \sigma)$ *is said to be* $rg\alpha c$**-homeomorphism** *if both* f *and* f^{-1} *are* $rg\alpha$*-irresolute. We say that spaces* $(X,\ \tau)$ *and* $(Y,\ \sigma)$ *are* $rg\alpha c$*-homeomorphic if there exists a* $rg\alpha c$*-homeomorphism from* $(X,\ \tau)$ *onto* $(Y,\ \sigma)$.

We denote the family of all $rg\alpha c$*-homeomorphisms of a topological space* $(X,\ \tau)$ *onto itself by* $rg\alpha c\text{-}h(X,\ \tau)$.

Theorem 2.6. *Every* $rg\alpha c$*-homeomorphism is an* $rg\alpha$*-homeomorphism but not conversely.*

Proof. Let $f : (X,\ \tau) \to (Y,\ \sigma)$ be an $rg\alpha c$-homeomorphism. Then f and f^{-1} are $rg\alpha$-irresolute and f is bijection. By Theorem 3.2 in [23], f and f^{-1} are $rg\alpha$-continuous. Therefore f is $rg\alpha$-homeomorphism. ■

The converse of the above Theorem is not true in general as seen from the following example.

Example 2.9 *Consider* $X = Y = \{a,\ b,\ c\}$ *with* $\tau = \{X,\ \phi,\ \{a\},\ \{b\},\ \{a,\ b\}\}$ *and* $\sigma = \{Y,\ \phi,\ \{a,\ b\}\}$. *Let* $f : (X,\ \tau) \to (Y,\ \sigma)$ *be the identity map. Then* f *is* $rg\alpha$*-homeomorphism but it is not* $rg\alpha c$*- homeomorphism, since* f *is not* $rg\alpha$*-irresolute.*

Theorem 2.7. *Every* $rg\alpha c$*-homeomorphism is* rwg*-homeomorphism but not conversely.*

Proof. Proof follows from Theorems 2.6. and 2.4. ■

Example 2.10 *Consider* $X = Y = \{a,\ b,\ c\}$ *with* $\tau = \{X,\ \phi,\ \{a\},\ \{b\},\ \{a,\ b\}\}$ *and* $\sigma = \{Y,\ \phi,\ \{a,\ b\}\}$. *Let* $f : (X,\ \tau) \to (Y,\ \sigma)$ *be the identity map. Then* f *is* rwg*-homeomorphism but it is not* $rg\alpha c$*- homeomorphism, since* f *is not* $rg\alpha$*-irresolute.*

Remark 2.3. $rg\alpha c$*-homeomorphism and* w^**-homeomorphism are independent as seen from the following example.*

Example 2.11 *Let* $X = Y = \{a,\ b,\ c\}$ *with* $\tau = \{X,\ \phi,\ \{a\},\ \{b\},\ \{a,\ b\}\}$ *and* $\sigma = \{Y,\ \phi,\ \{a,\ b\}\}$. *Let* $f : (X,\ \tau) \to (Y,\ \sigma)$ *be the identity map. Then* f *is* w^**-homeomorphism but it is not* $rg\alpha c$*- homeomorphism, since* f *is not* $rg\alpha$*-irresolute.*

Example 2.12 *Consider* $X = Y = \{a,\ b,\ c\}$ *with topologies* $\tau = \{X,\ \phi,\ \{a\}\}$ *and* $\sigma = \{Y,\ \phi,\ \{a,\ b\}\}$. *Let* $f : (X,\ \tau) \to (Y,\ \sigma)$ *be the identity map. Then* f *is* $rg\alpha c$*-homeomorphism but it is not* w^**-homeomorphism, since* f *is not* w*-irresolute.*

Remark 2.4. *From the above discussions and known results we have the following implications*
In the following diagram, by
$A \to B$ *we mean* A *implies* B *but not conversely and*
$A \nleftrightarrow B$ *means* A *and* B *are independent of each other.*

