Estimation of Stress-Strength Reliability for
Kumaraswamy Exponential Distribution
Based on Upper Record Values

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Abstract

In this paper, we consider the maximum likelihood estimator for
the reliability function for the kumaraswamy exponential distribution
based on upper record values. Exact interval estimation for reliability
function is conducted. Simulations study are conducted to investigate
the theoretical results.

Keywords: Kumaraswamy exponential, Upper record values, Reliability
function, Maximum Likelihood estimation, Confidence interval

1 Introduction

In the reliability literature, the stress-strength term refers to a compo-
nent which has a random strength $X$ and is subject to a random stress $Y$.
The component fails if the stress applied to it exceeds the strength, while the
component works whenever $Y < X$. Thus, $R = P(Y < X)$ is a measure of
component reliability. Stress-strength models have attracted many statisti-
cians for many years due to their applicability in different and diverse areas
such as engineering, economics, quality control and, in the last thirty years,
there have been numerous applications to medical problems and clinical trials
(e.g. Gupta and Gupta, 1990, Adimari and Chiogna, 2006) and to behavioral and educational studies (Klein and Moeschberger, 2003) where left-truncated models are applied. The reliability and its estimation have been well studied under different distributional assumptions on $X$ and $Y$. Kotz et al. (2003) provide an excellent review of theory and applications in this area.

Record values and associated statistics are of great interest in several real-life applications involving weather, economic, sports data "Olympic records or world records in sport" and life test. Also in industry, it might be interesting to determine the minimum failure stress of the products sequentially. The statistical study of record values started with Chandler (1952), he formulated the theory of record values as a model for successive extremes in a sequence of independently and identically random variables. Feller (1966) gave some examples of record values with respect to gambling problems. Resnick (1973) discussed the asymptotic theory of records. Theory of record values and its distributional properties have been extensively studied in the literature, for example, see, Resnick (1987), Ahsanullah (1995), Nagaraja (1988), Balakrishnan and Ahsanullah (1994), Raqab et al (2008), Soliman et al. (2010), Amin (2012) and Nadar et al (2013).

Let $\{X_n, n \geq 1\}$ be a sequence of independent and identically distributed random variables with cumulative distribution function (cdf) $F(x)$ and the corresponding (pdf) $f(x)$. Set $Y_n = \max(X_1, X_2, ..., X_n)$, $n \geq 1$. We say $X_i$ is a record value of $\{X_n\}$ if $Y_j > Y_{j-1}$. Thus the record values are the relative maxima of the sequence. By definition, $X_1$ is a record value. The random variables $U(0) = 1$ and $U(m) = \min\{j : j > U(m-1), X_j > X_{U(m-1)}\}$, are called the upper record times, and the sequence $\{U(m), m > 0\}$ is called the sequence of upper record times and the sequences $R_m = X_{U(m)}$ is called the sequences of upper record values.

Let $R_0, R_1, ..., R_m$ be the first $(m+1)^{th}$ upper record values from the distribution function $F(x; \theta)$ and pdf $f(x; \theta)$, then the density function of the upper record values, was found in Ahsanullah (1995) as follows

$$f_{R_m}(r_m; \theta) = \frac{1}{\Gamma(m+1)} \left[- \ln F(r_m; \theta)\right]^m f(r_m; \theta), \ -\infty < r_m < \infty, \ m = 0, 1, 2, ..., \quad (1.1)$$

while the likelihood function based on the first $(m+1)^{th}$ upper record values $R_0, R_1, ..., R_m$ is given by

$$L(\theta) = \prod_{i=0}^{m-1} \frac{f(r_i; \theta)}{F(r_i; \theta)} f(r_m; \theta) \quad -\infty < r_0 < r_1 < ... < r_m < \infty. \quad (1.2)$$

