Abstract

In this paper, we examine the motion of unsteady MHD flow of a viscous, electrically conducting, incompressible fluid flowing past an infinite porous plate subjected to convective surface boundary conditions. The coupled non-linear partial differential equations governing the problem are solved numerically using the explicit finite difference method. The effects of thermo-physical parameters such as Grashof number, ion-slip parameter, hall parameter and time as well as cooling of the plate by free convection currents on the thermal boundary layer of the laminar fluid flow are discussed graphically. The solution of this research provides an alternative way of thermal energy storage and a very useful source of information for researchers on the subject of hydromagnetic stokes free convection fluid flows.

Mathematics Subject Classification: 76W05

Keywords: Magnetohydrodynamics, magnetic field, electric current, porous plate

1Corresponding author
Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
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<tbody>
<tr>
<td>$B_0$</td>
<td>Magnetic field vector, [wm$^{-2}$]</td>
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<td>$B_i$</td>
<td>Convective heat exchange parameter</td>
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<tr>
<td>$d$</td>
<td>Electric displacement vector, [cm$^{-2}$]</td>
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<tr>
<td>$E_c$</td>
<td>Eckert number</td>
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<td>$E$</td>
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<td>$f_B$</td>
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<tr>
<td>$g$</td>
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<tr>
<td>$G_r$</td>
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<tr>
<td>$M$</td>
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<td>Heat transfer coefficient,</td>
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<td>$J$</td>
<td>Current density,[Am$^{-2}$]</td>
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<td>$k$</td>
<td>Thermal conductivity, [wm$^{-1}$k$^{-1}$]</td>
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<td>$T$</td>
<td>Dimensionless temperatures</td>
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<td>$T_f$</td>
<td>Hot fluid temperature, [k]</td>
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<tr>
<td>$v$</td>
<td>Velocity vector, [ms$^{-2}$]</td>
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<td>$U$</td>
<td>Velocity of the plate, [m/s]</td>
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<tr>
<td>$u_0$</td>
<td>Dimensionless suction velocity</td>
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<tr>
<td>$v_0^*$</td>
<td>Dimensional suction velocity, [m/s]</td>
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<tr>
<td>$x_1, y_1$</td>
<td>Dimensional cartesian coordinates, [m]</td>
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<tr>
<td>$u_1, v_1$</td>
<td>Dimensional velocity components of $v$ along x and y direction, [ms$^{-1}$]</td>
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<tr>
<td>$Z$</td>
<td>Complex velocity</td>
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<tr>
<td>$\overline{Z}$</td>
<td>Complex conjugate of $Z$</td>
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1. Introduction

