Fuzzy $T$-Ideals in BP-Algebras

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Abstract

Motivated by the study of fuzzy $T$–ideals in several algebras, that arise from the propositional calculi, in this paper, we define the notion of Fuzzy T-ideals and L-fuzzy T-ideals in BP-algebras and discuss their properties.

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1 Introduction

Y. Imai and K. Iseki introduced two new classes of abstract algebra, BCK algebras and BCI algebras ([8], [9], [10]). In 2002, J. Neggers and H.S. Kim [11] introduced the notion of $\beta$-algebras. In [15], the authors introduced a new class of algebras: TM-algebras and claimed that the class of TM-algebras is a generalization of the classes of BCK and BCI algebras. However, in [2], the
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authors proved that the class of TM-algebras is not a generalization of the class of BCK and BCI algebras, by giving several counter examples. In 2012 Sun Shin Ahn and Jeong Soon Han introduced the notion of BP-Algebras [14].

Lofti A. Zadeh [18] introduced the theory of fuzzy sets. Gogun extended the notion of fuzzy sets to the notion of L-fuzzy sets [7]. The study of fuzzy algebraic structures was started with the introduction of the concept of fuzzy subgroups in 1977, by Rosenfeld [13]. O.G. Xi [17] applied the concept of fuzzy sets to BCK algebras and got some results in 1991. In [7], we studied the fuzzy subalgebras of a BP-algebras. In [16], the notion of fuzzy T-ideals in TM-algebras is discussed. In [1] and [12] Fuzzy T-ideals and L-fuzzy T-ideals in β-algebras were discussed. Motivated by these works, in this paper, we introduce the concept of Fuzzy T-ideals and L-fuzzy T-ideals in BP-Algebras and discuss some of their properties.

2 Preliminares

In this section we recall some basic definitions that are needed for our work.

Definition 2.1 [14] A BP algebra \((X, *, 0)\) is a non-empty set \(X\) with a constant \(0\) and a binary operation \(*\) satisfying the following conditions: for all \(x, y, z \in X\),

1. \(x*x = 0\)
2. \(x*(x*y) = y\)
3. \((x*z)*(y*z) = x*y\)

Definition 2.2 [14] Let \(S\) be a non-empty subset of a BP-algebra \(X\). Then \(S\) is called a BP-sub algebra of \(X\) if \(x*y \in S\), for all \(x, y \in X\).

Definition 2.3 [14] Let \((X, *, 0)\) be a BP-Algebra. A non-empty subset \(I\) of \(X\) is called an ideal of \(X\) if it satisfies the following conditions:

1. \(0 \in I\)
2. \(x*y \in I\) and \(y \in I\) implies \(x \in I\), \(\forall x, y \in X\).

Definition 2.4 An ideal \(I\) of a BP-ideal of \(X\) is said to be closed if \(0 * x \in I\), \(\forall x \in I\).

Definition 2.5 Let \((X, *, 0)\) be a BP-algebra. A non-empty subset \(I\) of \(X\) is called a T-ideal of \(X\) if it satisfies the following conditions
1. \( 0 \in I \)
2. \((x \ast y) \ast z \in I \) and \( y \in I \) implies \( x \ast z \in I \), \( \forall x, y, z \in X \).

**Definition 2.6** [3] A fuzzy sub set \( \mu \) of a BP-algebra \((X, \ast, 0)\) is called a fuzzy BP sub algebra of \( X \) if, for all \( x, y \in X \) the following condition is satisfied
\[
\mu(x \ast y) \geq \min\{\mu(x), \mu(y)\}, \forall x, y \in X.
\]

**Definition 2.7** [5] A fuzzy sub set \( \mu \) of a BP-algebra \((X, \ast, 0)\) is called a L-fuzzy BP sub algebra of \( X \) if, for all \( x, y \in X \) the following condition is satisfied
\[
\mu(x \ast y) \geq \{\mu(x) \land \mu(y)\}, \forall x, y \in X.
\]

### 3 Fuzzy T-ideals in BP-Algebras

In this section, we introduce the notion of fuzzy T-ideals and prove some simple results.

