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# Estimation in Constant-Stress Accelerated Life Testing for Birnbaum-Saunders Distribution under Censoring

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**Abstract**

Accelerated life testing is a method for estimating the reliability of products at normal use conditions from the failure data obtained at the severe conditions. In this paper, constant-stress accelerated life testing is considered when the lifetime of a product follows a two- parameter Birnbaum-Saunders distribution. Based on type-II censoring. A certain conditions (stress) are assumed to affect the scale parameter through the inverse power law model. Maximum likelihood estimators (MLEs) of an accelerated life tests model are derived. Prediction of the scale parameter under usual conditions is obtained. Moreover, the reliability function at a certain mission time under the same conditions is also predicted. Simulation results are carried out to study the precision of the MLEs for the parameters involved.

**Keywords**: Accelerated life testing; Reliability; Constant stress; Birnbaum-Saunders distribution, Maximum likelihood estimation; Inverse power law model; Type-II censoring

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## 1. Introduction

Accelerated life testing (ALT) is often used in order to induce failures of very high reliability devices. In such tests, the units are tested under accelerated conditions (stress) that is extremely more severe than normal use conditions encountered in practice.

So, the time necessary to test a sample of such devices under normal conditions very costly, take a long time. So, ALT save time and cost. Stress would induce early failure of the tested units. Over stress may be in the form of temperature, voltage, pressure, load, humidity, vibration,…etc, or some combination of them.

In a constant stress accelerated test, each unit is run at a prespecified constant stress level which does not vary with time. In use, most products such as semiconductors and microelectronics, capacitors, lamps …etc, run at a constant stress. This type of stress is widely used and preferred because the stress is constant in most applications, it is much easier to apply and quantify constant stress and models for constant stress are available, widely publicized and empirically verified.

Determining a relationship between stress and scale parameter of the lifetime distribution, extrapolation to the normal use conditions will be carried. This relationship is known as the acceleration model. It is assumed that changing the stress from one level to another affects the value of the parameter only and not the functional form of the lifetime distribution. This is a major assumption of ALT.

In the literature for ALT, Viertl [1988] gave a briefer overview of the available statistical methods for ALT. Nelson [1990] introduced a comprehensive review of the theory and practice of ALT. Meeker and Escobar [1993] briefly reviewed the basic statistical and other ideas behind accelerated testing. They provided an overview of some current statistical research to improve accelerated tested planning and methods. In a constant stress ALT, was applied by several authors,( for example, see Owen [1997], Aly [1997], Abdel-Ghaly et al. [1998] , Al-Hussaini and Abd-Hamid[2004,2006], Watkins and John [2008] and Abd-Hamid and Al-Hussaini [2009] ).

Birnbaum and Saunders [1969a,b] proposed a two-parameter failure time distribution for fatigue failure caused under cyclic loading. Fatigue failure based on a model which assumes that failure is due to the development and growth of a dominant crack. This distribution is known as the two-parameter Birnbaum-Saunders (BS) distribution or as the fatigue life distribution. As Artur et al. [2011] indicate, this distribution is an attractive alternative to the weibull, gamma and log-normal models, since its derivation considers the basic characteristics of the fatigue process.

Statistical analysis for the BS distribution has for the most part, been limited to complete data by several authors,( for example, see Chang and Tang [1993], Dupuis and Mills [1998], Kevin [1999] , Xu and Tang [2010,2011]). Recently, little work has been based on censored data, (for example, see Rieck [1995],Jeng [2003], Ng et al. [2006],Wang et al[2006] and Artur et al. [2011] ). Constant-stress ALT for two-parameter BS distribution is introduced by Owen [1997].

In this paper, we discuss the MLEs of constant-stress ALT for two-parameter BS distribution under type-II censored data. This model is described in detail in section 2, the value of the scale parameter and the reliability function at a mission time $t_0$ under usual stress are predicted. In section 3 explains the simulation studies for illustrating the theoretical results. Finally, conclusions are included in section 4.

## 2. Model description and maximum likelihood estimation

The lifetime T is assumed to have a two parameters BS distribution with the shape and scale parameters α and β respectively. So, the probability density function of T is

$$f(t;\alpha,\beta)=\frac{1}{2\alpha\sqrt{2\pi\beta}}\frac{(t+\beta)}{\left(t^{\frac{3}{2}}\right)}exp\left[\frac{-1}{2\alpha^2}\left(\frac{t}{\beta}+\frac{\beta}{t}-2\right)\right], t\geq o\,, \alpha,\beta>o \qquad (1)$$

The scale parameter $\beta$ is the median of the $B$S distribution which has a wide use in reliability studies. The cumulative distribution function is as follows:

$$F(t;\alpha,\beta)=\Phi\left(\frac{1}{\alpha}\sqrt{\left(\frac{t}{\beta}\right)}-\sqrt{\left(\frac{\beta}{t}\right)}\right), t\geq o\,, \alpha,\beta>o \qquad (2)$$

Where Φ (.) denotes the standard normal distribution.

And the reliability function of $BS$ distribution in (1) takes the form:

$$R(t)=1-\Phi\left(\frac{1}{\alpha}\left(\sqrt{\left(\frac{t}{\beta}\right)}-\sqrt{\left(\frac{\beta}{t}\right)}\right)\right), t\geq o\,, \alpha,\beta>o \qquad (3)$$

The scale parameters $\beta$ and the stress V are related by the inverse power law model, which is described in Mann et al. [1974].

$$\beta=CV^{-P} \qquad ,C,P>o \qquad (4)$$

Where C is the constant of proportionality, and P is the power of the applied stress, which are the parameters of this model.