The word Magnetohydrodynamics (MHD) refers to the branch of fluid mechanics which is concerned with the interaction of electrically conducting fluids and electromagnetic fields. Some of these fluids are mercury, salty water, molten iron, ionized gases (plasma) e.g solar atmosphere etc. The official birth of incompressible fluid magnetohydrodynamics was 1937 when Hartman and Lazarus [1] performed theoretical and experimental studies of MHD flows in ducts using mercury. It was observed that a force is produced on the fluid in the direction normal to both the applied electric and magnetic fields. Later, a Swedish electrical engineer Hannes Alfvén [2] in 1947 from his research on magnetohydrodynamics described astrophysical phenomenon as an independent scientific discipline. Thereafter several others [3, 7, 9] have continued to conduct research into various aspects of the problem of MHD flows with respect to application. In many engineering practical applications, the knowledge of MHD is very useful as it has been used to explain certain phenomena in the universe [2, 8]. This has led to intensive scientific research in the field of computational modeling of MHD fluid flows. MHD covers phenomena in electrically conducting fluid where the velocity of the fluid, \( \mathbf{V} \), and the magnetic field, \( \mathbf{B} \),Magnetic field vector are coupled. Any movement of a conducting material in a magnetic field, \( \mathbf{B} \), and electric field with currents, \( \mathbf{J} \),Current density experiences MHD force \( \sim \mathbf{J} \times \mathbf{B} \), known as the Lorentz force [1]. When a viscous electrically conducting liquid flows in the presence of a transverse magnetic field, electromagnetic forces such as the Lorentz force acts on the fluid particles thereby altering the geometry of their motion. This motion of the particles creates viscous dissipation in the fluid which affects the overall motion of the fluid. Fluid flow over a plane surface is very important in areas such as extrusion and wire drawing [11]. It is therefore important to
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understand the heat and flow characteristics of the process so that the finished product meets the desired quality specifications. A wide variety of problems dealing with heat and fluid flow over porous and non-porous surfaces have been studied with both Newtonian and non-Newtonian fluids and with the inclusion of imposed magnetic fields and power law variation of the velocity [3, 5, 7, 10]. MHD boundary layer with heat and mass transfer past vertical plates are found in many engineering and geophysical applications such as geothermal reservoirs, thermal insulation and enhanced oil recovery [3, 4, 6, 7]. Several interesting computational studies of steady MHD boundary layer flows with heat and mass transfer have appeared in recent years [11]. Convective heat transfer studies are very important since it has been found to be very useful in processes involving high temperatures such as gas turbines, nuclear, thermal energy storage, heat exchange design and the power and the cooling systems [9]. In this study, our objective is to investigate the MHD heat and mass transfer over an infinite non-porous plate subjected to constant heat flux with ion-slip current and convective surface boundary condition. We shall consider an unsteady two dimensional laminar boundary layer fluid flow in the presence of uniform transverse magnetic field coupled with electric field normal to the magnetic field.

2. Mathematical Analysis

The unsteady MHD fluid flow for an infinite horizontal porous plate for an incompressible viscous fluid taking into account ion-slip current and constant heat flux is considered. The $x_1$-axis is taken along the plate in the horizontal direction and the $y_1$-axis is taken along the direction of magnetic field (figure 1 above). A uniform transverse magnetic field $B_0$ is imposed along $y_1$ axis and the plate is taken to be electrically non-conducting. At a time $t_1 > 0$, the plate starts moving impulsively in its own plane with velocity $U$ and there is constant supply of heat to the plate. Since the plate is infinite in extent, the physical variables are functions of $y_1$ and $t_1$ only. The density of the fluid is taken to be constant. In the presence of strong magnetic fields, the ion-slip and hall currents significantly affect the flow. Our study considers a model a model equation governing the flow of the fluid of the form:

$$\frac{1}{\rho}\frac{\partial \mathbf{V}}{\partial t_1} + (\nabla \cdot \mathbf{V})\mathbf{V} = -\nabla P + \mu \nabla^2 \mathbf{V} + g_\beta (T - T_\infty) + \mathbf{J} \times \mathbf{B} \quad (1)$$

$$\rho C_p \frac{\partial T}{\partial t_1} + (\mathbf{V} \cdot \nabla)T = K \nabla^2 T + Q + \mu \phi + \frac{J^2}{\sigma} \quad (2)$$

Under the usual Boussinesq and Oberbeck approximations, and considering that heat transfer can reasonably cause significant changes in the transport
proportions in an incompressible viscous fluid flow, the above set of equations (1) to (2) reduces to:

\[
\frac{\partial u_1}{\partial t_1} - v_0 \frac{\partial u_1}{\partial y_1} = g \beta_T (T - T_\infty) + \frac{\mu}{\rho} \frac{\partial^2 u_1}{\partial y_1^2} + \frac{B_{y_1}}{\rho} J_{z_1} \quad (3)
\]

\[
\frac{\partial w_1}{\partial t_1} - v_0 \frac{\partial w_1}{\partial y_1} = \frac{\mu}{\rho} \frac{\partial^2 w_1}{\partial y_1^2} - \frac{B_{y_1}}{\rho} J_{x_1} \quad (4)
\]

\[
\frac{\partial T}{\partial t_1} - v_0 \frac{\partial T}{\partial y_1} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y_1^2} + \frac{\mu}{\rho c_p} (\frac{\partial u_1}{\partial y_1})^2 + (\frac{\partial w_1}{\partial y_1})^2 + \frac{Q}{\rho c_p} \quad (5)
\]

we apply Fourier’s law of heat conduction.

\[
\frac{\partial T}{\partial y_1} = -\frac{q}{k} \quad (6)
\]

qHeat flux vector For simplicity, internal heat generation in the system is assumed to be given as \(\frac{q}{kT}Q\).