**Definition 3.1** [4] A fuzzy subset \( \mu \) in a BP-algebra \( X \) is called a fuzzy ideal of \( X \) if it satisfies the following conditions:
1. \( \mu(0) \geq \mu(x) \)
2. \( \mu(x) \geq \min\{\mu(x \ast y), \mu(y)\} \), \( \forall x, y \in X \).

**Definition 3.2** A fuzzy subset \( \mu \) in a BP-algebra \( X \) is called a fuzzy T-ideal of \( X \) if it satisfies the following conditions:
1. \( \mu(0) \geq \mu(x) \)
2. \( \mu(x \ast z) \geq \min\{\mu(x \ast y) \ast z, \mu(y)\}, \forall x, y, z \in X \).

**Theorem 3.3** A fuzzy set \( \mu \) of a BP-algebra \( X \) is a fuzzy subalgebra if and only if for every \( t \in [0,1], \mu_t \) is either empty or a subalgebra of \( X \).

**Proof:** Assume that \( \mu \) is a fuzzy subalgebra of \( X \) and \( \mu_t \neq \phi \). Then for any \( x, y \in \mu_t \), we have,
\[
\mu(x \ast y) \geq \min\{\mu(x), \mu(y)\} \geq t.
\]
Therefore \( x \ast y \in \mu_t \).
Hence \( \mu_t \) is a subalgebra of \( X \).
Conversely, Let \( \mu_t \) be a subalgebra of \( X \).
Let \( x, y \in X \). Take \( t = \min\{\mu(x), \mu(y)\} \)
Then by assumption \( \mu_t \) is a subalgebra of \( X \) implies \( x \ast y \in \mu_t \)
Therefore \( \mu(x \ast y) \geq \min\{\mu(x), \mu(y)\} \geq t \).
Hence \( \mu \) is a subalgebra of \( X \).
Theorem 3.4 Any subalgebra of a BP-algebra $X$ can be realized as a level subalgebra of some fuzzy subalgebra of $X$.

Proof: Let $\mu$ be a subalgebra of a given BP-algebra $X$ and let $\mu$ be a fuzzy set in $X$ defined by

$$
\mu(x) = \begin{cases} t & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}
$$

where $t \in (0,1)$ is fixed. It is clear that $\mu_t = A$.

Let $x, y \in X$. If $x, y \in A$ then $x \ast y \in A$ also,

Hence $\mu(x) = \mu(y) = \mu(x \ast y) = t$ and $\mu(x \ast y) \geq \min \{ \mu(x), \mu(y) \}$,

If $x, y \in A$ then $\mu(x) = \mu(y) = 0$ implies $\mu(x \ast y) \geq \min \{ \mu(x), \mu(y) \} = 0$.

If at most one of $x$ and $y$ belongs to $A$, then at least one of $\mu(x)$ and $\mu(y)$ is equal to 0.

Therefore, $\min \{ \mu(x), \mu(y) \} = 0$ so that $\mu(x \ast y) \geq 0$.

Theorem 3.5 Two level subalgebras $\mu_s, \mu_t$ ($s < t$) of a fuzzy subalgebra are equal if and only if there is no $x \in X$ such that $s \leq \mu(x) < t$.

Proof: Let $\mu_s = \mu_t$ for some $s < t$. If there exists $x \in X$, such that $s \leq \mu(x) < t$. If $x \in \mu_s$, then $\mu(x) \geq s$ and $\mu(x) \geq t$, since $\mu(x)$ does not lie between $s$ and $t$. Thus $x \in \mu_t$, which implies $\mu_s \mu_t$.

Also $\mu_t \mu_s$.

Therefore $\mu_s = \mu_t$

Theorem 3.6 Every fuzzy $T$-ideal $\mu$ of a BP-algebra $X$ is order reversing, that is if $x \leq y$ then: $\mu(x) \geq \mu(y) \forall x, y \in X$.