Adepoju and Chukwu (2015) defined the Kumaraswamy exponential distribution as follows

$$F(x, \lambda, a, b) = 1 - \left[1 - (1 - e^{-\lambda x})^a\right]^b, \ x > 0 \quad (1.3)$$
where $\lambda > 0$ is a scale parameter and the other positive parameters, $a$ and $b$ are shape parameters. The corresponding pdf and hazard rate function are

\[
f(x, \lambda, a, b) = ab\lambda e^{-\lambda x}(1 - e^{-\lambda x})^{a-1}\left[1 - (1 - e^{-\lambda x})^a\right]^{b-1}, \quad x \geq 0, \tag{1.4}
\]

and

\[
h(x, \lambda, a, b) = \frac{ab\lambda e^{-\lambda x}(1 - e^{-\lambda x})^{a-1}}{[1 - (1 - e^{-\lambda x})^a]}, \tag{1.5}
\]

respectively.

In this paper considers statistical inference for $R = P(Y < X)$ when the observed data $X$ and $Y$ are drawn from independent Kw-E distribution based on upper record values. There are some works on $R$ on the basis of record data. For generalized exponential distribution, Baklizi (2008a) compared likelihood and Bayesian estimation of stress-strength parameter using lower record values. Also, in the one and two-parameter exponential distribution, Baklizi (2008b) estimated $P(Y < X)$ based on record values. Nadar and Kizilaslan (2014) used maximum likelihood and Bayesian approaches to obtain the estimation of $P(Y < X)$ based on a set of upper record values from Kumaraswamy distribution.

The rest of the article is organized as follows. In Section 2, we derive the maximum likelihood estimator for the reliability function $R$ of the Kumaraswamy exponential distribution based on upper record values. In Section 3, we obtain the exact interval estimation for the reliability function $R$ of the Kw-E distribution based on upper records when the scale parameter $\lambda$ and shape parameter $a$ are known. Simulation study are obtained for the theoretical results in Section 4. Finally we conclude the paper in Section 5.

\section{Maximum Likelihood Estimator for the Reliability Function}

Let $X$ and $Y$ be independent random variables from (Kw-E) distribution (1.3) with parameters $(a, \lambda, b_1)$ and $(a, \lambda, b_2)$ respectively. The stress-strength reliability function $R$ is given by

\[
R = P(Y < X) = \int_0^\infty \int_0^x f(x)f(y)dydx = \frac{b_2}{b_1 + b_2}. \tag{2.1}
\]

Let $R_0, R_1, R_2, ..., R_n$ represent the first $(n + 1)$ upper record values arising from a sequence $\{X_i\}$ of iid Kw-E distribution (1.3) with parameters $(a, \lambda, b_1)$ and $S_0, S_1, S_2, ..., S_m$ be the first $(m + 1)$ upper record values arising from a sequence $\{Y_i\}$ of iid Kw-E distribution (1.3) with parameters $(a, \lambda, b_2)$ where
$(a, \lambda)$ are unknown. The Likelihood functions are given by, see Arnold et al. (1998),

\[
L_1(a, \lambda, b_1 | r) = (a \lambda b_1)^{n+1} \prod_{i=0}^{n} \frac{e^{-\lambda r_i} [K(\lambda, r_i)]^{a-1}}{T(a, \lambda, r_i)} [T(a, \lambda, r_n)]^{b_1} \tag{2.2}
\]

and

\[
L_2(a, \lambda, b_2 | s) = (a \lambda b_2)^{m+1} \prod_{i=0}^{m} \frac{e^{-\lambda s_i} [K(\lambda, s_i)]^{a-1}}{T(a, \lambda, s_i)} [T(a, \lambda, s_m)]^{b_2}, \tag{2.3}
\]

where $r = (r_0, r_1, r_2, ..., r_n)$, $s = (s_0, s_1, s_2, ..., s_m)$. Let $\phi = (a, \lambda, b_1, b_2)$, the joint likelihood and the joint log likelihood is given by

\[
L(\phi | r, s) = L_1(a, \lambda, b_1 | r) L_2(a, \lambda, b_2 | s) \tag{2.4}
\]