We apply the following initial and boundary conditions.

For \(t_1 \leq 0\) :

\[
u_1(y_1, t_1) = 0, \quad w_1(y_1, t_1) = 0 \quad (7)
\]

\[T(y_1, t_1) = T_\infty \quad (8)
\]

For \(t_1 > 0\) :

\[
u_1(0, t_1) = U, \quad w_1(0, t_1) = 0, \quad \frac{\partial T}{\partial y_1} = -\frac{q}{k}, \quad (9)
\]

\[
u_1(\infty, t_1) = 0, \quad w_1(\infty, t_1) = 0, \quad T(\infty, t_1) = T_\infty \quad (10)
\]
Considering the effects of ion-slip, the generalized Ohm’s law is given as

\[ J = \sigma (E + V \times B) - \frac{\omega_e \tau_e}{B_0} (J \times B) - \frac{\omega_e \tau_e \beta_n}{B_0^2} [(J \times B) \times B] \tag{11} \]

where \( B, E, V, J, \sigma, \omega, \tau, \beta_n \) are respectively the magnetic field vector, the electric field vector, \( (E = [E_{x1}, E_{y1}, E_{z1}]) \), the fluid velocity \( (V = [u_1, v_1, w_1]) \), the current density vector, the conductivity of the fluid, the cyclotron frequency, the electron collision time and the ion-slip parameter. For partially ionized fluids, the magnetic Reynolds number may be neglected since the induced magnetic field is negligible in comparison to the applied magnetic field but the effects of viscous dissipation in the fluid is taken into account. For a short circuit problem, the induced magnetic fields are neglected and the electric field vector along \( x_1 \)-axis and \( z_1 \)-directions respectively are constant everywhere in the flow. In view of the above assumptions, equation (11) can be expressed into components form along the \( x_1 \) and \( z_1 \) directions only since \( J_{y1} = 0 \). Therefore equating the \( x_1 \) and \( z_1 \) components of equation (11) and then solving for the current densities \( J_{x1} \) and \( J_{z1} \), we have

\[ J_{x1} = \frac{\sigma \{(1 + \beta_m \beta_n)(E_{x1} - B_0 w_1) + \beta_m (E_{z1} + B_0 u_1)\}}{(1 + \beta_m \beta_n)^2 + \beta_m^2} \tag{12} \]

\[ J_{z1} = \frac{\sigma \{(1 + \beta_m \beta_n)(E_{z1} + B_0 u_1) - \beta_m (E_{x1} - B_0 w_1)\}}{(1 + \beta_m \beta_n)^2 + \beta_m^2} \tag{13} \]

where \( \beta_m = \omega_e \tau_e \) is the hall current parameter. Substituting equations (12) and (13) into equations (3) and (4) we have

\[ \frac{\partial u_1}{\partial t} - v_0^* \frac{\partial u_1}{\partial y_1} = g \beta_T (T_1 - T) + \mu \frac{\partial^2 u_1}{\partial y_1^2} - \frac{\sigma B_0 \{(1 + \beta_m \beta_n)(E_{z1} + B_0 u_1) - \beta_m (E_{x1} - B_0 w_1)\}}{\rho \{(1 + \beta_m \beta_n)^2 + \beta_m^2\}} \tag{14} \]

\[ \frac{\partial w_1}{\partial t} - v_0^* \frac{\partial w_1}{\partial y_1} = \frac{\mu \partial^2 w_1}{\rho \partial y_1^2} - \frac{\sigma B_0 \{(1 + \beta_m \beta_n)(E_{x1} - B_0 w_1) + \beta_e (E_{z1} + B_0 u_1)\}}{\rho \{(1 + \beta_m \beta_n)^2 + \beta_m^2\}} \tag{15} \]