Proof: Let $x, y \in X$ such that $x \leq y$.

Therefore $x \ast y = 0$

Now,

$$
\mu(x) = \mu(x \ast 0) \\
\geq \min \{ \mu(x \ast y \ast 0), \mu(y) \} \\
= \min \{ \mu(0 \ast 0), \mu(y) \} \\
= \min \{ \mu(0), \mu(y) \} \\
= \mu(y)
$$

Theorem 3.7 A fuzzy set $\mu$ in a BP-algebra $X$ is a fuzzy $T$-ideal if and only if it is a fuzzy ideal of $X$.

Proof: Let $\mu$ be a fuzzy $T$-ideal of $X$

$\mu(0) \geq \mu(x)$
Let $\mu$ be a fuzzy set in a BP-algebra $X$ and let $t \in \text{Im}(\mu)$. Then $\mu$ is a fuzzy T-ideal of $X$ if and only if the level subset, $\mu_t = \{ x \in X / \mu(x) \geq t \}$ is a T-ideal of $X$, which is called a level T-ideal of $\mu$.

**Proof:** Assume that $\mu$ is a fuzzy T-ideal of $X$.

Clearly, $0 \in \mu_t$

Let $(x \ast y) \ast z \in \mu_t$ and $y \in \mu_t$

$$\Rightarrow \mu((x \ast y) \ast z) \geq t \text{ and } \mu(y) \geq t.$$  

Now, $\mu(x \ast z) \geq \min\{(\mu(x \ast y) \ast z), \mu(y)\}$

This shows that $\mu_t$ is T-ideal of $X$.

Conversely, Let $\mu_t$ be T-ideal of $X$ for any $t \in [0,1]$. Suppose, assume that there exist some $x_o \in X$ such that $\mu(0) < \mu(x_o)$.

Take $s = \frac{1}{2}[\mu(0) + \mu(x_o)]$

$\Rightarrow s \mu(x_o)$ and $0 \leq \mu(0) < s < 1$.

$\Rightarrow x_o \in \mu_s$ and $0 \notin \mu_s$, which is a contradiction, since $\mu_s$ is a T-ideal of $X$.

Therefore, $\mu(0) \geq \mu(x) \forall \ x \in X$.

If possible, assume that $x_o, y_o, z_o \in X$ such that

$\mu(x_o \ast z_o) \geq \min\{(\mu(x_o \ast y_o) \ast z_o), \mu(y_o)\}$

$\Rightarrow s > \min\{(\mu(x_o \ast y_o) \ast z_o), \mu(y_o)\}$

$\Rightarrow s > \mu(x_o \ast z_o), s < \min\{(\mu(x_o \ast y_o) \ast z_o) \text{ and } s < \mu(y_o)\}$

$\Rightarrow x_o \ast z_o \notin \mu_s$, which is a contradiction, since $\mu_s$ is a T-ideal of $X$.

Therefore, $\mu(x \ast z) \geq \min\{(\mu(x \ast y) \ast z), \mu(y)\} \forall x, y, z \in X$.

**Definition 3.9** Let $X$ and $Y$ be BP-algebras. A mapping $f : X \rightarrow Y$ is said to be a homomorphism if it satisfies: $f(x \ast y) = f(x) \ast f(y)$, for all $x, y \in X$.
Definition 3.10 Let $f: X \rightarrow X$ be an endomorphism and $\mu$ a fuzzy set in $X$. We define a new fuzzy set in $X$ by $\mu_f$ in $X$ by $\mu_f(x) = \mu(f(x))$, for all $x$ in $X$.

Theorem 3.11 Let $f$ be an endomorphism of a BP-algebra $X$. If $\mu$ is a fuzzy $T$-ideal of $X$, then so is $\mu_f$.