and

\[
\ln L(\phi | r, s) = (n + 1) \ln(a \lambda b_1) + (m + 1) \ln(a \lambda b_2) - \lambda \left[ \sum_{i=0}^{n} r_i + \sum_{j=0}^{m} s_j \right] + (a - 1) \left[ \sum_{i=0}^{n} \ln K(\lambda, r_i) + \sum_{j=0}^{m} \ln K(\lambda, s_j) \right] + b_1 \ln T(a, \lambda, r_n)
\]

\[- \left[ \sum_{i=0}^{n} \ln T(a, \lambda, r_i) + \sum_{j=0}^{m} \ln T(a, \lambda, s_j) \right] + b_2 \ln T(a, \lambda, s_m), \tag{2.5}
\]

respectively. The maximum likelihood estimators of $b_1, b_2, a$ and $\lambda$, say $\hat{b}_1, \hat{b}_2, \hat{a}$ and $\hat{\lambda}$ respectively, can be obtained as a solution of

\[
\frac{\partial \ln L(\phi | r, s)}{\partial b_1} = \frac{n + 1}{b_1} + \ln T(a, \lambda, r_n) = 0, \tag{2.6}
\]

\[
\frac{\partial \ln L(\phi | r, s)}{\partial b_2} = \frac{m + 1}{b_2} + \ln T(a, \lambda, s_m) = 0, \tag{2.7}
\]

\[
\frac{\partial \ln L(\phi | r, s)}{\partial a} = \frac{n + m + 2}{a} + \sum_{i=0}^{n} \ln K(\lambda, r_i) + \sum_{j=0}^{m} \ln K(\lambda, s_j)
\]

\[+ \sum_{i=0}^{n} \frac{[K(\lambda, r_i)]^a \ln K(\lambda, r_i)}{T(a, \lambda, r_i)} + \sum_{j=0}^{m} \frac{[K(\lambda, s_j)]^a \ln K(\lambda, s_j)}{T(a, \lambda, s_j)}
\]

\[- b_1 \frac{[K(\lambda, r_n)]^a \ln K(\lambda, r_n)}{T(a, \lambda, r_n)} - b_2 \frac{[K(\lambda, s_m)]^a \ln K(\lambda, s_m)}{T(a, \lambda, s_m)} = 0, \tag{2.8}
\]
and

\[
\frac{\partial \ln L(\varphi | r, s)}{\partial \lambda} = \frac{n + m + 2}{\lambda} + (a - 1) \left[ \sum_{i=0}^{n} \frac{r_i e^{-\lambda r_i}}{K(\lambda, r_i)} + \sum_{j=0}^{m} \frac{s_j e^{-\lambda s_j}}{K(\lambda, s_j)} \right] - \sum_{i=0}^{n} r_i \nonumber \\
- \sum_{j=0}^{m} s_j + a \left[ \sum_{i=0}^{n} \frac{r_i e^{-\lambda r_i} [K(\lambda, r_i)]^{a-1}}{T(a, \lambda, r_i)} + \sum_{j=0}^{m} \frac{s_j e^{-\lambda s_j} [K(\lambda, s_j)]^{a-1}}{T(a, \lambda, s_j)} \right] \\
- \frac{b_1 a r_n e^{-\lambda r_n} [K(\lambda, r_n)]^{a-1}}{T(a, \lambda, r_n)} - \frac{b_2 a s_m e^{-\lambda s_m} [K(\lambda, s_m)]^{a-1}}{T(a, \lambda, s_m)} = 0. \tag{2.9}
\]

from (2.6) and (2.7) we get

\[
\hat{b}_1 = \frac{-(n + 1)}{\ln T(\hat{a}, \hat{\lambda}, r_n)}, \tag{2.10}
\]

\[
\hat{b}_2 = \frac{-(m + 1)}{\ln T(\hat{a}, \hat{\lambda}, s_n)}. \tag{2.11}
\]

