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C-Prime Fuzzy Ideals of $R^n[x]$

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Abstract

We characterize c-prime fuzzy ideals of the polynomial ring $(R^n[x], +, \cdot)$ using the notion of ideal symmetry.

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1 Introduction

Kedukodi, Kuncham, and Bhavanari [6] introduced the notion of fuzzy graph of a nearring N with respect to a level ideal which helped to depict graphically the fuzzy character concealed algebraically in the examples of 3-prime fuzzy ideals of N. If μ is 3-prime fuzzy ideal of N, then (N, μ, σ, t) has a special type of symmetry called the ideal symmetry. In this paper we take N as the polynomial ring $(R^n[x], +, \cdot)$. $(R^n[x], +, \cdot)$ is a commutative ring with unit element. The notions of 3-prime fuzzy ideal and c-prime fuzzy ideal coincide in commutative rings (Kedukodi, Kuncham, and Bhavanari [5]). Using these interconnections, we characterize c-prime fuzzy ideals $(R^n[x], +, \cdot)$.

2 Preliminary Notes

(Biggs [1]) Let R^n represent the complete set of 2^n binary words of length n. A binary code of length n is a subset C of R^n. A code C is linear if $\widehat{a}, \widehat{b} \in C \Rightarrow \widehat{a} + \widehat{b} \in C$ (The addition + is the modular addition in Z_2). A code C is called cyclic if it is a linear code and if

$$a_0 a_1 a_2 ... a_{n-1} \in C \Rightarrow a_{n-1} a_0 a_1 a_2 ... a_{n-2} \in C$$

We denote by $R^n[x]$ the ring of polynomials modulo $x^n - 1$, with coefficients in Z_2. Working modulo $x^n - 1$ is same as replacing x^n by 1, x^{n+1} by x, x^{n+2} by x^2, and so on. We have a bijective correspondence between R^n and $R^n[x]$. If $a(x)$ and $b(x)$ in $R^n[x]$ correspond to $\widehat{a}, \widehat{b}$ in R^n then $a(x) + b(x)$ corresponds to $\widehat{a} + \widehat{b}$ and $xa(x)$ corresponds to the first cyclic shift.

Theorem 2.1. *(Biggs [1]) A code in R^n is cyclic if and only if it corresponds to an ideal in $R^n[x]$.*

Definition 2.2. *(Kedukodi, Kuncham, and Bhavanari [6])Let $\mu : N \to (0, 1]$ be a fuzzy ideal of N with thresholds α and β. Let $t \in (\alpha, \beta]$ be fixed. Define $\sigma : N \times N \to [0, 1]$ as follows:*

$$\sigma(x, y) = \begin{cases} \mu(x) \wedge \mu(y) & x \neq y \text{ and } (xNy \subseteq \mu_t \text{ or } yNx \subseteq \mu_t) \\ 0 & \text{Otherwise} \end{cases}$$

Then the fuzzy graph (N, μ, σ) is called the fuzzy graph of N with respect to the level ideal μ_t. We denote the fuzzy graph of N with respect to the level ideal μ_t by (N, μ, σ, t). The fuzzy graph (N, μ, σ, t) is called ideal symmetric if for every pair of vertices x, y in (N, μ, σ, t) with an edge between them, either $[\sigma(x, z) > 0 \ \forall \ z \neq x;\ z \in N]$ or $[\sigma(y, z) > 0 \ \forall \ z \neq y;\ z \in N]$.

3 Main Result

Theorem 3.1. *Let $\mu : R^n[x] \to [0, 1]$ be a fuzzy subset of $R^n[x]$ and $\Delta = \{\mu_t \mid t \in (\alpha, \beta]\}$. Let each level set μ_t in Δ be such that for every $x \in N$, $x^2 \in \mu_t$ implies $x \in \mu_t$. Let $\Upsilon = \{C \mid C$ is a code in $R^n\}$ such that $\Delta \equiv \Upsilon$. Then μ is a c-prime fuzzy ideal of $R^n[x]$ if and only if every code $C \in \Upsilon$ is cyclic and $(R^n[x], \mu, \sigma, t)$ is ideal symmetric for all $t \in (\alpha, \beta]$.*

Proof. By Theorem 2.1, we get μ is a fuzzy ideal of $R^n[x]$ if and only if every code $C \in \Upsilon$ is cyclic. $R^n[x]$ is a commutative ring with unit element 1. By Corollary 3.11 of Kedukodi, Kuncham, and Bhavanari [6], μ is c-prime if and only if $(R^n[x], \mu, \sigma, t)$ is ideal symmetric for all $t \in (\alpha, \beta]$. □

Example 3.2. *Let* R^4 *represent the complete set of 16 binary words of length* 4. *Let* $N = R^4[x] = \{0, 1, x, x^2, x^3, 1+x+x^2, 1+x+x^3, 1+x^2+x^3, x+x^2+x^3, 1+x, 1+x^2, x+x^2, 1+x^3, x+x^3, x^2+x^3, 1+x+x^2+x^3\}$. *(Graham and Harary [3]) The elements of the ring* R^4 *(hence elements of* $R^4[x]$*) can be represented by a hypercube of dimension* 4 *denoted by* Q_4. Q_4 *is the graph of* 16 *nodes such that each node is uniquely labeled with an* $4-$*bit binary string and two nodes are adjacent whenever their labels vary in exactly one bit. The sketch of* Q_4 *can be found in Graham and Harary [3]. Let* $A = \{1+x, 1+x^2, x+x^2, 1+x^3, x+x^3, x^2+x^3\}$ *and* $B = \{1, x, x^2, x^3, 1+x+x^2, 1+x+x^3, 1+x^2+x^3, x+x^2+x^3\}$. *Define a fuzzy subset* $\mu : N \to [0,1]$ *by*

$$\mu(y) = \begin{cases} 0.8 & \text{if } y \in \{0, 1+x+x^2+x^3\} \\ 0.6 & \text{if } y \in A \\ 0.4 & \text{if } y \in B \end{cases}$$