Proof:

\[ \mu_f(x) = \mu(f(x)) \leq \mu(0) \]

Let $x, y, z \in X$. Then $\mu_f(x \ast z) = \mu(f(x \ast z))$

\[ = \mu(f(x) \ast f(z)) \]

\[ \geq \min \{\mu((f(x) \ast f(y)) \ast f(z)), \mu(f(y))\} \]

\[ = \min \{\mu((f(x) \ast f(y)) \ast (z)), \mu(f(y))\} \]

\[ = \min \{\mu(f((x \ast y) \ast z)), \mu(f(y))\} \]

\[ = \min \{\mu_f((x \ast y) \ast z), \mu_f(y)\} \].

Hence $\mu_f$ is a fuzzy $T$-ideal of $X$.

4 Cartesian product of fuzzy $t$-ideals of bp-algebras

In this section, we introduce the notion of Cartesian product of two fuzzy $T$-ideals and study its properties.

Definition 4.1 Let $\mu$ and $\lambda$ be the fuzzy set in a set $X$. the Cartesian product $\mu \times \lambda: X \times X \rightarrow [0, 1]$ is defined by $(\mu \times \lambda)(x, y) = \min \{\mu(x), \lambda(y)\} \forall x, y \in X$.

Theorem 4.2 If $\mu$ and $\lambda$ are fuzzy $T$-ideal in a BP algebra of $X$, then $\mu \times \lambda$ is a fuzzy $T$-ideal in $X \times X$.

Proof: For any $(x, y) \in X \times X$, we have,

\[ (\mu \times \lambda)(0, 0) = \min \\{\mu(0), \lambda(0)\} \]

\[ \geq \min \{\mu(x), \lambda(y)\} \]

\[ = (\mu \times \lambda)(x, y) \]
Let \((x_1, x_2), (y_1, y_2)\) and \((z_1, z_2)\) \in X \times X.

\[
(\mu \times \lambda)((x_1, x_2) \times (z_1, z_2)) = (\mu \times \lambda)((x_1 \ast z_1, x_2 \ast z_2)) \\
= \{\mu (x_1 \times z_1), \mu (x_2 \times z_2)\} \\
\geq \{\min(\mu(x_1 \times y_1) \times z_1), \mu(y_1)\}, \\min(\lambda(x_2 \times y_2) \times z_2), \lambda(y_2)\} \\
= \min\{\min(\mu(x_1 \times y_1) \times z_1), \mu(y_1)\}, \\min(\lambda(x_2 \times y_2) \times z_2), \lambda(y_2)\} \\
= \min\{\mu(x_1 \times y_1) \times (x_2 \times y_2) \times (z_1, z_2)), (\mu \times \lambda)(y_1, y_2)\} \\
= \min\{\mu(x_1 \times (y_1 \times y_2) \times (z_1, z_2)), (\mu \times \lambda)(y_1, y_2)\}
\]

Hence \(\mu \times \lambda\) is a fuzzy T-ideal in \(X \times X\).

**Theorem 4.3** Let \(\mu\) and \(\lambda\) be fuzzy sets in a BP-algebra \(X\) such that \(\mu \times \lambda\) is a is a fuzzy T-ideal of fuzzy T-ideal of \(X \times X\). Then

1. Either \(\mu(0) \geq \mu(x)\) or \(\lambda(0) \geq \lambda(x)\) for all \(x \in X\)
2. If \(\mu(0) \geq \mu(x)\) for all \(x \in X\), then either \(\lambda(0) \geq \mu(x)\) or \(\lambda(0) \geq \lambda(x)\)
3. If \(\lambda(0) \geq \lambda(x)\) for all \(x \in X\), then either \(\mu(0) \geq \mu(x)\) or \(\mu(0) \geq \lambda(x)\)
4. Either \(\mu\) or \(\lambda\) is a fuzzy T-ideal of \(X\).