Assume that the life testing experiment is conducted under K higher stress $V_j$, j=1…k and assume that $V_u$ be the normal use conditions such that $V_u<V_1<V_2<\ldots<V_k$. At each $V_j$ there are $n_j$ units put on test. The total number of units in the experiments is $N=\sum_{j=1}^{k} n_j$. When a type-II censoring is adopted at each stress level, the experiment terminates once the number of failure $r_j$ selected of observations $n_j$ are reached. The lifetime $t_{ij}$ at stress $V_j$, i=1,.. .$r_j$ and j=1,..k, are assumed the two parameters BS distribution with the density:

$$f(t_{ij};\alpha,\beta_j)=\frac{1}{2\alpha\sqrt{2\pi\beta_j}}\frac{(t_{ij}+\beta_j)}{(t_{ij})^{\frac{3}{2}}}exp\left(-\frac{1}{2\alpha^2}\left(\frac{t_{ij}}{\beta_j}+\frac{\beta_j}{t_{ij}}-2\right)\right),t_{ij}\geq 0 \quad (5)$$

The likelihood function of the experiment is considered to have the following form:

$$L(\alpha,C,P)=\prod_{j=1}^{k}\left[\frac{n_j!}{(n_j-r_j)!}\right]\left[\prod_{i=1}^{r_j}f\left(t_{ij};\alpha,C,P\right)\right]\left[1-F\left(t_{(r_jj)}\right)\right]^{n_j-r_j} \quad (6)$$

From a statistical point of view, the method of maximum likelihood yields estimators with good statistical properties. Using lnL to denote the natural logarithm of L $(\alpha,C,P)$, then we have

$$lnL=constant-\sum_{j=1}^{k}r_j\left[ln\alpha+\frac{lnC}{2}\right]+\frac{P}{2}\sum_{j=1}^{k}r_j\,lnV_j+\sum_{j=1}^{k}\sum_{i=1}^{r_j}ln\,(t_{ij}+CV_j^{-P})$$

$$-\frac{1}{2\alpha^2}\sum_{j=1}^{k}\sum_{i=1}^{r_j}\left(\frac{t_{ij}}{CV_j^{-P}}+\frac{CV_j^{-P}}{t_{ij}}-2\right)+\sum_{j=1}^{k}(n_j-r_j)W(r_jj) \quad (7)$$

Where $W(r_jj)=\ln\left[1-\Phi\left(\omega_{(r_jj)}\right)\right]$ , $\omega_{(r_jj)}=\frac{1}{\alpha}\left(\sqrt{\frac{t_{(r_jj)}}{CV_j^{-P}}}-\sqrt{\frac{CV_j^{-P}}{t_{(r_jj)}}}\right)$

MLEs of α, C and P are obtained by getting the partial derivatives of lnL with respect to α, C and P respectively , as follows:

$$\frac{\partial lnL}{\partial\alpha}=-\sum_{j=1}^{k}r_j\left(\frac{1}{\alpha}\right)+\left(\frac{1}{\alpha^3}\right)\sum_{j=1}^{k}\sum_{i=1}^{r_j}\left(\frac{t_{ij}}{CV_j^{-P}}+\frac{CV_j^{-P}}{t_{ij}}-2\right)+\sum_{j=1}^{k}(n_j-r_j)\frac{\partial W(r_jj)}{\partial\alpha} \quad (8)$$

$$\frac{\partial lnL}{\partial C}=-\sum_{j=1}^{k}r_j\left(\frac{1}{2C}\right)+\sum_{j=1}^{k}\sum_{i=1}^{r_j}\frac{V_j^{-P}}{(t_{ij}+CV_j^{-P})}-\frac{1}{2\alpha^2}\sum_{j=1}^{k}\sum_{i=1}^{r_j}\left(\frac{V_j^{-P}}{t_{ij}}-\frac{t_{ij}}{C^2V_j^{-P}}\right)+\sum_{j=1}^{k}(n_j-r_j)\frac{\partial W(r_jj)}{\partial C} \quad (9)$$

$$\frac{\partial lnL}{\partial P}=\frac{1}{2}\sum_{j=1}^{k}r_j\ln V_J-\sum_{j=1}^{k}\ln V_J\sum_{i=1}^{r_j}\frac{C}{\left(t_{ij}V_j^{P}+C\right)}-\frac{1}{2\alpha^2}\sum_{j=1}^{k}\ln V_j\sum_{i=1}^{r_j}\left(\frac{t_{ij}}{CV_j^{-P}}-\frac{CV_j^{-P}}{t_{ij}}\right)+\sum_{j=1}^{k}(n_j-r_j)\frac{\partial W(r_jj)}{\partial P} \quad (10)$$

$Where\ :\ \frac{\partial W(r_jj)}{\partial \alpha}=H\left(\omega_{(r_jj)}\right)\frac{1}{\alpha}\left(\omega_{(r_jj)}\right)$

$$\frac{\partial W(r_jj)}{\partial C}=H\left(\omega_{(r_jj)}\right)\frac{1}{2\alpha}\left(\sqrt{\frac{t_{(r_jj)}}{C^3V_j^{-P}}}+\sqrt{\frac{V_j^{-P}}{Ct_{(r_jj)}}}\right)$$

$$\frac{\partial w(r_jj)}{\partial P}=H\left(\omega_{(r_jj)}\right)\frac{-(\ln V_J)}{2\alpha}\left(\sqrt{\frac{t_{(r_jj)}}{CV_j^{-P}}}+\sqrt{\frac{CV_j^{-P}}{t_{(r_jj)}}}\right)$$

$$Such\ that:\ H(y)=\frac{\emptyset(y)}{1-\Phi(y)}$$

To obtain $\widehat{\alpha}$, $\widehat{C}$ and $\widehat{P}$ at which the log likelihood function is maximized, equating the equations (8-10) to zero, Since the closed form solution to thesis equations do not exist. The Newton-Raphson iteration will be used to solve these equations numerically. To estimate the variance-covariance matrix of the estimated parameters, we use the second derivatives of the logarithm of the likelihood function defined in equation (6).The second derivatives are used to get the information matrix, and hence the asymptotic variance-covariance matrix is its inverse. The Fisher-information can be expressed in the form:

$$I=\begin{bmatrix}\frac{-\partial^2 lnL}{\partial\alpha^2} & \frac{-\partial^2 lnL}{\partial\alpha\partial C} & \frac{-\partial^2 lnL}{\partial\alpha\partial P}\\ \frac{-\partial^2 lnL}{\partial\alpha\partial C} & \frac{-\partial^2 lnL}{\partial C^2} & \frac{-\partial^2 lnL}{\partial C\partial P}\\ \frac{-\partial^2 lnL}{\partial\alpha\partial P} & \frac{-\partial^2 lnL}{\partial C\partial P} & \frac{-\partial^2 lnL}{\partial P^2}\end{bmatrix} \quad (11)$$