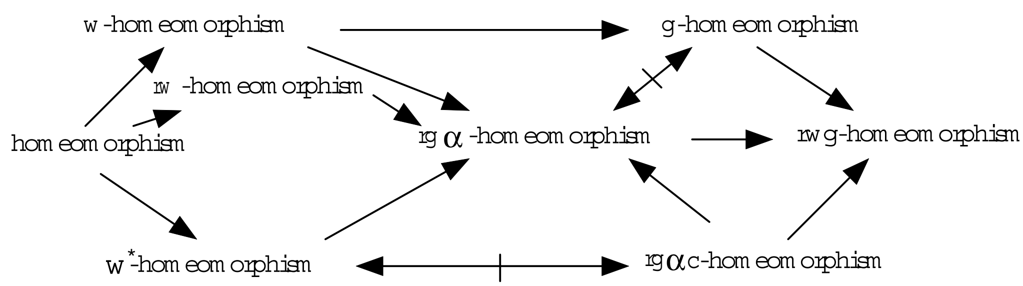

fig-1

Theorem 2.8. *Let* $f : (X,\ \tau) \to (Y,\ \sigma)$ *and* $g : (Y,\ \sigma) \to (Z,\ \eta)$ *are* $rg\alpha c$*-homeomorphisms, then their composition* $g \circ f : (X,\ \tau) \to (Z,\ \eta)$*is also* $rg\alpha c$*-homeomorphism.*

Proof. Let U be a $rg\alpha$-open set in $(Z,\ \eta)$. Since g is $rg\alpha$-irresolute, $g^{-1}(U)$ is $rg\alpha$-open in $(Y,\ \sigma)$. Since f is $rg\alpha$-irresolute, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is $rg\alpha$ open set in $(X,\ \tau)$. Therefore $g \circ f$ is $rg\alpha$-irresolute. Also for a $rg\alpha$-open set G in $(X,\ \tau)$, we have $(g \circ f)(G) = g(f(G)) = g(W)$, where $W = f(G)$. By hypothesis, $f(G)$ is $rg\alpha$-open in $(Y,\ \sigma)$ and so again by hypothesis, $g(f(G))$ is a $rg\alpha$-open set in $(Z,\ \eta)$. That is $(g \circ f)(G)$ is a $rg\alpha$-open set in $(Z,\ \eta)$ and therefore $(g \circ f)^{-1}$ is $rg\alpha$-irresolute. Also $g \circ f$ is a bijection. Hence $g \circ f$ is $rg\alpha c$-homeomorphism. ■

Theorem 2.9. *The set* $rg\alpha c$*-*$h(X,\ \tau)$ *is a group under the composition of maps.*

Proof. Define a binary operation $* : rg\alpha c\text{-}h(X,\ \tau) \times rg\alpha c\text{-}h(X,\ \tau) \to rg\alpha c\text{-}h(X,\ \tau)$ by $f * g = g \circ f$ for all $f,\ g \in rg\alpha c\text{-}h(X,\ \tau)$ and $\circ$ is the usual operation of composition of maps. Then by Theorem 2.8., $g \circ f \in rg\alpha c\text{-}h(X,\ \tau)$. We know that the composition of maps is associative and the identity map $I : (X,\ \tau) \to (X,\ \tau)$ belonging to $rg\alpha c\text{-}h(X,\ \tau)$ serves as the identity element.

If $f \in rg\alpha c$-$h(X,\ \tau)$, then $f^{-1} \in rg\alpha c$-$h(X,\ \tau)$ such that $f \circ f^{-1} = f^{-1} \circ f = I$ and so inverse exists for each element of $rg\alpha c$-$h(X,\ \tau)$. Therefore ($rg\alpha c$-$h(X,\ \tau),\ \circ)$ is a group under the operation of composition of maps. ■

Theorem 2.10. *Let $f : (X,\ \tau) \rightarrow (Y,\ \sigma)$ be a $rg\alpha c$-homeomorphism. Then f induces an isomorphism from the group $rg\alpha c$-$h(X,\ \tau)$ onto the group $rg\alpha c$-$h(Y,\ \sigma)$.*

Proof. Using the map f, we define a map $\Psi_f : rg\alpha c$-$h(X,\ \tau) \rightarrow rg\alpha c$-$h(Y,\ \sigma)$ by $\Psi_f(h) = f \circ h \circ f^{-1}$ for every $h \in rg\alpha c$-$h(X,\ \tau)$. Then Ψ_f is a bijection. Further, for all $h_1 h_2 \in rg\alpha c$-$h(X,\ \tau)$, $\Psi_f(h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = \Psi_f(h_1) \circ \Psi_f(h_2)$. Therefore Ψ_f is a homeomorphism and so it is an isomorphism induced by f. ■

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