Also \(\hat{a}\), and \(\hat{\lambda}\) can be obtained as a solution of the following nonlinear equations

\[
\frac{n + m + 2}{\hat{a}} + \sum_{i=0}^{n} \ln K(\hat{\lambda}, r_i) + \sum_{j=0}^{m} \ln K(\hat{\lambda}, s_j) + \sum_{i=0}^{n} \left[ \frac{K(\hat{\lambda}, r_i)}{T(\hat{a}, \hat{\lambda}, r_i)} \right]^{\hat{a}} \ln K(\hat{\lambda}, r_i) \nonumber \\
+ \sum_{j=0}^{m} \left[ \frac{K(\hat{\lambda}, s_j)}{T(\hat{a}, \hat{\lambda}, s_j)} \right]^{\hat{a}} \ln K(\hat{\lambda}, s_j) + \frac{(n + 1)}{T(\hat{a}, \hat{\lambda}, r_n)} \ln K(\hat{\lambda}, r_n) + \frac{(m + 1)}{T(\hat{a}, \hat{\lambda}, s_m)} \ln K(\hat{\lambda}, s_m) = 0, \tag{2.12}
\]
and
\[
\frac{n + m + 2}{\hat{\lambda}} + (\hat{a} - 1) \left[ \sum_{i=0}^{n} \frac{r_i e^{-\hat{\lambda} r_i}}{K(\hat{\lambda}, r_i)} + \sum_{j=0}^{m} \frac{s_j e^{-\hat{\lambda} s_j}}{K(\hat{\lambda}, s_j)} \right] - \left[ \sum_{i=0}^{n} r_i + \sum_{j=0}^{m} s_j \right]
\]
\[
+\hat{a} \left[ \sum_{i=0}^{n} \frac{r_i e^{-\hat{\lambda} r_i}}{T(\hat{a}, \hat{\lambda}, r_i)} K(\hat{\lambda}, r_i)^{\hat{a}-1} + \sum_{j=0}^{m} \frac{s_j e^{-\hat{\lambda} s_j}}{T(\hat{a}, \hat{\lambda}, s_j)} K(\hat{\lambda}, s_j)^{\hat{a}-1} \right]
\]
\[
- \frac{(n + 1)\hat{a} r_n e^{-\hat{\lambda} r_n}}{T(\hat{a}, \hat{\lambda}, r_n) \ln T(\hat{a}, \hat{\lambda}, r_n)} e^{-\hat{\lambda} r_n} 
- \frac{(m + 1)\hat{a} s_m e^{-\hat{\lambda} s_m}}{T(\hat{a}, \hat{\lambda}, s_m) \ln T(\hat{a}, \hat{\lambda}, s_m)} e^{-\hat{\lambda} s_m}
\]
\[
= 0. \quad (2.13)
\]

After, \( \hat{a} \) and \( \hat{\lambda} \) are obtained, \( \hat{b}_1 \), and \( \hat{b}_2 \) can be obtained from Equations (2.10) and (2.11), respectively. Therefore the maximum likelihood estimator of \( R \) is given as
\[
\hat{R} = \frac{\hat{b}_2}{\hat{b}_1 + \hat{b}_2} \quad (2.14)
\]