The inverse of I is the asymptotic variance-covariance matrix of $\left(\widehat{\alpha}, \widehat{C} \text{ and } \widehat{P}\right)$.

$$v=I^{-1} \quad (12)$$

Then, the second partial derivates are given as follows:

$$\frac{\partial^2 lnL}{\partial\alpha^2}=\sum_{j=1}^{k}r_j\frac{1}{\alpha^2}-\frac{3}{\alpha^4}\sum_{j=1}^{k}\sum_{i=1}^{r_j}\left(\frac{t_{ij}}{CV_j^{-P}}+\frac{CV_j^{-P}}{t_{ij}}-2\right)+\sum_{j=1}^{k}(n_j-r_j)\frac{\partial^2 W(r_jj)}{\partial\alpha^2} \quad (13)$$

$$\frac{\partial^2 lnL}{\partial C^2}=\sum_{j=1}^{k}r_j\frac{1}{2C^2}-\sum_{j=1}^{k}\sum_{i=1}^{r_j}\left(\frac{1}{t_{ij}V_j^{P}+C}\right)^2-\frac{1}{\alpha^2}\sum_{j=1}^{k}\sum_{i=1}^{r_j}\left(\frac{t_{ij}}{C^3V_j^{-P}}\right)+\sum_{j=1}^{k}(n_j-r_j)\frac{\partial^2 W(r_jj)}{\partial C^2} \quad (14)$$

$$\frac{\partial^2 lnL}{\partial P^2} = C\sum_{j=1}^{k}\left(\ln V_j\right)^2\sum_{i=1}^{r_j}\left(\frac{t_{ij}V_j^P}{\left(t_{ij}V_j^P + C\right)^2}\right) - \frac{1}{2\alpha^2}\sum_{j=1}^{k}\left(\ln V_j\right)^2\sum_{i=1}^{r_j}\left(\frac{t_{ij}}{CV_j^{-P}} + \frac{CV_j^{-P}}{t_{ij}}\right) + \sum_{j=1}^{k}(n_j - r_j)\frac{\partial^2 W(r_j j)}{\partial P^2} \tag{15}$$

$$\frac{\partial^2 lnL}{\partial\alpha\partial C} = \frac{1}{\alpha^3}\sum_{j=1}^{k}\sum_{i=1}^{r_j}\left(\frac{V_j^{-P}}{t_{ij}} - \frac{t_{ij}}{C^2V_j^{-P}}\right) + \sum_{j=1}^{k}(n_j - r_j)\frac{\partial^2 W(r_j j)}{\partial\alpha\partial C} \tag{16}$$

$$\frac{\partial^2 lnL}{\partial\alpha\partial P} = \frac{1}{\alpha^3}\sum_{j=1}^{k}\left(\ln V_j\right)\sum_{i=1}^{r_j}\left(\frac{t_{ij}}{CV_j^{-P}} - \frac{CV_j^{-P}}{t_{ij}}\right) + \sum_{j=1}^{k}(n_j - r_j)\frac{\partial^2 W(r_j j)}{\partial\alpha\partial P} \tag{17}$$

$$\frac{\partial^2 lnL}{\partial C\partial P} = -\sum_{j=1}^{k}\ln V_j - \sum_{i=1}^{r_j}\frac{t_{ij}V_j^P}{\left(t_{ij}V_j^P + C\right)^2} + \frac{1}{2\alpha^2}\sum_{j=1}^{k}\left(\ln V_j\right)\sum_{i=1}^{r_j}\left(\frac{V_j^{-P}}{t_{ij}} + \frac{t_{ij}}{C^2V_j^{-P}}\right) + \sum_{j=1}^{k}(n_j - r_j)\frac{\partial^2 W(r_j j)}{\partial C\partial P} \tag{18}$$

*Where* :

$$\frac{\partial^2 W(r_j j)}{\partial\alpha^2} = H^`\left(\omega_{(r_j j)}\right)\frac{1}{\alpha}\left(\omega_{(r_j j)}\right) - H\left(\omega_{(r_j j)}\right)\frac{1}{\alpha^2}\left(\omega_{(r_j j)}\right)$$

$$\frac{\partial^2 W(r_j j)}{\partial C^2} = H^`\left(\omega_{(r_j j)}\right)\frac{1}{2\alpha}\left(\sqrt{\frac{t_{(r_j j)}}{C^3V_j^{-P}}} + \sqrt{\frac{V_j^{-P}}{Ct_{(r_j j)}}}\right) - H\left(\omega_{(r_j j)}\right)\frac{1}{4\alpha}\left(3\sqrt{\frac{t_{(r_j j)}}{C^5V_j^{-P}}} + \sqrt{\frac{V_j^{-P}}{C^3t_{(r_j j)}}}\right)$$

$$\frac{\partial^2 W(r_j j)}{\partial P^2} = H^`\left(\omega_{(r_j j)}\right)\frac{-\left(\ln V_j\right)}{2\alpha}\left(\sqrt{\frac{t_{(r_j j)}}{CV_j^{-P}}} + \sqrt{\frac{CV_j^{-P}}{t_{(r_j j)}}}\right) - H\left(\omega_{(r_j j)}\right)\frac{\left(\ln V_j\right)^2}{4\alpha}\left(\sqrt{\frac{t_{(r_j j)}}{CV_j^{-P}}} - \sqrt{\frac{CV_j^{-P}}{t_{(r_j j)}}}\right)$$