Let $\alpha = 0.4$ *and* $\beta = 0.6$. *Then* μ *is a fuzzy ideal of* $N = R^4[x]$. *Take* $t = 0.6$. *Then* $\mu_t = \mu_{0.6}$

$$= \{0, 1+x, 1+x^2, x+x^2, 1+x^3, x+x^3, x^2+x^3, 1+x+x^2+x^3\}.$$

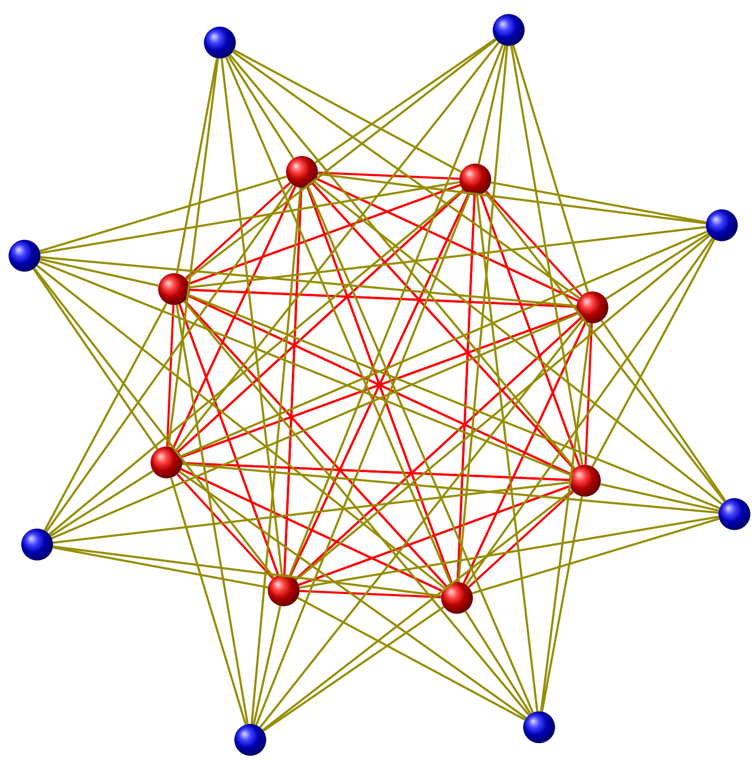
We form a table for σ *as follows:*

$\sigma(y,z)$	$z = 0$	$z = 1+x+x^2+x^3$	$z = 1+x$
$y = 0$	0	0.8	0.6
$y = 1+x+x^2+x^3$	0.8	0	0.6
$y = 1+x$	0.6	0.6	0
$y = 1+x^2$	0.6	0.6	0.6
$y = x+x^2$	0.6	0.6	0.6
$y = 1+x^3$	0.6	0.6	0.6
$y = x+x^3$	0.6	0.6	0.6
$y = x^2+x^3$	0.6	0.6	0.6
$y \in B$	0.4	0.4	0.4

$\sigma(y,z)$	$z=1+x^2$	$z=x+x^2$
$y=0$	0.6	0.6
$y=1+x+x^2+x^3$	0.6	0.6
$y=1+x$	0.6	0.6
$y=1+x^2$	0	0.6
$y=x+x^2$	0.6	0
$y=1+x^3$	0.6	0.6
$y=x+x^3$	0.6	0.6
$y=x^2+x^3$	0.6	0.6
$y\in B$	0.4	0.4

$\sigma(y,z)$	$z=1+x^3$	$z=x+x^3$	$z=x^2+x^3$	$z\in B$
$y=0$	0.6	0.6	0.6	0.4
$y=1+x+x^2+x^3$	0.6	0.6	0.6	0.4
$y=1+x$	0.6	0.6	0.6	0.4
$y=1+x^2$	0.6	0.6	0.6	0.4
$y=x+x^2$	0.6	0.6	0.6	0.4
$y=1+x^3$	0	0.6	0.6	0.4
$y=x+x^3$	0.6	0	0.6	0.4
$y=x^2+x^3$	0.6	0.6	0	0.4
$y\in B$	0.4	0.4	0.4	0

$(N,\mu,\sigma,0.6)$ *is the network shown below:*



Theorem 3.1 gives an interesting way of ascertaining whether a fuzzy subset $\mu : R^n[x] \to [0,1]$ is a c-prime fuzzy ideal or not.

Consider the fuzzy subset μ of $R^4[x]$ defined in Example 3.2. Let $\alpha = 0.4$ and $\beta = 0.6$. The possible level subset for $t \in (0.4, 0.6]$ is

$$F = \{0, 1+x, 1+x^2, x+x^2, 1+x^3, x+x^3, x^2+x^3, 1+x+x^2+x^3\}.$$

Hence $\Delta = \{F\}$. Note that $y^2 \in F$ implies $y \in F$.

Corresponding to the set F, the binary 4-tuple code is

$$C_F = \{0000, 0011, 0101, 0110, 1001, 1010, 1100, 1111\}.$$

Hence $\Upsilon = \{C_F\}$. Now we have $\Delta \equiv \Upsilon$.

Note that

(i) every code $C \in \Upsilon$ is cyclic; and

(ii) $(R^n[x], \mu, \sigma, t)$ is ideal symmetric for all $t \in (0.4, 0.6]$.

Thus by Theorem 3.1, μ is a c-prime fuzzy ideal of $R^4[x]$.

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