**Proof:** \(\mu \times \lambda\) is a fuzzy p-ideal of \(X \times X\). Therefore

\[
(\mu \times \lambda)(0, 0) \geq (\mu \times \lambda)(x, y) \text{ for all } (x, y) \in X \times X
\]

And

\[
(\mu \times \lambda)((x_1, x_2), (z_1, z_2)) \geq \min(\mu \times \lambda)((x_1, x_2) \ast (z_1, z_2)) \ast ((y_1, y_2) \ast (z_1, z_2)), (\mu \times \lambda)(y_1, y_2)
\]

for all \((x_1, x_2), (y_1, y_2)\) and \((z_1, z_2)\) \in X \times X.

Suppose that \(\mu(0) < \mu(x)\) and \(\lambda(0) < \lambda(y)\) for some \(x, y \in X\).

\[
Then: (\mu \times \lambda)(x, y) = \min\{\mu(x), \lambda(y)\} \\
\geq \min\{\mu(0), \lambda(0)\} \\
= (\mu \times \lambda)(0, 0), \text{ which is a contradiction.}
\]

Therefore either \(\mu(0) \geq \mu(x)\) or \(\lambda(0) \geq \lambda(x)\) for all \(x \in X\).

Assume that there exist \(x, y \in X\) such that: \(\lambda(0) < \mu(x)\) and \(\lambda(0) < \lambda(y)\).

Then

\[
(\mu \times \lambda)(0, 0) = \min\{\mu(0), \lambda(0)\} \\
= \lambda(0) \text{ and hence} \\
(\mu \times \lambda)(x, y) = \min\{\mu(x), \lambda(y)\} > \lambda(0) \\
= (\mu \times \lambda)(0, 0), \text{ a contradiction.}
\]
Hence if \(\mu(0) \geq \mu(x)\) \(\forall x \in X\), then either \(\lambda(0) \geq \mu(x)\) or \(\lambda(0) \geq \lambda(x)\).

Similarly we can prove that if \(\lambda(0) \geq \lambda(x)\) for all \(x \in X\), then either \(\mu(0) \geq \mu(x)\) or \(\mu(0) \geq \lambda(x)\).

First we prove that \(\lambda\) is a fuzzy T-ideal of \(X\).

Since, by (i), either \(\mu(0) \geq \mu(x)\) or \(\lambda(0) \geq \lambda(x)\) for all \(x \in X\).

Assume that \(\lambda(0) \geq \lambda(x)\) for all \(x \in X\).

It follows from (iii) that either \(\mu(0) \geq \mu(x)\) or \(\mu(0) \geq \lambda(x)\).

If \(\mu(0) \geq \lambda(x)\) for any \(x \in X\), then:

\[
\lambda(x) = \min\{\mu(0), \lambda(x)\} = (\mu \times \lambda)(0, x).
\]

\[
= \min\{\mu(0), \lambda(x)\} = (\mu \times \lambda)(0, x) \geq \min\{(\mu \times \lambda)((0, x) \ast (0, z)) \ast ((0 \ast y) \ast (0, z)), (\mu \times \lambda)(0, y)\}
\]

\[
= \min\{(\mu \times \lambda)((0 \ast 0), (x \ast z)) \ast ((0 \ast 0), (y \ast z)), (\mu \times \lambda)(0, y)\}
\]

\[
= \min\{(\mu \times \lambda)((0 \ast 0) \ast (0 \ast 0)), ((x \ast z) \ast (y \ast z)), (\mu \times \lambda)(0, y)\}
\]

\[
= \min\{\lambda((x \ast z) \ast (y \ast z)), (\mu \times \lambda)(0, y)\}\lambda(x)
\]

Hence \(\lambda\) is a fuzzy T-ideal of \(X\).

Now we will prove that \(\mu\) is a fuzzy T-ideal of \(X\).

Let \(\mu(0) \geq \mu(x)\).

By (ii) either \(\lambda(0) \geq \mu(x)\) or \(\lambda(0) \geq \lambda(x)\).