$$\frac{\partial^2 W(r_j j)}{\partial\alpha\partial C} = H^`\left(\omega_{(r_j j)}\right)\frac{1}{\alpha}\left(\omega_{(r_j j)}\right) - H\left(\omega_{(r_j j)}\right)\frac{1}{2\alpha}\left(\sqrt{\frac{t_{(r_j j)}}{C^3V_j^{-P}}} + \sqrt{\frac{V_j^{-P}}{Ct_{(r_j j)}}}\right)$$

$$\frac{\partial^2 W(r_j j)}{\partial\alpha\partial P} = H^`\left(\omega_{(r_j j)}\right)\frac{1}{\alpha}\left(\omega_{(r_j j)}\right) + H\left(\omega_{(r_j j)}\right)\frac{\left(\ln V_j\right)}{2\alpha}\left(\sqrt{\frac{t_{(r_j j)}}{CV_j^{-P}}} + \sqrt{\frac{CV_j^{-P}}{t_{(r_j j)}}}\right)$$

$$\frac{\partial^2 W(r_j j)}{\partial C\partial P} = H^`\left(\omega_{(r_j j)}\right)\frac{1}{2\alpha}\left(\sqrt{\frac{t_{(r_j j)}}{C^3V_j^{-P}}} + \sqrt{\frac{V_j^{-P}}{Ct_{(r_j j)}}}\right) + H\left(\omega_{(r_j j)}\right)\frac{\left(\ln V_j\right)}{4\alpha}\left(\sqrt{\frac{t_{(r_j j)}}{C^3V_j^{-P}}} - \sqrt{\frac{V_j^{-P}}{Ct_{(r_j j)}}}\right)$$

*Such that* : $H^`(y) = -yH(y) + H^2(y)$

By getting Fisher-information matrix, it can be said that the MLEs of $\alpha$, $C$ and $P$ have an asymptotic variance-covariance matrix defined by equation (12) and by substituting $\widehat{\alpha}$ for $\alpha$, $\widehat{C}$ for C and $\widehat{P}$ for P .

The approximate confidence intervals of the parameters are derived based on the asymptotic distribution of the MLEs of the elements of the vector of unknown parameters $\Theta = (\alpha, C, P)$·

It is known that the asymptotic distribution of the MLEs of $(\widehat{\Theta} - \Theta)/\sqrt{var(\widehat{\Theta})}$ can be approximated by a standard normal distribution, where $var\left(\widehat{\Theta}\right)$ is estimated as the asymptotic variance, obtained from(12). Then, the approximate $100(1-\gamma)\%$ two sided confidence interval for α, C, P are, respectively, given by:

$$\widehat{\alpha} \pm Z_{\gamma/2}\sqrt{\mathrm{var}(\widehat{\alpha})} \quad , \widehat{C} \pm Z_{\gamma/2}\sqrt{\mathrm{var}(\widehat{C})} \quad and\ \widehat{P} \pm Z_{\gamma/2}\sqrt{\mathrm{var}(\widehat{P})} \qquad (19)$$

Where $Z_{\gamma/2}$ is the $100(\gamma/2)$ standard normal percentile.

## 3. Simulation Studies

In this section simulation studies are conducted to investigate the performances of the MLEs through their absolute relative bias (RABias) and mean square error (MSE).Using the invariance property of MLEs, we can estimate the MLEs of scale parameter $\beta_j$ through the following equation

$$\hat{\beta}_j = \hat{C}V_j^{-\hat{P}} \qquad ,C,P > 0\ , \qquad j = 1,2,3 \qquad (20)$$

The Simulation procedures were described as follows:

Step1. 1000 random samples of sizes 90, 150, 210, 300 and 450 were generated from the BS distribution. Different initial values are selected for all sets of $(\alpha, C \text{ and } P)$. There are only three different levels of stress, k=3, the stress values are selected as $(V_1 = .9, V_2 = 1.4, V_3 = 1.9)$ , $n_j = \frac{n}{3}$ and $r_j = 60\% n_j$.

Step2. For all sample sizes, the parameters of the model were estimated under type-II censored samples, where $r_j = 60\% n_j$ for all sets of $(\alpha_0, C_0 \text{and } P_0)$.

Step3.All equations were solved by using the Newton-Raphson method. The estimate of the scale parameter $\beta_j$ was obtained from equation (20).

Step4. The RABias and MSE were tabulated for all sets of $(\alpha_0, C_0 \text{and } P_0)$.

Step5. The confidence limit with confidence level $(1-\gamma)=0.95$ for all sets of $(\alpha_0, C_0$ and $P_0)$ were obtained.

Step6. Using the invariance property of MLEs, the MLEs of the scale parameter $\beta_u$ of BS distribution at usual stress $V_u = .5$, can be estimated by using the following equation:

$$\hat{\beta}_u = \hat{C}.5^{-\hat{P}} \qquad ,C,P>0 \quad j=1,2,3 \tag{21}$$

Also, the MLEs of the reliability function at mission time $(t_1 = 1.5, t_2 = 1.8\,, t_3 = 2.1\ and\ t_4 = 2.4)$ are predicted under the same usual condition for all sets of parameters.

Simulation results are summarized in Tables (1-9). Tables (1-3) give the estimators, RABias and MSE. The approximate confidence intervals of the MLEs at 95% are presented in Tables (4-6). Tables (7-9) give the estimated scale parameter under $V_u = 0.5$. And the reliability function are predicted under $V_u$=0.5.

## 4. Conclusion

The classical inference procedure for the unknown parameters of the constant-stress ALT model for the BS distribution when the data are type-II censored. It is observed that the maximum likelihood estimators cannot be obtained in closed form and we have proposed to use the Newton-Raphson as an iterative method to compute them. The value of the scale parameter $\beta_j$ under usual stress is predicted and the reliability function at mission times is estimated. The performances of the estimators are investigated by simulation study and from results of Tables (1-6), it is observed the following:

1. The maximum likelihood estimators for the all sets of initial values of parameters have good statistical properties for all sample sizes. As the sample size increases the MSE of estimators decreases.
2. As the sample size increases the interval of the estimators decreases.
3. As the stress increases it is evident that the MSE of the estimated scale parameter $\beta_j$ tend to decrease.
4. As the value of $\alpha_0\ and\ C_0$ decrease, it is evident that the MSE of the estimates decrease.
5. It is evident that the MSE of the estimates do not rely on varying the value of $P_0$.

