Foreign Equity Option Pricing under Bivariate Time-Varying Coefficient Jump-Diffusion Model

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Abstract
This paper is dedicated to the study of the foreign equity option pricing under bivariate time-varying coefficient jump-diffusion model. Economic variables are not carved on tablets of stone, they change over time. Hence we allow the returns and variance of the equity price and foreign exchange rate are time-varying functions. Foreign equity options (quanto options) have become more and more popular in international financial markets, where the payoff depending on the equity price in one currency but the actual payoff is done in another currency. In this paper, we use a bivariate Bernoulli distribution and a bivariate Laplace distribution to model the jump indicators and jump sizes, respectively. The distribution of return is analysed by the \( \dot{I} \)tô formula and normal asymmetric Laplace distribution. The pricing formula of foreign equity call option is proposed which is based on domestic currency under the risk-neutral measure. The numerical results show that the jump correlation is significant to the foreign equity option prices.

Keywords: Foreign equity option; Time-varying coefficient; Jump-diffusion model; Jump correlation; Pricing formula

1 Introduction

With the development of the integration of the world economy and the promotion of information technology, the speed of investments globalization has
been also increasing rapidly in recent years. Foreign equity options traded in international markets provide investors with an efficacious tool to manage transnational risks from different financial markets. Therefore, foreign equity options become a popular tool where the payoff depending on the equity price in one currency but the actual payoff is done in another currency.

Many previous studies dealing with the foreign equity options usually based on the Black-Scholes framework and model the dynamics of equity prices and foreign exchange rates with Brownian motions, for example, Dravid et al. [2], Reiner [5], Ho et al. [15] and Kowk and Wong [18]. In this specification, there are tail-fatness and asymmetry in the return distribution. Hence the existing literature proposed many methods to extend the basic model. Xu [16] extended the basic model in the way which used the Gram-Charlier series expansion to foreign equity option pricing. To explain the effect of kurtosis and skewness of return in the European option pricing, Jarrow and Rudd [12] proposed this method, and it has been applied by Longstaff [6], Backus et al. [4] and Topaloglou et al. [10]. Beyond the traditional Black-Scholes framework, there are many other models to price the foreign equity options. Duan and Wei [7] assumed a bivariate nonlinear GARCH model for pricing foreign currency and cross-currency options. Huang and Hung [13] priced foreign equity options under Lévy processes. Xu et al. [17] proposed a mixed model which combined stochastic volatility and jumps to simulate tail-fatness. Ulyah et al. [14] priced foreign equity options with a bivariate constant coefficient jump-diffusion model.

According to Xu [17] and Ulyah et al. [14], when the expiry date is short, the contribution from jumps is far greater than that from stochastic volatility. Many studies showed constant coefficient jump-diffusion model. However, economic environment changes with time, empirical results show that jump-diffusion model with time-varying coefficient can better fit the data in the financial markets. Therefore, this paper proposed a bivariate time-varying coefficient jump-diffusion model where the drift terms and the diffusion terms are all depend on time. And this paper can be seemed as an extension of Ulyah et al. [14]. We investigated the correlation between jumps from the two assets, which is most important to the foreign equity option prices. The foreign equity option value is calculated under the bivariate jump-diffusion model.

The rest of this paper is organized as follows. In Section 2, we introduce the model and analyzes the jumps in our model. In Section 3, we investigate the foreign equity option prices in the form of a European call option and derive the pricing formula. In the Section 4, we provide several numerical examples. Finally, in Section 5, we conclude the paper.
2 Model and analysis of jumps

In this section, we proposed a jump-diffusion model as follows

\[
\begin{align*}
  \text{d}S_t &= \mu_1(t)S_t \text{d}t + \sigma_1(t)S_t \text{d}W_t^{(1)} + \text{d}J_t^{(1)}, \\
  \text{d}F_t &= \mu_2(t)F_t \text{d}t + \sigma_2(t)F_t \text{d}W_t^{(2)} + \text{d}J_t^{(2)},
\end{align*}
\]

where \( S_t \) is the equity price and \( F_t \) is foreign exchange rate, \( \mu_1(\cdot) \) and \( \mu_2(\cdot) \) denote the drift terms which depend on \( t \), \( \sigma_1(\cdot) \) and \( \sigma_2(\cdot) \) are diffusion terms which also depend on \( t \), \( W_t^{(1)} \) and \( W_t^{(2)} \) are the standard Brownian motions while \( \text{d}W_t^{(1)} \text{d}W_t^{(2)} = \rho \text{d}t \), \( J_t^{(1)} \) and \( J_t^{(2)} \) are jumps.