The electric field vector \( E_{z1} \) along the \( z_1 \)-axis produces force equivalent to \(-B_0 U\) according to the usual Fleming’s law of electromagnetic induction, where \( U \) is the plate velocity. The effect of electric field vector \( E_{x1} \) on the flow is infinitesimal and so \( E_{z1} \approx 0 \). Equations (14) and (15) are therefore reduced to equations (16) and (17) respectively, and together with equation (5), we have

\[ \frac{\partial u_1}{\partial t} - v_0^* \frac{\partial u_1}{\partial y_1} = g \beta_T (T - T) + \nu \frac{\partial^2 u_1}{\partial y_1^2} - \]
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\[
\frac{\partial w}{\partial t} - v_0^* \frac{\partial w}{\partial y} = \frac{\nu}{\rho} \frac{\partial^2 w}{\partial y^2} - \frac{\sigma B_0^2 \{(1 + \beta_m \beta_n)(w_1 - U) - \beta_m w_1\}}{\rho \left[(1 + \beta_m \beta_n)^2 + \beta_m^2\right]} \tag{16}
\]

\[
\frac{\partial T}{\partial t} - v_0^* \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q}{c_p} + \nu \left[ 1 - \frac{M^2}{\left[(1 + \beta_m \beta_n)^2 + \beta_m^2\right]} \right] \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 \tag{17}
\]

\[
\frac{\partial \theta}{\partial t} - v_0^* \frac{\partial \theta}{\partial y} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial y^2} - \frac{B_i \theta}{P_r} + \frac{E_c}{\rho c_p} \left[ 1 - \frac{M^2}{\left[(1 + \beta_m \beta_n)^2 + \beta_m^2\right]} \right] \frac{\partial w}{\partial y} \frac{\partial Z}{\partial y} \tag{18}
\]

On introducing the dimensionless quantities,

\[
u = \frac{u_1}{U}, \quad w = \frac{w_1}{U}, \quad v_0 = \frac{v_0^*}{U}, \quad y = \frac{y_1 U}{\nu}
\]

\[
t = \frac{t_1 U^2}{\nu}, \quad G_r = \frac{\nu g \theta \beta_T}{U^3 \left(\frac{q}{k T}\right)}, \quad B_i = \frac{Q \nu}{k U^2}, \quad E_c = \frac{U}{c_p \left(\frac{q}{k T}\right)}
\]

\[
\theta = \frac{T - T_\infty}{(\frac{q}{k T})}, \quad M = [(\frac{\sigma v}{\rho})^2 \frac{B_0}{U}], \quad P_r = \frac{\nu c_p}{k}
\]

equations (16) to (18) become

\[
\frac{\partial \mathbf{u}}{\partial t} - v_0 \frac{\partial \mathbf{u}}{\partial y} = G_r \theta + \frac{\partial^2 \mathbf{u}}{\partial y^2} - \frac{M^2 \{(1 + \beta_m \beta_n)(u - 1) + \beta_m w\}}{\left[(1 + \beta_m \beta_n)^2 + \beta_m^2\right]} \tag{19}
\]

\[
\frac{\partial \mathbf{w}}{\partial t} - v_0 \frac{\partial \mathbf{w}}{\partial y} = \frac{\partial^2 \mathbf{w}}{\partial y^2} - \frac{M^2 \{(1 + \beta_m \beta_n)w - \beta_m (u - 1)\}}{\left[(1 + \beta_m \beta_n)^2 + \beta_m^2\right]} \tag{20}
\]

\[
\frac{\partial \theta}{\partial t} - v_0 \frac{\partial \theta}{\partial y} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial y^2} - \frac{B_i \theta}{P_r} + \frac{E_c}{c_p} \left[ 1 - \frac{M^2}{\left[(1 + \beta_m \beta_n)^2 + \beta_m^2\right]} \right] \frac{\partial w}{\partial y} \frac{\partial Z}{\partial y} \tag{21}
\]