3 Interval Estimation for the Reliability Function when \( \lambda \) and \( a \) are Known

In this case we consider without loss of generality that the scale parameter \( \lambda = \lambda_0 \), and \( a = a_0 \). Therefore, the \( R_0, R_1, R_2, ..., R_n \) be a set of upper records from Kw-E distribution with parameters \( (a_0, \lambda_0, b_1) \) and \( S_0, S_1, S_2, ..., S_m \) be independent set of upper record from Kw-E distribution with parameters \( (a_0, \lambda_0, b_2) \), then the maximum likelihood estimators for \( b_1 \), and \( b_2 \), say \( \hat{b}_{11} \), and \( \hat{b}_{21} \), respectively, are given from the following equations
\[
\hat{b}_{11} = -\frac{(n + 1)}{\ln T(a_0, \lambda_0, r_n)}, \quad (3.1)
\]
\[
\hat{b}_{21} = -\frac{(m + 1)}{\ln T(a_0, \lambda_0, s_n)}, \quad (3.2)
\]
and the maximum likelihood estimator for the reliability function is given by
\[
\hat{R}_1 = \frac{\hat{b}_{21}}{\hat{b}_{11} + \hat{b}_{21}}. \quad (3.3)
\]
To study the the distribution of the maximum likelihood estimator for \( \hat{R}_1 \), we must obtain the distribution of \( \hat{b}_{11} \) and \( \hat{b}_{21} \). Consider \( Z_1 = \frac{-\frac{(n + 1)}{\ln T(a_0, \lambda_0, r_n)}}{\ln T(a_0, \lambda_0, r_n)} \), one
can show that the probability distribution of $Z_1$ is recognized as the inverted gamma distribution with parameters $(n+1)$ and $(n+1)b_1$. From the properties of the inverted gamma we show that $\frac{2(n+1)b_1}{Z_1}$ has a chi-square distribution with $2(n+1)$ degrees of freedom. Similarly $Z_2 = \frac{-(m+1)}{\ln T(a_0,\lambda_0,s_m)}$ has inverted gamma distribution with parameters $(m+1)$ and $(m+1)b_2$ and $\frac{2(m+1)b_2}{Z_2}$ has a chi-square distribution with $2(m+1)$ degrees of freedom. Now since the random variables $Z_1$ and $Z_2$ are independent, then one can show that

$$\frac{Z_1}{Z_2} \sim \frac{b_1}{b_2} F_{2(m+1),2(n+1)}$$

(3.4)

Therefore, after some simple argument we can show that the distribution of $\hat{R}_1$ is

$$\frac{1}{1 + \frac{Z_1}{Z_2} F_{2(m+1),2(n+1)}}$$

and $F_{2(m+1),2(n+1)}$ is F-distribution with $2(m+1)$ and $2(m+1)$ degrees of freedom. Based on the sampling distribution of $\hat{R}_1$, and after some transformation $100(1-\alpha)$% confidence interval for the stress strength reliability function for the Kw-E distribution based on upper record values is $(L,U)$ where.

$$L = \left[1 + \frac{Z_1}{Z_2 F_{2(m+1),2(n+1)}}\right]^{-1} \quad \text{and} \quad U = \left[1 + \frac{Z_1}{Z_2 F_{2(m+1),2(n+1)}}\right]^{-1}$$

(3.5)

### 4 Simulation Study

In this section our purpose to render numerical analysis for average mean square error AMSE of MLE and confidence intervals (C.I) for reliability function $R$. The software package MathCAD (2001) is used for the simulation study. The following steps are considered

1- Generate $N = 10000$ sets of uniform $(0,1)$ random variables, then use the usual transformation technique to get the corresponding sets of $X$-samples from the Kw-E distribution with parameters $a$, $\lambda$, $b_1$.

2- Choose from each vector the first $(n+1)$, $n= 2, 4, 6$ upper record values $r_0, r_1, r_2, \ldots, r_n$.

3- By the same manner, the set of $Y$- samples from the Kw-E distribution and the first $(m+1)$, $m= 2, 4, 6$ upper record values $s_0, s_1, s_2, \ldots, s_n$ for different values of $a$, $\lambda$ and $b_2$ are selected.

4- The validity of the estimate of $R$ is discussed by measures the average mean square error of the estimate and the average confidence interval of the estimate.

5- Tables 1 to 4 contains the following

i) Different combinations of the sample sizes and the distribution parameters.

ii) Simulation results for the maximum likelihood estimators $\hat{R}$ and $\hat{R}_1$ for the
reliability function $R$

iii) The average mean square error (AMSE) for the MLE of $\hat{R}$ and $\hat{R}_1$

iv) The 95% and 99% confidence interval for the reliability function $R$ and the average probability interval length