In order to make our model suited pricing foreign equity option, we suggest a bivariate Bernoulli distribution and a bivariate asymmetric Laplace distribution to model joint jump indicators and joint jump sizes, respectively.

First of all, we propose the bivariate Bernoulli distribution. As introduced in Dai et al.[3], the random vector \((Y_1, Y_2)\) follows a bivariate Bernoulli distribution, then its probability density function can be given by

\[
P(Y_1 = y_1, Y_2 = y_2) = q_{12}q_{00}q_{10}q_{01},
\]

where \( q_{ij} = P(Y_1 = i, Y_2 = j) \), \( q_{00} + q_1 + q_2 + q_{12} = 1 \).

For the sake of convenience, we follow Ulyah et al.[14] which proposed an alternative parametrization \((Y_1, Y_2) \sim BB^*(p_1, p_2, \rho)\), where

\[
p_1 = q_{12} + q_1, p_2 = q_{12} + q_2, \rho = \frac{q_{12} - p_1p_2}{\sqrt{p_1(1-p_1)p_2(1-p_2)}}.
\]

Secondly, we suggest the bivariate asymmetric Laplace distribution. As defined in Kotz et al.[1], the random vector \((Y_1, Y_2) \sim BAL(m_1, n_1, m_2, n_2, \delta)\), their joint probability density function can be written as

\[
p(y_1, y_2) = \frac{1}{\pi n_1 n_2 \sqrt{1 - \delta^2}} \exp \left\{ \frac{(m_1 n_2 / n_1 - m_2 \delta)y_1 + (m_1 n_1 / n_2 - m_1 \delta)y_2}{n_1 n_2 (1 - \delta^2)} \right\}
\times K_0(D \sqrt{y_1^2 n_2 / n_1 - 2 \delta y_1 y_2 + y_2^2 n_1 / n_2}),
\]

where

\[
K_0(z) = \int_0^\infty \cos z \sinh y \, dy,
\]

\[
D = \frac{1}{n_1 n_2 (1 - \delta^2)} \sqrt{2 n_1 n_2 (1 - \delta^2) - 2 m_1 m_2 \delta + m_1^2 n_2 / n_1 + m_2^2 n_1 / n_2}.
\]
It is more simple to make the characteristic function take the form as follows

\[
E[e^{ia_1Y_1 + ia_2Y_2}] = \left[ 1 + \frac{a_1^2 n_1^2}{2} + \frac{a_2^2 n_2^2}{2} + a_1 a_2 n_1 n_2 \delta - i(a_1 m_1 + a_2 m_2) \right]^{-1}. \tag{2.5}
\]

And its expectation, standard deviation and covariance take the following format

\[
E(Y_i) = m_i, \ Var(Y_i) = n_i^2 + m_i^2, i = 1, 2 \nonumber \]

\[
Cov(Y_1, Y_2) = n_1 n_2 \delta + m_1 m_2. \nonumber
\]

Then we can get the correlation coefficient

\[
\rho = Corr(Y_1, Y_2) = \frac{n_1 n_2 \delta + m_1 m_2}{\sqrt{(n_1^2 + m_1^2)(n_2^2 + m_2^2)}}. \tag{2.6}
\]

According to Ulyah et al.[14], a bivariate asymmetric Laplace distribution has marginal distributions which are asymmetric Laplace distributions. If \(Y\) follows \(AL(m, n)\), then pdf of \(Y\) define as below

\[
f(y) = \begin{cases} (m^2 + 2n^2)^{-\frac{1}{2}} \exp(-2(\sqrt{m^2 + 2n^2} + m)^{-1}y), & y \geq 0 \\ (m^2 + 2n^2)^{\frac{1}{2}} \exp(-2(\sqrt{m^2 + 2n^2} - m)^{-1}y), & y < 0. \end{cases} \tag{2.7}
\]