In order to reduce the number of equations at the present level, we apply complex transformation as follows:

\[
Z = u + iw - 1, \quad \bar{Z} = u - iw - 1.
\]

We combine equations (19) and (20), and simplify equation (21) yielding

\[
\frac{\partial \bar{Z}}{\partial t} - v_0 \frac{\partial \bar{Z}}{\partial y} = \frac{\partial^2 \bar{Z}}{\partial y^2} + G_r \theta - \frac{M^2 \{(1 + \beta_m \beta_n) - i \beta_m\}}{\left[(1 + \beta_m \beta_n)^2 + \beta_m^2\right]} \bar{Z} \tag{22}
\]

\[
\frac{\partial \theta}{\partial t} - v_0 \frac{\partial \theta}{\partial y} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial y^2} - \frac{B_i \theta}{P_r} + \frac{E_c}{c_p} \frac{\partial Z}{\partial y} \frac{\partial \bar{Z}}{\partial y} \tag{23}
\]

Non-dimensionalizing initial and boundary conditions, we get

For \( t \leq 0 \):

\[
Z(y, 0) = 0, \quad \theta(y, 0) = 0,
\]

For \( t > 0 \):

\[
Z(0, t) = 1, \quad \theta(0, t) = 1,
\]
\[ Z(\infty, t) = 0, \theta(\infty, t) = 0, \]

### 3. Computational method

As the exact solution of equations (22) and (23) together with initial and boundary conditions is not possible, we now solve them numerically by finite difference method. Equations (22) and (23) in finite difference form become

\[
\frac{Z_{i,j+1} - Z_{i,j}}{\Delta t} = u_0 \frac{Z_{i,j} - Z_{i-1,j}}{\Delta y} + \frac{(Z_{i-1,j} - 2Z_{i,j} + Z_{i+1,j})}{(\Delta y)^2} + Gr \theta_{i,j} - M^2 \left\{ (1 + \beta_m \beta_n) - i \beta_m \right\} Z_{i,j} \]

\[
\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t} = u_0 \frac{Z_{i,j} - Z_{i-1,j}}{\Delta y} + \frac{(\theta_{i,j} - 2\theta_{i,j} + \theta_{i+1,j})}{Pr(\Delta y)^2} - \frac{Bi}{Pr} \theta_{i,j} + \frac{Ec}{\Delta y} \frac{(Z_{i,j} - Z_{i-1,j})(\bar{Z}_{i,j} - \bar{Z}_{i-1,j})}{\Delta y} \]

where "i" and "j" refer to "y" and "t" respectively. Also \( \Delta t = t_{j+1} - t_j \) and \( \Delta y = y_{i+1} - y_i \). We have \( \Delta y = 0.01 \) and \( \Delta t = 0.00004 \). Although the boundary conditions for \( t > 0 \) applies to \( y = \infty \), (where \( \infty \) in this context refers to a large number) we shall regard \( y = 1.0 \). That is, \( i = 101 \) as corresponding to \( y = \infty \) and therefore \( Z(101, j) = \theta(101, j) = 0 \). This is chosen because \( Z, \theta \), tend to zero at around \( y = 1.0 \). The temperature at \( y = 0, (i = 0) \) has to change rapidly to 1 from its value of zero at \( t < 0 \) since the plate is impulsively set into motion by velocity \( U \), hence \( Z_{0,j} = 1 \). The temperature of the plate wall changes gradually due to the effect of the constant heat flux at the plate \( (y = 0) \) hence \( \theta_{0,0} = 0 \). Therefore the following set of initial conditions are applied. For \( t \leq 0 \): For all \( i \): \( Z_{i,0} = 0, \theta_{i,0} = 0 \)