From results of Tables (7-9), it is observed the following:

1. As the sample size increases, the reliability function increases.

2. As the values of $C_0$ and $P_0$ increase at the same mission time, the reliability function increases.
3. There is an inverse proportional relationship between the shape parameter $\alpha_0$ and the reliability function.

Finally, as future work, estimation the parameters of the constant partially ALT with censoring data should be addressed.

**Table 1**
The Estimates, Relative Bias and MSE of the parameters $(\alpha, C, P, \beta_1, \beta_2, \beta_3)$ under type II censoring

| n | Parameters | $(\alpha_0 = .25, C_0 = 1.5, P_0 = 1)$ | | | $(\alpha_0 = 1, C_0 = 1.5, P_0 = 1)$ | | |
|---|---|---|---|---|---|---|---|
| | | Estimators | RABias | MSE | Estimators | RABias | MSE |
| 90 | $\alpha$ | 0.243 | 0.028 | 0.0007 | 0.968 | 0.032 | 0.013 |
| | C | 1.497 | 0.0019 | 0.0037 | 1.508 | 0.005 | 0.05 |
| | P | 0.998 | 0.0024 | 0.008 | 1.013 | 0.013 | 0.11 |
| | $\beta_1$ | 1.664 | 0.0019 | 0.006 | 1.684 | 0.011 | 0.085 |
| | $\beta_2$ | 1.07 | 0.0014 | 0.001 | 1.068 | 0.003 | 0.016 |
| | $\beta_3$ | 0.789 | 0.0002 | 0.001 | 0.79 | 0.0003 | 0.018 |
| 150 | $\alpha$ | 0.245 | 0.018 | 0.0004 | 0.984 | 0.016 | 0.007 |
| | C | 1.498 | 0.0016 | 0.002 | 1.492 | 0.006 | 0.032 |
| | P | 0.995 | 0.0047 | 0.005 | 0.993 | 0.007 | 0.064 |
| | $\beta_1$ | 1.663 | 0.0019 | 0.003 | 1.66 | 0.004 | 0.053 |
| | $\beta_2$ | 1.071 | 0.0002 | 0.0006 | 1.064 | 0.006 | 0.009 |
| | $\beta_3$ | 0.791 | 0.0015 | 0.0007 | 0.789 | 0.001 | 0.009 |
| 210 | $\alpha$ | 0.248 | 0.009 | 0.0003 | 0.994 | 0.006 | 0.005 |
| | C | 1.499 | 0.0005 | 0.001 | 1.503 | 0.002 | 0.023 |
| | P | 0.998 | 0.002 | 0.003 | 0.999 | 0.0009 | 0.049 |
| | $\beta_1$ | 1.666 | 0.0006 | 0.002 | 1.973 | 0.004 | 0.038 |
| | $\beta_2$ | 1.071 | 0.00007 | 0.0004 | 1.072 | 0.0007 | 0.007 |
| | $\beta_3$ | 0.79 | 0.0009 | 0.0005 | 0.793 | 0.004 | 0.008 |
| 300 | $\alpha$ | 0.248 | 0.007 | 0.0002 | 0.988 | 0.012 | 0.004 |
| | C | 1.499 | 0.0005 | 0.001 | 1.497 | 0.002 | 0.015 |
| | P | 0.999 | 0.0009 | 0.002 | 1.001 | 0.001 | 0.033 |
| | $\beta_1$ | 1.666 | 0.0006 | 0.002 | 1.665 | 0.0008 | 0.025 |
| | $\beta_2$ | 1.071 | 0.0003 | 0.0003 | 1.068 | 0.004 | 0.005 |
| | $\beta_3$ | 0.79 | 0.0002 | 0.0004 | 0.788 | 0.002 | 0.005 |

**Table 1 (continued)**
The Estimates, Relative Bias and MSE of the parameters $(\alpha, C, P, \beta_1, \beta_2, \beta_3)$ under type II censoring

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 450 | α | 0.249 | 0.003 | 0.0001 | 0.992 | 0.008 | 0.003 |
| | C | 1.5 | 0.0002 | 0.0007 | 1.499 | 0.0008 | 0.011 |
| | P | 0.998 | 0.002 | 0.0016 | 1 | 0.00003 | 0.023 |
| | $\beta_1$ | 1.666 | 0.0004 | 0.001 | 1.667 | 0.00009 | 0.018 |
| | $\beta_2$ | 1.072 | 0.0003 | 0.0002 | 1.07 | 0.002 | 0.003 |
| | $\beta_3$ | 0.79 | 0.0009 | 0.0002 | 0.789 | 0.0004 | 0.003 |

**Table 2**
The Estimates, Relative Bias and MSE of the parameters $(\alpha, C, P, \beta_1, \beta_2, \beta_3)$ under type II censoring