Then, the alternative parametrization of asymmetric Laplace distribution can be written as

\[
\xi^+ = 2(\sqrt{m^2 + 2n^2} + m)^{-1}, \xi^- = 2(\sqrt{m^2 + 2n^2} - m)^{-1},
\]

which denoted by \(AL^*(\xi^+, \xi^-)\). Hence the alternative parametrization distribution \(BAL^*(\xi_1^+, \xi_1^-, \xi_2^+, \xi_2^-; \rho)\) can be defined as

\[
\xi_i^+ = \frac{2}{\sqrt{m^2 + 2n^2} + m}, \xi_i^- = \frac{2}{\sqrt{m^2 + 2n^2} - m}, i = 1, 2. \tag{2.8}
\]

Finally, with expiration time \(T\), we use Itô calculus solve the stochastic differential equation (2.1), then the equity price and foreign exchange rate are as follows

\[
\begin{align*}
S_T &= S_0 \exp \left\{ \int_0^T (\mu_1(\tau) - \frac{1}{2} \sigma_1^2(\tau)) \, d\tau + \int_0^T \sigma_1(\tau) \, dW_\tau^{(1)} + X_1 Y_1 \right\} \\
F_T &= F_0 \exp \left\{ \int_0^T (\mu_2(\tau) - \frac{1}{2} \sigma_2^2(\tau)) \, d\tau + \int_0^T \sigma_2(\tau) \, dW_\tau^{(2)} + X_2 Y_2 \right\},
\end{align*} \tag{2.9}
\]

where \(X_1, X_2\) and \(Y_1, Y_2\) are the jump indicators and jump sizes of equity price and foreign exchange rate which is valid on \([0, T]\).
Hence our model can be defined as

\[(X_1, X_2) \sim BB^*(p_1, p_2, \rho_X), (Y_1, Y_2) \sim BAL^*(\xi_1^+, \xi_1^-, \xi_2^+, \xi_2^-, \rho_Y),\]

where \(\rho_X\) and \(\rho_Y\) represent the correlation coefficients of \(X_1, X_2\) and \(Y_1, Y_2\), respectively.

**Remark 2.1** If \(X_1 = X_2 = 1\), the joint jump size of \(Y_1 + Y_2\) follows \( \text{AL}^*(\xi_{12}^+, \xi_{12}^-)\), where

\[
\xi_{12}^+ = \frac{1}{n_{12}^+} \left( \sqrt{m_{12}^2 + 2n_{12}^2 - n_{12}} \right), \quad \xi_{12}^- = \frac{1}{n_{12}^-} \left( \sqrt{m_{12}^2 + 2n_{12}^2 + n_{12}} \right), \quad (2.10)
\]

in which

\[
m_{12} &= \frac{1}{\xi_1^+} + \frac{1}{\xi_2^+} - \frac{1}{\xi_1^-} - \frac{1}{\xi_2^-}, \\
n_{12}^2 &= n_{12}^2 + n_{12}^2 + 2n_{12}^2 n_{12} \delta \\
 &= 2 \left[ \frac{1}{\xi_1^+ \xi_1^-} + \frac{1}{\xi_2^+ \xi_2^-} - \left( \frac{1}{\xi_1^+} - \frac{1}{\xi_1^-} \right) \left( \frac{1}{\xi_2^+} - \frac{1}{\xi_2^-} \right) \right. \\
 &\quad \left. + \rho_Y \left( \frac{1}{(\xi_1^+)^2} + \frac{1}{(\xi_1^-)^2} \right) \left( \frac{1}{(\xi_2^+)^2} + \frac{1}{(\xi_2^-)^2} \right)^{\frac{1}{2}} \right].
\]

### 3 Foreign equity option pricing formula

Before pricing foreign equity options, we introduce the distribution function which is necessary to the following discussion.