Table (1): MLE’s and confidence interval for the reliability function when $\lambda = 0.5$, $a = 0.5$, $b_1 = 1.5$ and $b_2 = 0.5$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$m$</th>
<th>$\hat{R}$</th>
<th>$\hat{R}_1$</th>
<th>95% C.I</th>
<th>99% C.I</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>0.328</td>
<td>0.302</td>
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<td>[0.043, 0.714]</td>
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<tr>
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<td>$(8.21 \times 10^{-5})$</td>
<td>$(1.16 \times 10^{-4})$</td>
<td>0.526</td>
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<tr>
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<td></td>
<td>0.318</td>
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<td>$(4.26 \times 10^{-5})$</td>
<td>$(3.88 \times 10^{-5})$</td>
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<td>0.653</td>
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<td>[0.12, 0.625]</td>
<td>[0.086, 0.733]</td>
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<td>$(3.62 \times 10^{-5})$</td>
<td>$(2.01 \times 10^{-5})$</td>
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<td>0.647</td>
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<tr>
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<td>0.247</td>
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<td>[0.038, 0.595]</td>
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<td>$(2.59 \times 10^{-5})$</td>
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<td>0.32</td>
<td>[0.089, 0.511]</td>
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<td>$(1.08 \times 10^{-5})$</td>
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<td>0.561</td>
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<td>$(1.05 \times 10^{-5})$</td>
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<td>0.463</td>
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* The values between brackets are the AMSE
Table (2): MLE’s and confidence interval for the reliability function when $\lambda = 0.5$, $a = 0.5$, $b_1 = 0.5$ and $b_2 = 0.5$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$m$</th>
<th>$\hat{R}$</th>
<th>$\hat{R}_1$</th>
<th>95% C.I</th>
<th>99% C.I</th>
</tr>
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<td>[0.119,0.884]</td>
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<td>(5.15×10$^{-5}$)</td>
<td>(1.91×10$^{-5}$)</td>
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<td>0.401</td>
<td>[0.156,0.702]</td>
<td>[0.1,0.773]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4.21×10$^{-5}$)</td>
<td>(2.38×10$^{-4}$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.368</td>
<td>0.443</td>
<td>[0.205,0.702]</td>
<td>[0.148,0.77]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.48×10$^{-5}$)</td>
<td>(1.16×10$^{-4}$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.13×10$^{-5}$)</td>
<td>(1.02×10$^{-4}$)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The values between brackets are the AMSE
Table (3): MLE’s and confidence interval for the reliability function when $\lambda = 0.5$, $a = 0.5$, $b_1 = 0.5$ and $b_2 = 0.5$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$m$</th>
<th>$\hat{R}$</th>
<th>$\hat{R}_1$</th>
<th>95% C.I</th>
<th>99% C.I</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>0.68</td>
<td>0.525</td>
<td>[0.181,0.813]</td>
<td>[0.116,0.88]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(5.26×10^{-5})</td>
<td>(5.18×10^{-5})</td>
<td>0.632</td>
<td>0.765</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>0.527</td>
<td>0.493</td>
<td>[0.238,0.822]</td>
<td>[0.17,0.885]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.94×10^{-5})</td>
<td>(2.82×10^{-5})</td>
<td>0.584</td>
<td>0.715</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>0.532</td>
<td>0.566</td>
<td>[0.291,0.839]</td>
<td>[0.221,0.897]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.74×10^{-5})</td>
<td>(1.95×10^{-5})</td>
<td>0.548</td>
<td>0.676</td>
</tr>
</tbody>
</table>

| 4   | 2   | 0.337     | 0.457      | [0.178,0.762] | [0.115,0.83] |
|     |     | (5.47×10^{-5}) | (4.68×10^{-5}) | 0.584    | 0.715    |
| 4   |     | 0.349     | 0.501      | [0.237,0.77] | [0.172,0.834] |
|     |     | (4.35×10^{-5}) | (3.23×10^{-5}) | 0.532    | 0.661    |
| 6   |     | 0.518     | 0.547      | [0.292,0.79] | [0.225,0.848] |
|     |     | (3.58×10^{-5}) | (1.27×10^{-5}) | 0.498    | 0.623    |

| 6   | 2   | 0.08      | 0.497      | [0.158,0.704] | [0.101,0.775] |
|     |     | (4.72×10^{-5}) | (3.23×10^{-5}) | 0.547    | 0.674    |
| 4   |     | 0.417     | 0.412      | [0.213,0.712] | [0.155, 0.778] |
|     |     | (2.46×10^{-5}) | (2.02×10^{-5}) | 0.498    | 0.623    |
| 6   |     | 0.381     | 0.434      | [0.265,0.734] | [0.205,0.794] |
|     |     | (1.04×10^{-5}) | (1.33×10^{-5}) | 0.468    | 0.59     |