Assume that \(\lambda(0) \geq \mu(x)\),

\[
Then \mu(x) = \min\{\mu(x), \lambda(0)\} = (\mu \times \lambda)(x, 0), \mu(x)
\]

\[
= \min\{\mu(x), \lambda(0)\} = (\mu \times \lambda)(x, 0) \geq \min\{(\mu \times \lambda)(((x, 0) \ast (z, 0)) \ast ((y \ast 0) \ast (z, 0))), (\mu \lambda)(y, 0)\}
\]

\[
= \min\{(\mu \times \lambda)(((x \ast z), (0 \ast 0)) \ast ((y \ast z), (0 \ast 0))), (\mu \lambda)(y, 0)\}
\]

\[
= \min\{(\mu \times \lambda)(((x \ast z) \ast (y \ast z)), ((0 \ast 0) \ast (0 \ast 0))), (\mu \lambda)(y, 0)\}
\]

\[
= \min\{\mu((x \ast z) \ast (y \ast z)), \mu(y)\}
\]

Hence \(\mu\) is a fuzzy T-ideal of \(X\).

5 L-Fuzzy T-ideals in BP-Algebras

Analogous to the notion of fuzzy T-ideals in section 3, in this section we study L-fuzzy T-ideals.
Definition 5.1 [6] A L-fuzzy subset \( \mu \) in a BP-algebra \( X \) is called a L-fuzzy ideal of \( X \) if it satisfies the following conditions:

1. \( \mu(0) \geq \mu(x) \)
2. \( \mu(x \ast z) \geq \{ \mu(x \ast y) \land \mu(y) \} \forall x, y \in X \)

Definition 5.2 A fuzzy subset \( \mu \) in a BP-algebra \( X \) is called a L-fuzzy T-ideal of \( X \) if it satisfies the following conditions:

1. \( \mu(0) \geq \mu(x) \)
2. \( \mu(x \ast z) \geq \{ (\mu(x \ast y) \ast z) \land \mu(y) \} \forall x, y, z \in X \).

Theorem 5.3 Every L-fuzzy T-ideal \( \mu \) of a BP-algebra \( X \) is order reversing if \( x \leq y \) then \( \mu(x) \geq \mu(y) \forall x, y \in X \).

Proof: Let \( x, y \in X \) such that \( x \leq y \).

Therefore \( x \ast y = 0 \)

Now, \( \mu(x) = \mu(x \ast 0) \geq \{ (\mu(x \ast y) \ast 0) \land \mu(y) \} \)

\( = \{ (\mu(0 \ast 0) \land \mu(y) \} \)

\( = \{ (\mu(0) \land \mu(y) \} \)

\( = \mu(y) \)

Theorem 5.4 A L-fuzzy set \( \mu \) in a BP-algebra \( X \) is a L-fuzzy T-ideal if and only if it is a L-fuzzy ideal of \( X \).

Proof: Let \( \mu \) be a fuzzy T-ideal of \( X \)

\( \mu(0) \geq \mu(x) \)

\( \mu(x \ast z) \geq \{ (\mu(x \ast y) \ast z) \land \mu(y) \} \forall x, y, z \in X \).

By putting \( z = 0 \) in (ii) we have \( \mu(x) \geq \{ \mu(x \ast y) \land \mu(y) \} \)
Hence \( \mu \) is a fuzzy ideal of \( X \).

Conversely, \( \mu \) is a fuzzy ideal of \( X \).

Then, \( \mu(x \ast z) \geq \{ (\mu(x \ast y) \ast z) \land \mu(y) \} \forall x, y, z \in X \).

\( = \{ (\mu(x \ast y) \ast z) \land \mu(y) \} \forall x, y, z \in X \), which proves the result.