**Lemma 3.1.** If \(A\) follows \(N(\mu, \sigma)\), \(B\) follows \( \text{AL}^*(\xi^+, \xi^-)\), \(Z = A + B\) follows a normal asymmetric Laplace distribution, i.e. \(Z \sim NAL(\mu, \sigma, \xi^+, \xi^-)\), then

\[
f_{NAL}(z; \mu, \sigma, \xi^+, \xi^-) = \frac{\xi^+ \xi^-}{\xi^+ + \xi^-} e^{-\left( z - \mu - \frac{1}{2} \sigma^2 \xi^+ \right) \xi^+} \Phi \left( \frac{z - \mu - \sigma^2 \xi^+}{\sigma} \right) \\
+ \frac{\xi^+ \xi^-}{\xi^+ + \xi^-} e^{-\left( z - \mu - \frac{1}{2} \sigma^2 \xi^- \right) \xi^-} \Phi \left( \frac{z - \mu + \sigma^2 \xi^-}{\sigma} \right), \quad (3.1)
\]

\[
F_{NAL}(z; \mu, \sigma, \xi^+, \xi^-) = 1 - \frac{\xi^+ \xi^-}{\xi^+ + \xi^-} e^{\left( \mu + \frac{1}{2} \sigma^2 \xi^+ \right) \xi^+} \Psi \left( z, -\xi^+, \mu + \sigma^2 \xi^+, \sigma \right) \\
- \frac{\xi^+ \xi^-}{\xi^+ + \xi^-} e^{\left( \mu - \frac{1}{2} \sigma^2 \xi^- \right) \xi^-} \Psi \left( z, \xi^-, \mu + \sigma^2 \xi^-, -\sigma \right), \quad (3.2)
\]
where $f_{\text{NAL}}$ and $F_{\text{NAL}}$ are the probability density function and distribution function respectively, and $\Phi(z)$ denotes the distribution function of standard normal distribution, and

$$
\Psi(z, \alpha, \beta, \gamma) = -\frac{1}{\alpha} \left[ e^{\alpha z} \Phi \left( \frac{z - \beta}{\gamma} \right) + \frac{\gamma}{\alpha \beta + \frac{1}{2} \alpha^2 \gamma^2} \Phi \left( -\frac{z - \beta - \alpha \gamma^2}{\gamma} \right) \right].
$$

(3.3)

The proofs of this Lemma can be seen in Ulyah et al.[14].

Then we introduce the distribution of return. Let $Q_T$ denotes the log return of equity price measured in domestic currency, we plugged (2.9) into

$$
Q_T = \ln \frac{P_T}{P_0},
$$

where $P_t$ denote the equity price struck in the domestic currency, then

$$
Q_T = \ln S_T + \ln F_T - \ln S_0 - \ln F_0
$$

$$
= \int_0^T \left[ \mu_1(\tau) + \mu_2(\tau) - \frac{1}{2}(\sigma_1^2(\tau) + \sigma_2^2(\tau)) \right] d\tau
$$

$$
+ \int_0^T \sigma_1(\tau) dW^{(1)}_\tau + \int_0^T \sigma_2(\tau) dW^{(2)}_\tau + X_1 Y_1 + X_2 Y_2
$$

$$
= \int_0^T \left[ \mu_1(\tau) + \mu_2(\tau) - \frac{1}{2}(\sigma_1^2(\tau) + \sigma_2^2(\tau)) \right] d\tau
$$

(3.4)

$$
+ \int_0^T \sigma_1(\tau) dW^{(1)}_\tau + \int_0^T \sigma_2(\tau) dW^{(2)}_\tau
$$

$$
\begin{cases}
Y_1, & \text{with probability } q_1 \\
Y_2, & \text{with probability } q_2 \\
Y_1 + Y_2, & \text{with probability } q_{12} \\
0, & \text{with probability } q_{00},
\end{cases}
$$

where $q_1, q_2, q_{12}, q_{00}$ can be derived from (2.3),

$$
q_{12} = p_1 p_2 + \rho \sqrt{p_1 (1 - p_1) p_2 (1 - p_2)},
$$

$$
q_1 = p_1 - q_{12}, q_2 = p_2 - q_{12},
$$

$$
q_{00} = 1 - q_1 - q_2 - q_{12}.
$$

From the above analysis, the moment generating function of $Q_T$ can be
obtained as follows

\[ m = E(e^{Q_T}) \]

\[ = E \left\{ \exp \left\{ \int_0^T \left[ \mu_1(\tau) + \mu_2(\tau) - \frac{1}{2} \left( \sigma_1^2(\tau) + \sigma_2^2(\tau) \right) \right] d\tau \right\} \]

\[ + \int_0^T \sigma_1(\tau) dW^{(1)}_\tau + \int_0^T \sigma_2(\tau) dW^{(2)}_\tau + J \right\} \}