| n | Parameters | $(\alpha_0 = .25, C_0 = 1, P_0 = 1)$ | | | $(\alpha_0 = .25, C_0 = 2, P_0 = 1)$ | | |
|---|---|---|---|---|---|---|---|
| | | Estimators | RABias | MSE | Estimators | RABias | MSE |
| 90 | α | 0.243 | 0.028 | 0.0007 | 0.243 | 0.028 | 0.0007 |
| | C | 0.997 | 0.003 | 0.001 | 1.997 | 0.001 | 0.007 |
| | P | 0.999 | 0.001 | 0.008 | 1 | 0.0004 | 0.009 |
| | $\beta_1$ | 1.108 | 0.002 | 0.002 | 2.22 | 0.001 | 0.012 |
| | $\beta_2$ | 0.713 | 0.002 | 0.0005 | 1.426 | 0.002 | 0.002 |
| | $\beta_3$ | 0.525 | 0.002 | 0.0005 | 1.051 | 0.002 | 0.002 |
| 150 | α | 0.245 | 0.019 | 0.0004 | 0.247 | 0.014 | 0.0004 |
| | C | 0.998 | 0.002 | 0.0009 | 1.996 | 0.002 | 0.004 |
| | P | 0.998 | 0.002 | 0.005 | 1 | 0.0004 | 0.005 |
| | $\beta_1$ | 1.109 | 0.002 | 0.002 | 2.219 | 0.002 | 0.007 |
| | $\beta_2$ | 0.713 | 0.001 | 0.0003 | 1.426 | 0.002 | 0.001 |
| | $\beta_3$ | 0.526 | 0.0004 | 0.0003 | 1.051 | 0.001 | 0.001 |
| 210 | α | 0.247 | 0.014 | 0.0003 | 0.247 | 0.012 | 0.0003 |
| | C | 0.999 | 0.0009 | 0.0006 | 1.999 | 0.0003 | 0.003 |
| | P | 1.002 | 0.002 | 0.004 | 1..003 | 0.003 | 0.004 |
| | $\beta_1$ | 1.11 | 0.0006 | 0.001 | 2.223 | 0.0001 | 0.005 |
| | $\beta_2$ | 0.713 | 0.002 | 0.0002 | 1.427 | 0.001 | 0.0008 |
| | $\beta_3$ | 0.525 | 0.002 | 0.0002 | 1.051 | 0.002 | 0.0009 |
| 300 | α | 0.247 | 0.011 | 0.0002 | 0.248 | 0.006 | 0.0002 |
| | C | 0.999 | 0.001 | 0.0005 | 2 | 0.0001 | 0.002 |
| | P | 0.999 | 0.001 | 0.002 | 1.003 | 0.003 | 0.002 |
| | $\beta_1$ | 1.11 | 0.001 | 0.0009 | 2.223 | 0.0005 | 0.003 |
| | $\beta_2$ | 0.714 | 0.0008 | 0.0001 | 1.427 | 0.001 | 0.0006 |
| | $\beta_3$ | 0.526 | 0.0003 | 0.0001 | 1.051 | 0.002 | 0.0007 |

**Table 2 (continued)**
The Estimates, Relative Bias and MSE of the parameters $(\alpha, C, P, \beta_1, \beta_2, \beta_3)$ under type II censoring

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 450 | α | 0.248 | 0.007 | 0.0001 | 0.248 | 0.007 | 0.0001 |
| | C | 1 | 0.00002 | 0.0003 | 1.999 | 0.0004 | 0.001 |
| | P | 1 | 0.0001 | 0.002 | 1.001 | 0.0007 | 0.002 |
| | $\beta_1$ | 1.111 | 0.00002 | 0.0005 | 2.222 | 0.0003 | 0.002 |
| | $\beta_2$ | 0.714 | 0.00004 | 0.00009 | 1.428 | 0.0007 | 0.0004 |
| | $\beta_3$ | 0.526 | 0.00019 | 0.0001 | 1.052 | 0.002 | 0.0004 |

**Table 3**
The Estimates, Relative Bias and MSE of the parameters $(\alpha, C, P, \beta_1, \beta_2, \beta_3)$ under type II censoring

| n | Parameters | $(\alpha_0 = .25, C_0 = 1.5, P_0 = .5)$ | | | $(\alpha_0 = .25, C_0 = 1.5, P_0 = 1.5)$ | | |
|---|---|---|---|---|---|---|---|
| | | Estimators | RABias | MSE | Estimators | RABias | MSE |
| 90 | α | 0.242 | 0.031 | 0.0007 | 0.243 | 0.026 | 0.0008 |
| | C | 1.496 | 0.003 | 0.004 | 1.498 | 0.001 | 0.003 |
| | P | 0.499 | 0.003 | 0.009 | 1.501 | 0.0005 | 0.008 |
| | $\beta_1$ | 1.577 | 0.002 | 0.006 | 1.755 | 0.001 | 0.006 |
| | $\beta_2$ | 1.265 | 0.002 | 0.002 | 0.904 | 0.002 | 0.0008 |
| | $\beta_3$ | 1.087 | 0.001 | 0.002 | 0.572 | 0.002 | 0.0007 |
| 150 | α | 0.247 | 0.012 | 0.0004 | 0.245 | 0.018 | 0.0004 |
| | C | 1.495 | 0.003 | 0.002 | 1.497 | 0.002 | 0.002 |
| | P | 0.497 | 0.006 | 0.005 | 1.5 | 0.0003 | 0.005 |
| | $\beta_1$ | 1.576 | 0.003 | 0.003 | 1.753 | 0.002 | 0.004 |
| | $\beta_2$ | 1.265 | 0.002 | 0.0008 | 0.903 | 0.003 | 0.0004 |
| | $\beta_3$ | 1.087 | 0.001 | 0.001 | 0.571 | 0.002 | 0.0004 |
| 210 | α | 0.248 | 0.009 | 0.0003 | 0.246 | 0.014 | 0.0003 |
| | C | 1.501 | 0.0004 | 0.002 | 1.497 | 0.002 | 0.002 |
| | P | 0.502 | 0.005 | 0.004 | 1.5 | 0.0002 | 0.004 |
| | $\beta_1$ | 1.582 | 0.0008 | 0.002 | 1.754 | 0.002 | 0.003 |
| | $\beta_2$ | 1.267 | 0.0006 | 0.0006 | 0.904 | 0.002 | 0.0003 |
| | $\beta_3$ | 1.087 | 0.001 | 0.001 | 0.572 | 0.002 | 0.0003 |

**Table 3 (continued)**
The Estimates, Relative Bias and MSE of the parameters $(\alpha, C, P, \beta_1, \beta_2, \beta_3)$ under type II censoring