Theorem 5.5 Let \( \mu \) be a fuzzy set in a BP-algebra \( X \) and let \( t \in \text{Im}(\mu) \). Then \( \mu \) is a fuzzy T-ideal of \( X \) if and only if the level subset, \( \mu \ast t = \{ x \in X / \mu(x) \geq t \} \) is a T-ideal of \( X \), which is called a level T-ideal of \( \mu \).
Proof: Assume that \( \mu \) is a fuzzy T-ideal of \( X \).
Clearly, \( 0 \in \mu_t \)
Let \((x*y)*z \in \mu_t \) and \( y \in \mu_t \)
Then \( \mu((x*y)*z) \geq t \) and \( \mu(y) \geq t \).
Now, \( \mu(x*z) \geq \{ (\mu(x*y)*z) \land \mu(y) \} \)
\( \geq \{t,t\} \)
\( = \ t \)

Hence \( \mu_t \) is T-ideal of \( X \).
Conversely, Let \( \mu_t \) be T-ideal of \( X \) for any \( t \in [0,1] \). Suppose, assume that there exist some \( x_o \in X \) such that \( \mu(0) < \mu(x_o) \).

Takes \( s = \frac{1}{2} \left[ \mu(0) + \mu(x_o) \right] \)
\( \Rightarrow s < \mu(x_o) \) and \( 0 \leq \mu(0) < s < 1 \).
\( \Rightarrow x_o \in \mu_s \) and \( 0 \notin \mu_s \), which is a contradiction, since \( \mu_s \) is a T-ideal of \( X \).

Therefore, \( \mu(0) \geq \mu(x) \) \( \forall x \in X \).
If possible, assume that \( x_o, y_o, z_o \in X \) such that
\( \mu(x_o*z_o) \geq \{ (\mu(x_o*y_o)*z_o) \land \mu(y_o) \} \)
Takes \( s = \frac{1}{2} \left[ \mu(x_o*z_o) + \{ (\mu(x_o*y_o)*z_o) \land \mu(y_o) \} \right] \)
\( \Rightarrow s > \mu(x_o*z_o) \) and \( s < \{ (\mu(x_o*y_o)*z_o) \land \mu(y_o) \} \)
\( \Rightarrow s > \mu(x_o*z_o), s < \{ (\mu(x_o*y_o)*z_o) \land \mu(y_o) \} \)
\( \Rightarrow x_o*z_o \notin \mu_s \), which is a contradiction, since \( \mu_s \) is a T-ideal of \( X \).
Therefore, \( \mu(x*z) \geq \{ (\mu(x*y)*z) \land \mu(y) \} \) \( \forall x, y, z \in X \).

Definition 5.6 Let \( \mu \) and \( \lambda \) be the fuzzy set in a set \( X \). the Cartesian product \( \mu \times \lambda : X \times X \to [0,1] \) is defined by \( (\mu \times \lambda)(x,y) = \min \{ \mu(x), \lambda (y) \} \) \( \forall x, y \in X \).

Theorem 5.7 If \( \mu \) and \( \lambda \) are fuzzy T-ideal in a BP algebra of \( X \), then \( \mu \times \lambda \) is a fuzzy T-ideal in \( X \times X \).

Proof: For any \( (x, y) \in X \times X \), we have,
\( \mu \times \lambda(0,0) = \{ \mu(0) \land \lambda(0) \} \)
\( \geq \{ \mu(x) \land \lambda(y) \} \)
\( = \ (\mu \times \lambda)(x, y) \)
Let \((x_1, x_2), (y_1, y_2)\) and \((z_1, z_2)\) \(\in X \times X\).

\[
(\mu \times \lambda)((x_1, x_2) \ast (z_1, z_2)) = (\mu \times \lambda)((x_1 \ast y_1, x_2 \ast y_2) \ast (z_1, z_2))
\]

\[
\geq \left\{ \mu(x_1 \ast y_1) \wedge \mu(y_1) \right\} \wedge \left\{ \lambda(x_2 \ast y_2) \ast (z_1, z_2) \right\} \wedge \left\{ \lambda(y_2) \right\}
\]

Hence \(\mu \times \lambda\) is a fuzzy T-ideal in \(X \times X\).

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**References**


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