\[ = \exp \left\{ \int_0^T \left[ \mu_1(\tau) + \mu_2(\tau) - \frac{1}{2} \left( \sigma_1^2(\tau) + \sigma_2^2(\tau) \right) \right] d\tau + \frac{1}{2} \int_0^T \sigma_1^2(\tau) + \sigma_2^2(\tau) \right\} \]

\[ \times \left\{ \frac{q_1}{(1 - \frac{1}{\xi_1})(1 + \frac{1}{\xi_1})} + \frac{q_2}{(1 - \frac{1}{\xi_2})(1 + \frac{1}{\xi_2})} + \frac{q_{12}}{(1 - \frac{1}{\xi_{12}})(1 + \frac{1}{\xi_{12}})} + q_{00} \right\} , \]  

where \( J \) denotes the last component of \( Q_T \).

Because \( \int_0^T [\mu_1(\tau) + \mu_2(\tau) - \frac{1}{2} (\sigma_1^2(\tau) + \sigma_2^2(\tau))] d\tau + \int_0^T \sigma_1(\tau) dW^{(1)}_\tau + \int_0^T \sigma_2(\tau) dW^{(2)}_\tau \)

follows a normal distribution with mean equal to \( u = \int_0^T [\mu_1(\tau) + \mu_2(\tau) - \frac{1}{2} (\sigma_1^2(\tau) + \sigma_2^2(\tau))] d\tau \) and variance \( v = \int_0^T (\sigma_1^2(\tau) + \sigma_2^2(\tau)) d\tau \), we can obtain the

probability density function of \( Q_T \) as below

\[ f_{Q_T}(x) = q_1 f_{NAL}(x; u, \sqrt{v}, \xi_1^+, \xi_1^-) + q_2 f_{NAL}(x; u, \sqrt{v}, \xi_2^+, \xi_2^-) + q_{12} f_{NAL}(x; u, \sqrt{v}, \xi_{12}^+, \xi_{12}^-) + q_{00} \phi \left( \frac{x - u}{\sqrt{v}} \right) . \]  

(3.6)

Based on the above discussion, we can gain the European foreign equity option pricing formula which expressed in the from of a call option. According to Bajgrowicz et al[11], under the risk-neutral measure, we consider arbitrage-free model, and assume \( r(t) \) is the risk-free rate of interest, \( d(t) \) is the dividend yield of equity. Consequently, according to the martingale condition

\[ e^{\int_0^T (r(\tau) - d(\tau)) d\tau} = E(e^{Q_T}) , \]

we can get

\[ u = \int_0^T (r(\tau) - d(\tau)) d\tau - \frac{1}{2} \int_0^T (\sigma_1^2(\tau) + \sigma_2^2(\tau)) d\tau \]

\[ - \ln \left[ \frac{q_1}{(1 - \frac{1}{\xi_1})(1 + \frac{1}{\xi_1})} + \frac{q_2}{(1 - \frac{1}{\xi_2})(1 + \frac{1}{\xi_2})} + \frac{q_{12}}{(1 - \frac{1}{\xi_{12}})(1 + \frac{1}{\xi_{12}})} + q_{00} \right] . \]

(3.7)

Therefore, we can obtain the pricing formula of a foreign equity option which is based on domestic currency.

**Theorem 3.2.** Given such a European foreign equity call option has the following payoff at expiration time \( T \)

\[ (P_T - K_d)^+ \]
where $K_d$ is the strike price in domestic currency. Then the price is given as follows

$$C = e^{-\int_0^T (r(t) - d(t)) dt} \left[ q_1 \Upsilon(P_0, K_d; u, \sqrt{v}, \xi^+, \xi^-) + q_2 \Upsilon(P_0, K_d; u, \sqrt{v}, \xi^2, \xi_2) + q_{12} \Upsilon(P_0, K_d; u, \sqrt{v}, \xi_{12}, \xi_{12}) + q_0 \left( P_0 e^{\mu + \frac{1}{2} \sigma^2} \Phi \left( -\frac{\ln(K_d/P_0) - u - v}{\sqrt{v}} \right) - K_d \Phi \left( -\frac{\ln(K_d/P_0) - u}{\sqrt{v}} \right) \right) \right],$$