* The values between brackets are the AMSE
Table (4): MLE’s and confidence interval for the reliability function when
$\lambda = 0.5$, $a = 0.5$, $b_1 = 1.5$ and $b_2 = 0.5$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$m$</th>
<th>$\hat{R}$</th>
<th>$\hat{R}_1$</th>
<th>95% C.I</th>
<th>99% C.I</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>0.474</td>
<td>0.272</td>
<td>[0.071,0.602]</td>
<td>[0.043,0.719]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(1.84 \times 10^{-5})$</td>
<td>$(7.23 \times 10^{-5})$</td>
<td>0.53</td>
<td>0.675</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>0.388</td>
<td>0.295</td>
<td>[0.095,0.608]</td>
<td>[0.065,0.721]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(1.33 \times 10^{-5})$</td>
<td>$(4.61 \times 10^{-5})$</td>
<td>0.519</td>
<td>0.66</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>0.421</td>
<td>0.332</td>
<td>[0.124,0.641]</td>
<td>[0.089,0.74]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(1.12 \times 10^{-5})$</td>
<td>$(3.52 \times 10^{-5})$</td>
<td>0.517</td>
<td>0.651</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>0.301</td>
<td>0.263</td>
<td>[0.068,0.591]</td>
<td>[0.042,0.621]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(2.68 \times 10^{-5})$</td>
<td>$(4.53 \times 10^{-5})$</td>
<td>0.451</td>
<td>0.579</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>0.454</td>
<td>0.236</td>
<td>[0.094,0.526]</td>
<td>[0.065,0.624]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(2.37 \times 10^{-5})$</td>
<td>$(3.74 \times 10^{-5})$</td>
<td>0.432</td>
<td>0.56</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>0.364</td>
<td>0.302</td>
<td>[0.121,0.552]</td>
<td>[0.089,0.64]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(1.55 \times 10^{-5})$</td>
<td>$(1.97 \times 10^{-5})$</td>
<td>0.431</td>
<td>0.551</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>0.309</td>
<td>0.193</td>
<td>[0.058,0.44]</td>
<td>[0.036,0.532]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(2.81 \times 10^{-5})$</td>
<td>$(4.75 \times 10^{-5})$</td>
<td>0.382</td>
<td>0.496</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>0.323</td>
<td>0.216</td>
<td>[0.079,0.438]</td>
<td>[0.055,0.525]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(1.84 \times 10^{-5})$</td>
<td>$(3.61 \times 10^{-5})$</td>
<td>0.359</td>
<td>0.471</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>0.404</td>
<td>0.263</td>
<td>[0.107,0.458]</td>
<td>[0.079,0.542]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(1.48 \times 10^{-5})$</td>
<td>$(1.72 \times 10^{-5})$</td>
<td>0.351</td>
<td>0.463</td>
</tr>
</tbody>
</table>

* The values between brackets are the AMSE

5 Conclusion

In this paper the estimation problem of the stress-strength reliability
$R = P(Y < X)$ when $X$ and $Y$ are independent Kw-E distribution based
on upper record values are discussed. The MLE and the confidence interval
for reliability function are obtained. We have presented a simulation study,
and observed the following results:
1- For fixed values $n$ the AMSE is decreases when $m$ increases, also for fixed
$m$ the AMSE decreases when $n$ increases.
2- The length of the confidence interval is decreases when $n$ or $m$ increases.

References

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Received: March 11, 2017; Published: March 26, 2017