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 300 | $\alpha$ | 0.248 | 0.009 | 0.0002 | 0.247 | 0.01 | 0.0002 |
| | C | 1.499 | 0.0004 | 0.001 | 1.499 | 0.0006 | 0.001 |
| | P | 0.502 | 0.004 | 0.002 | 1.501 | 0.0004 | 0.002 |
| | $\beta_1$ | 1.581 | 0.0001 | 0.002 | 1.756 | 0.0004 | 0.002 |
| | $\beta_2$ | 1.266 | 0.001 | 0.0004 | 0.905 | 0.0009 | 0.0002 |
| | $\beta_3$ | 1.086 | 0.001 | 0.0007 | 0.572 | 0.0009 | 0.0002 |
| 450 | $\alpha$ | 0.248 | 0.007 | 0.0001 | 0.248 | 0.007 | 0.0001 |
| | C | 1.499 | 0.0004 | 0.0007 | 1.5 | 0.0002 | 0.0007 |
| | P | 0.5 | 0.0010 | 0.002 | 1.5 | 0.0002 | 0.002 |
| | $\beta_1$ | 1.581 | 0.0002 | 0.001 | 1.757 | 0.00009 | 0.001 |
| | $\beta_2$ | 1.267 | 0.0006 | 0.0003 | 0.905 | 0.0003 | 0.0002 |
| | $\beta_3$ | 1.088 | 0.0007 | 0.0004 | 0.573 | 0.0003 | 0.0001 |

**Table 4:** Confidence bounds of the estimates at confidence levels .95 of the parameters $(\alpha, C, P)$ under type II censoring

| n | Parameters | $(\alpha_0 = .25, C_0 = 1.5, P_0 = 1)$ | | | $(\alpha_0 = 1, C_0 = 1.5, P_0 = 1)$ | | |
|---|---|---|---|---|---|---|---|
| | | Estimators | Variance | (Lower, Upper) | Estimators | Variance | (Lower, Upper) |
| 90 | $\alpha$ | 0.243 | 0.00056 | (0.196,0.289) | 0.968 | 0.009 | (0.783,1.152) |
| | C | 1.497 | 0.002 | (1.419,1.575) | 1.508 | 0.021 | (1.226,1.79) |
| | P | 0.998 | 0.004 | (0.873,1.122) | 1.013 | 0.055 | (0.552,1.475) |
| 150 | $\alpha$ | 0.245 | 0.00034 | (0.209,0.282) | 0.984 | 0.006 | (0.837,1.131) |
| | C | 1.498 | 0.001 | (1.436,1.559) | 1.492 | 0.013 | (1.267,1.716) |
| | P | 0.995 | 0.0024 | (0.897,1.093) | 0.993 | 0.035 | (0.628,1.359) |
| 210 | $\alpha$ | 0.248 | 0.00026 | (0.216,0.279) | 0.994 | 0.004 | (0.868,1.119) |
| | C | 1.499 | 0.0007 | (1.446,1.552) | 1.503 | 0.009 | (1.309,1.698) |
| | P | 0.998 | 0.0018 | (0.914,1.082) | 0.999 | 0.025 | (0.687,1.311) |
| 300 | $\alpha$ | 0.248 | 0.00018 | (0.222,0.275) | 0.988 | 0.003 | (0.883,1.093) |
| | C | 1.499 | 0.0005 | (1.455,1.544) | 1.497 | 0.007 | (1.334,1.66) |
| | P | 0.999 | 0.0013 | (0.929,1.069) | 1.001 | 0.018 | (0.74,1.262) |
| 450 | $\alpha$ | 0.249 | 0.00012 | (0.228,0.271) | 0.992 | 0.002 | (0.906,1.079) |
| | C | 1.5 | 0.0003 | (1.463,1.536) | 1.499 | 0.005 | (1.365,1.633) |
| | P | 0.998 | 0.0009 | (0.94,1.056) | 1 | 0.012 | (0.786,1.214) |

**Table 5**
Confidence bounds of the estimates at confidence levels .95 of the parameters $(\alpha, C, P)$ under type II censoring

| n | Parameters | $(\alpha_0 = .25, C_0 = 1, P_0 = 1)$ | | | $(\alpha_0 = .25, C_0 = 2, P_0 = 1)$ | | |
|---|---|---|---|---|---|---|---|
| | | Estimators | Variance | (Lower, Upper) | Estimators | Variance | (Lower, Upper) |
| 90 | α | 0.243 | 0.0006 | (0.197,0.289) | 0.243 | 0.0006 | (0.197,0.289) |
| | C | 0.997 | 0.0007 | (0.945,1.05) | 1.997 | 0.003 | (1.893,2.102) |
| | P | 0.999 | 0.004 | (0.874,1.123) | 1 | 0.004 | (0.876,1.125) |
| 150 | α | 0.245 | 0.0003 | (0.209,0.282) | 0.247 | 0.0004 | (0.210,0.283) |
| | C | 0.998 | 0.0004 | (0.957,1.039) | 1.996 | 0.002 | (1.914,2.079) |
| | P | 0.998 | 0.003 | (0.9,1.096) | 1 | 0.003 | (0.901,1.098) |
| 210 | α | 0.247 | 0.0002 | (0.215,0.278) | 0.247 | 0.0003 | (0.216,0.278) |
| | C | 0.999 | 0.0003 | (0.964,1.034) | 1.999 | 0.001 | (1.929,2.07) |
| | P | 1.002 | 0.002 | (0.918,1.085) | 1..003 | 0.002 | (0.919,1.086) |
| 300 | α | 0.247 | 0.0002 | (0.221,0.273) | 0.248 | 0.0002 | (0.222,0.275) |
| | C | 0.999 | 0.0002 | (0.969,1.028) | 2 | 0.0009 | (1.941,2.06) |
| | P | 0.999 | 0.001 | (0.929,1.069) | 1.003 | 0.001 | 0.933,1.074) |
| 450 | α | 0.248 | 0.0001 | (0.227,0.27) | 0.248 | 0.0001 | (0.227,0.270) |
| | C | 1 | 0.0001 | (0.976,1.024) | 1.999 | 0.0006 | (1.951,2.048) |
| | P | 1 | 0.0009 | (0.942,1.057) | 1.001 | 0.0009 | (0.943,1.058) |