(3.8)

where $\Phi(z)$ denotes the distribution function of standard normal distribution,

$$\Upsilon(\alpha, \beta; \mu, \sigma, \xi^+, \xi^-) = \frac{\xi^+ - \xi^-}{\xi^+ + \xi^-} \times$$

$$\left[ e^A \left( \alpha \Psi \left( \ln \frac{\beta}{\alpha}, 1 - \xi^+, \mu + \sigma^2 \xi^+, \sigma \right) - \beta \Psi \left( \ln \frac{\beta}{\alpha}, -\xi^+, \mu + \sigma^2 \xi^+, \sigma \right) \right) + e^{-B} \left( \alpha \Psi \left( \ln \frac{\beta}{\alpha}, 1 + \xi^-, \mu - \sigma^2 \xi^-, -\sigma \right) - \beta \Psi \left( \ln \frac{\beta}{\alpha}, -\xi^-, \mu - \sigma^2 \xi^-, -\sigma \right) \right) \right],$$

(3.9)

where $A = (\mu + \frac{1}{2} \sigma^2) \xi^+, B = (\mu - \frac{1}{2} \sigma^2) \xi^-$ and $\Psi(z, \alpha, \beta, \gamma)$ is defined by (3.3).

Proof. In general, the foreign equity option call pricing formula is as follows

$$C = e^{-\int_0^T (r(t) - d(t)) dt} E[(P_T - K_d)^+]$$

$$= e^{-\int_0^T (r(t) - d(t)) dt} E[(P_0 e^{R_T} - K_d)^+].$$

According to the probability density function of $R_T$ which is composed of a normal distribution whose probability is $q_{00}$ and three normal asymmetric Laplace distributions whose probability are $q_1, q_2$ and $q_{12}$, we can calculate the expectation as follows

$$E[(P_0 e^{Q_T} - K_d)^+] = E[\max(P_0 e^{Q_T} - K_d, 0)]$$

$$= \int_{\ln K_d/P_0}^{\infty} (P_0 e^{Q_T} - K_d) f_{Q_T} dQ_T.$$

Then we can denote the normal asymmetric Laplace distribution terms as the following function

$$E[(\alpha e^X - \beta)^+] = \Upsilon(\alpha, \beta; \mu, \sigma, \xi^+, \xi^-)$$

$$= \int_{\ln \frac{\beta}{\alpha}}^{\infty} (\alpha e^x - \beta) f_{N_{AL}}(x; \mu, \sigma, \xi^+, \xi^-) dx$$

$$= \frac{\xi^+ - \xi^-}{\xi^+ + \xi^-} \left[ e^A \left( \alpha \Psi \left( \ln \frac{\beta}{\alpha}, 1 - \xi^+, \mu + \sigma^2 \xi^+, \sigma \right) - \beta \Psi \left( \ln \frac{\beta}{\alpha}, -\xi^+, \mu + \sigma^2 \xi^+, \sigma \right) \right) + e^{-B} \left( \alpha \Psi \left( \ln \frac{\beta}{\alpha}, 1 + \xi^-, \mu - \sigma^2 \xi^-, -\sigma \right) - \beta \Psi \left( \ln \frac{\beta}{\alpha}, -\xi^-, \mu - \sigma^2 \xi^-, -\sigma \right) \right) \right],$$
where $X \sim NAL(\mu, \sigma, \xi^+, \xi^-)$. For the normal distribution term, it is similar to the Black-Scholes formula,

$$E_N = P_0 e^{\frac{1}{2}v}\Phi(-d_1) - K_d\Phi(-d_2),$$

where

$$d_1 = \frac{\ln \frac{P_0}{K_d} + u + \frac{1}{2}v}{\sqrt{v}}, \quad d_2 = \frac{\ln \frac{P_0}{K_d} + u - \frac{1}{2}v}{\sqrt{v}}.$$

Hence, the price of European foreign equity call option can be expressed as (3.8).