The boundary conditions for the velocity and temperature are given as follows: For \( j > 0 \) at \( y = 0 \):

\[ Z_{0,j} = 1, \theta_{0,j} = 0 \]

For \( j > 0 \) at \( y = \infty \):

\[ Z_{\infty,j} = 0, \theta_{\infty,j} = 0 \]

The velocity at the end of time step \( \Delta t \), viz. \( Z(i, j+1), i = 1, 2, ..., 100 \) is computed from equation (24) in terms of velocities and temperatures at points on earlier time step. Similarly, \( \theta(i, j+1) \) is computed from equation (25). This procedure is continued until \( j = 1200 \). i.e. up to time \( t = 0.048 \). Computations are carried out for water \( (Pr = 7.0) \). To test for convergence of the algorithm...
Figure 2: Temperature profiles for $\beta_n$ when $u_0 = 0.5$, $Gr = 0.2$, $Pr = 7.0$, $Ec = 0.01$, $t = 0.032$, $\beta_m = 1.0$, $m^2 = 5.0$, $Bi = 1.0$.

used in the present study, the program was run with smaller values of $\Delta t$, viz., $\Delta t = 0.000005, 0.00001$ and it was observed that, there were no significant changes in the results, which ensures that the finite difference technique used in this problem converges and is stable.

4. Results and discussion

For the purpose of discussing the results and in order to get a physical understanding of the problem, numerical calculations have been performed for the temperature profiles. The results are presented by use of graphs. The influence of various thermophysical parameters on the temperature profiles against coordinate $y$ are demonstrated in figures 2-6. From Figure 2 it is noted that the temperature profiles increase with an increase in ion slip parameter, $\beta_n$. Figure 3 displays that an increase in Grashof number, $Gr$, results in an increase in temperature profiles. It is observed from Figure 4 that an increase in hall parameter $\beta_m$ gives rise to a decrease in the temperature profiles. It also seen from Figure 5 that an increase in time, $t$, leads to a rise in temperature profiles of the fluid. Figure 6 reveals that the temperature profiles for the fluid is not affected by the withdrawal of the suction velocity, $u_0$.

5. Conclusion

From the present numerical investigation, we established that an increase in ion-slip parameter, $\beta_n$, time, $t$ or a decrease in Hall current parameter,
Figure 3: Temperature profiles for $Gr$ when $u_0 = 0.5$, $\beta_n = 0.2, Pr = 7.0$, $Ec = 0.01$, $t = 0.032$, $\beta_m = 1.0$, $m^2 = 5.0$, $Bi = 1.0$.

Figure 4: Temperature profiles for $\beta_m$ when $u_0 = 0.5$, $Gr = 0.2$, $\beta_n = 0.2$, $Pr = 7.0$, $Ec = 0.01$, $t = 0.032$, $m^2 = 5.0$, $Bi = 1.0$.

Figure 5: Temperature profiles for $t$ when $u_0 = 0.5$, $Gr = 0.2$, $\beta_n = 0.2$, $Pr = 7.0$, $Ec = 0.01$, $\beta_m = 1.0$, $m^2 = 5.0$, $Bi = 1.0$. 
Figure 6: Temperature profiles for $u_0$ when $Gc = 1.0$, $Gr = 0.2$, $\beta_n = 0.2$, $Pr = 7.0$, $Ec = 0.01$, $t = 0.032$, $\beta_m = 1.0$, $m^2 = 5.0$, $Bi = 1.0$.

$\beta_m$ leads to an increase in temperature profiles. It is surprising to realize that withdrawal as suction velocity has no effect on temperature profiles. For Prandtl number, $Pr = 7.0$, which corresponds to water, the temperature of the liquid generally increases from zero on the plate surface, attains a maximum value slightly away from the boundary region and then decreases to free stream value far away from the plate surface. This shows that in the presence of constant heat flux, cooling of the plate by free convection currents ($Gr > 0$) causes an increase in thermal boundary layer thickness.

References


Received: September 6, 2016; Published: January 25, 2017