**Table 6**
Confidence bounds of the estimates at confidence levels .95 of the parameters $(\alpha, C, P)$ under type II censoring

| n | Parameters | $(\alpha_0 = .25, C_0 = 1.5, P_0 = .5)$ | | | $(\alpha_0 = .25, C_0 = 1.5, P_0 = 1.5)$ | | |
|---|---|---|---|---|---|---|---|
| | | Estimators | Variance | (Lower, Upper) | Estimators | Variance | (Lower, Upper) |
| 90 | α | 0.242 | 0.0006 | (0.196,0.288) | 0.243 | 0.0006 | (0.197,0.290) |
| | C | 1.496 | 0.002 | (1.418,1.574) | 1.498 | 0.002 | (1.419,1.576) |
| | P | 0.499 | 0.004 | (0.375,0.623) | 1.501 | 0.004 | (1.376,1.625) |
| 150 | α | 0.247 | 0.0004 | (0.21,0.284) | 0.245 | 0.0004 | (0.209,0.282) |
| | C | 1.495 | 0.001 | (1.433,1.557) | 1.497 | 0.001 | (1.435,1.559) |
| | P | 0.497 | 0.003 | (0.399,0.596) | 1.5 | 0.003 | (1.403,1.598) |
| 210 | α | 0.248 | 0.0003 | (0.216,0.279) | 0.246 | 0.0003 | (0.215,0.278) |
| | C | 1.501 | 0.0007 | (1.448,1.554) | 1.497 | 0.0007 | (1.445,1.55) |
| | P | 0.502 | 0.002 | (0.419,0.586) | 1.5 | 0.002 | (1.417,1.584) |
| 300 | α | 0.248 | 0.0002 | (0.221,0.274) | 0.247 | 0.0002 | (0.221,0.274) |
| | C | 1.499 | 0.0005 | (1.455,1.544) | 1.499 | 0.0005 | (1.455,1.544) |
| | P | 0.502 | 0.001 | 0.432,0.572) | 1.501 | 0.001 | (1.431,1.571) |

**Table 6 (continued)**
Confidence bounds of the estimates at confidence levels .95 of the parameters $(\alpha, C, P)$ under type II censoring

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 450 | α | 0.248 | 0.0001 | (0.227,0.270) | 0.248 | 0.0001 | (0.227,0.270) |
| | C | 1.499 | 0.0003 | (1.463,1.536) | 1.5 | 0.0003 | (1.463,1.536) |
| | P | 0.5 | 0.0009 | (0.443,0.558) | 1.5 | 0.0009 | (1.443,1.558) |

**Table 7**
Estimated shape parameter and the reliability function at $V_u = .5$ under type II censoring

| | $(\alpha_0 = .25, C_0 = 1.5, P_0 = 1)$ | | | | | $(\alpha_0 = 1, C_0 = 1.5, P_0 = 1)$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 90 | 150 | 210 | 300 | 450 | 90 | 150 | 210 | 300 | 450 |
| $\hat{\beta}_u$ | 3 | 2.992 | 2.999 | 2.999 | 2.998 | 3.192 | 3.057 | 3.07 | 3.039 | 3.028 |
| $\hat{R}_u(t$ | .996 | .997 | .997 | .997 | .997 | .757 | .751 | .756 | .758 | .759 |
| $\hat{R}_u(t$ | .975 | .977 | .978 | .979 | .979 | .695 | .688 | .693 | .695 | .696 |
| $\hat{R}_u(t$ | .913 | .917 | .919 | .921 | .921 | .639 | .631 | .637 | .638 | .638 |
| $\hat{R}_u(t$ | .80 | .804 | .808 | .81 | .81 | .587 | .58 | .583 | .586 | .587 |

**Table 8**
Estimated shape parameter and the reliability function at $V_u = .5$ under type II censoring

| | $(\alpha_0 = .25, C_0 = 1, P_0 = 1)$ | | | | | $(\alpha_0 = .25, C_0 = 2, P_0 = 1)$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 90 | 150 | 210 | 300 | 450 | 90 | 150 | 210 | 300 | 450 |
| $\hat{\beta}_u$ | 2 | 1.997 | 2.004 | 1.998 | 2.001 | 4.011 | 4.001 | 4.013 | 4.014 | 4.003 |
| $\hat{R}_u(t$ | .865 | .867 | .873 | .872 | .874 | 1 | 1 | 1 | 1 | 1 |
| $\hat{R}_u(t$ | .652 | .654 | .661 | .658 | .662 | .999 | .999 | .999 | .999 | .999 |
| $\hat{R}_u(t$ | .417 | .417 | .423 | .419 | .422 | .993 | .994 | .995 | .995 | .995 |
| $\hat{R}_u(t$ | .234 | .232 | .235 | .231 | .233 | .975 | .977 | .978 | .979 | .98 |

**Table 9**
Estimated shape parameter and the reliability function at $V_u = .5$ under type II censoring

| | $(\alpha_0 = .25, C_0 = 1.5, P_0 = .5)$ | | | | | $(\alpha_0 = .25, C_0 = 1.5, P_0 = 1.5)$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 90 | 150 | 210 | 300 | 450 | 90 | 150 | 210 | 300 | 450 |
| $\hat{\beta}_u$ | 2.122 | 2.115 | 2.129 | 2.126 | 2.123 | 4.254 | 4.244 | 4.243 | 4.246 | 4.246 |
| $\hat{R}_u(t$ | 0.907 | 0.909 | 0.915 | 0.916 | 0.917 | 1 | 1 | 1 | 1 | 1 |
| $\hat{R}_u(t$ | 0.731 | 0.732 | 0.743 | 0.743 | 0.743 | 0.999 | 1 | 1 | 1 | 1 |
| $\hat{R}_u(t$ | 0.507 | 0.507 | 0.518 | 0.517 | 0.515 | 0.996 | 0.997 | .997 | 0.998 | 0.998 |
| $\hat{R}_u(t$ | 0.309 | 0.307 | 0.316 | 0.313 | 0.311 | 0.986 | 0.987 | 0.988 | 0.989 | 0.989 |

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