4 Numerical results

In this section, numerical results is performed to show the change of foreign equity option price and the influence of jump correlations. we can see how the price of European foreign equity call option change as $\rho_X$ and $\rho_Y$ move away from 0. we also provide the bias in foreign equity call option price which is caused by jump correlation, and it takes the form as below

$$Bias = (C_{\rho \neq 0} - C_{\rho = 0})/C_{\rho = 0}. \quad (4.1)$$

Some of the model parameters are taken from Ulyah et al.[14]: $\xi^+_1 = \xi^-_1 = \xi^+_2 = \xi^-_2 = 5$. We also need the following parameters and they will be used in the numerical analysis: $r(\tau) = 0.05 + 0.0001\tau$, $d(\tau) = 0.06 + 0.0001\tau$, $\sigma_1(\tau) = 0.09 - 0.0001\tau$, $\sigma_2(\tau) = 0.08 - 0.0001\tau$ and $K_d = 100$. As to the expiration times, we choose one week and one month. For the one week foreign equity option, we set $T = 1/50$, $\lambda_1 = 24$ and $\lambda_2 = 18$. For the one month foreign equity option, we choose $T = 1/12$, $\lambda_1 = 12$ and $\lambda_2 = 9$.

![Graph](image-url)
In Figure 1, the \( x \) label is the equity price struck in the domestic currency, the \( y \) label is European FEO call price, the expiration time is \( T = 1/12 \). Three different jump correlation were considered: \( \rho_X = \rho_Y = 0.0 \), \( \rho_X = \rho_Y = 0.4 \) and \( \rho_X = \rho_Y = 0.8 \). In Figure 2, the \( x \) label is the equity price struck in the domestic currency, the \( y \) label is European FEO call price, the expiration time is \( T = 1/50 \). Three different jump correlation were considered: \( \rho_X = \rho_Y = 0.0 \), \( \rho_X = \rho_Y = 0.4 \) and \( \rho_X = \rho_Y = 0.8 \). As shown in Figure 1 and Figure 2, the foreign equity option call prices are getting higher as \( P_0 \) become larger. As \( \rho_X \) and \( \rho_Y \) increase, the function of foreign equity option call pricing is getting higher. This indicates that the jump correlation has an impact on the price of foreign equity option.

Table 1: The bias of FEO call price (\( T = 1/50 \))

<table>
<thead>
<tr>
<th>( P_0 )</th>
<th>( \rho_X = \rho_Y = 0.0 )</th>
<th>( \rho_X = \rho_Y = 0.4 )</th>
<th>( \rho_X = \rho_Y = 0.8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_0 = 90 )</td>
<td>4.323</td>
<td>5.059</td>
<td>17.01</td>
</tr>
<tr>
<td>( P_0 = 100 )</td>
<td>6.992</td>
<td>7.482</td>
<td>7.02</td>
</tr>
<tr>
<td>( P_0 = 110 )</td>
<td>13.705</td>
<td>13.852</td>
<td>1.07</td>
</tr>
</tbody>
</table>

Table 2: The bias of FEO call price (\( T = 1/12 \))

<table>
<thead>
<tr>
<th>( P_0 )</th>
<th>( \rho_X = \rho_Y = 0.0 )</th>
<th>( \rho_X = \rho_Y = 0.4 )</th>
<th>( \rho_X = \rho_Y = 0.8 )</th>
</tr>
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<tbody>
<tr>
<td>( P_0 = 90 )</td>
<td>6.713</td>
<td>8.136</td>
<td>21.20</td>
</tr>
<tr>
<td>( P_0 = 100 )</td>
<td>10.461</td>
<td>11.680</td>
<td>11.65</td>
</tr>
<tr>
<td>( P_0 = 110 )</td>
<td>16.422</td>
<td>17.182</td>
<td>4.63</td>
</tr>
</tbody>
</table>

Figure 2: \( T = 1/50 \)
From Table 1 and Table 2, we can see that the bias of foreign equity call option price is getting larger as $\rho_X$ and $\rho_Y$ increase. The largest bias even reach 51.44% when $\rho_X = \rho_Y = 0.8$. All these results indicate that the jump correlation is significant to the foreign equity option prices.

5 Conclusions

In this paper, we study the foreign equity option pricing under bivariate time-varying coefficient jump-diffusion model. Considering economic variables changing over time, we allow the returns and variance of the equity price and foreign exchange rate are time-varying functions. We use a bivariate Bernoulli distribution and a bivariate Laplace distribution to model the jump indicators and jump sizes, respectively. The distribution of return is analysed by the Itô formula and normal asymmetric Laplace distribution. The pricing formula of foreign equity call option is proposed which is based on domestic currency under the risk-neutral measure. The numerical results show that the jump correlation is significant to the foreign equity option prices.

References


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