Int. J. Contemp. Math. Sciences, Vol. 8, 2013, no. 2, 93 - 116
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Fuzzy Linear Programming Model for Critical Path Analysis

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Abstract

In this paper, a new fuzzy linear programming model is proposed to find fuzzy critical path and fuzzy completion time of a fuzzy project. All the activities in the project network are represented by trapezoidal fuzzy numbers. A new representation of trapezoidal fuzzy number is introduced to reduce the constraints in the fuzzy linear programming model. Further, an example is illustrated which shows the advantages of using the proposed representation over the existing representation of trapezoidal fuzzy numbers and will present with great clarity the proposed technique and illustrate its application for fuzzy critical path problems occurring in real life situations.

Keywords: Fuzzy Critical Path Problem; Ranking function; Trapezoidal fuzzy number; LR fuzzy number; linear programming

1. Introduction

In today's highly competitive business environment, project management's ability to schedule activities and monitor progress strictly in cost, time and performance guidelines is becoming increasingly important to obtain competitive priorities such as on-time delivery and customization. In many situations, projects can be complicated and challenging to manage. Critical Path Method (CPM) is to identify critical activities on the critical path so that resources may be concentrated on these activities in order to reduce the project length time. Further implementation of CPM requires the availability of clear determined time duration for each activity. To deal with real life situations, Zadeh[1] introduced the concept of fuzzy set. There is always uncertainty in the network planning of time duration of activities. So, fuzzy critical path method introduced in late 1970s. So many approaches are proposed over the past years to find the fuzzy critical path. The first method called Fuzzy Programming Evaluation and review Technique (FPERT) was proposed by Chanas and Kamburowski [2]. They presented the project completion time in the form of fuzzy set in the time space. In paper [3], Gazdik developed a fuzzy network of unknown project to estimate the activity durations and used fuzzy algebraic operations to calculate the duration of the project and its critical path. A chapter of the book [4] is devoted to the critical path method in which activity times are represented by triangular fuzzy numbers. A new methodology to calculate the fuzzy completion project time is presented in the paper [5]. Nasution [6] proposed a method to compute total floats and the critical paths in a fuzzy project network. Yao and Lin [7] proposed a method for ranking fuzzy numbers without the need for any assumptions and have used both positive and negative fuzzy values to define ordering an it is applied to fuzzy project network. In [8], Dubois et al. extended the fuzzy arithmetic operational model to compute the latest starting time of each activity in a fuzzy project network. Chen[11] proposed an approach based on the extension principle and linear programming formulation to critical path analysis in fuzzy project networks. In [12], Chen and Hsueh presented a simple approach to solve the Fuzzy CPM problems on the basis of linear programming formulation and the fuzzy number ranking method that are more realistic than crisp ones. The problems of determining possible values of earliest and latest starting times of an activity in fuzzy project networks with minimal time intervals and vague durations that are presented by means of fuzzy interval or fuzzy numbers is introduced in the paper[13]. Kumar and Kaur [9] proposed a new fuzzy linear programming model using Jai Mehar Di (JMD) representation of triangular fuzzy numbers. In this paper, we propose a novel fuzzy linear programming model using new representation of trapezoidal fuzzy number. It is observed that the use of proposed representation is better than the existing representations of trapezoidal fuzzy numbers, to find the fuzzy optimal solution of fuzzy critical path problems. To illustrate the fuzzy linear programming model and to show the advantages of the proposed representation of trapezoidal fuzzy numbers, a numerical example is evaluated by representing all the constraints as existing and proposed type of trapezoidal fuzzy numbers.

In section 2, the background Information is presented. In that, basic definitions, existing representations of trapezoidal and ranking function are reviewed. In the section 3, linear programming of crisp critical path problems and also the linear programming formulation of fuzzy critical path problems are presented. The section 4 follows the method to find the fuzzy optimal solution of fuzzy critical path problems[9]. In section 5, a new representation of trapezoidal fuzzy number is mentioned. To illustrate the proposed method, a numerical example is taken and solved in section 6. Comparision study of existing model and proposed model is presented in section 7.

2. **Background information**

In this section, basic definitions of fuzzy sets and existing representations of trapezoidal fuzzy numbers, arithmetic operations and ranking functions are reviewed.

2.1. Basic definitions

In this section, some basic definition are reviewed [10].

Definition 1: The Characterstic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the Universal set X fall within a particular range i.e. $\mu_{\tilde{A}} : X \to [0,1]$. The allocated value indicates the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for each $x \in X$ is called a fuzzy set.

Definition 2: A fuzzy set $\tilde{A}$, defined on the Universal set of real numbers **R**, is said to be a fuzzy number if its membership function has the following characteristics:

(i) $\tilde{A}$ is convex i.e.,

$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \text{minimum}(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \quad \forall x_1, x_2 \in R, \forall \lambda \in [0,1]$.

(ii) $\tilde{A}$ is normal i.e., $\exists\, x_0 \in R$, such that $\mu_{\tilde{A}}(x_0) = 1$.

(iii) $\mu_{\tilde{A}}(x)$ is piecewise continuous.

Definition 3: A fuzzy number $\tilde{A}$, is called non-negative fuzzy number if $\mu_{\tilde{A}}(x) = 0 \; \forall x < 0$.

2.2. Existing representation of trapezoidal fuzzy numbers

Different representations of trapezoidal fuzzy numbers are presented in this section.

2.2.1. General representation of trapezoidal fuzzy numbers with membership function

Definition 4: A fuzzy number $\tilde{A} = (a, b, c, d)$ where $a \leq b \leq c \leq d$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{if } -\infty < x \le a \\ \dfrac{x-a}{b-a}, & \text{if } a < x \le b \\ 1, & \text{if } b < x \le c \\ \dfrac{d-x}{d-c}, & \text{if } c < x \le d \\ 0, & \text{if } d < x < \infty \end{cases}$$

Definition 5 : A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be zero trapezoidal fuzzy number iff $a = 0, b = 0, c = 0, d = 0$.

Definition 6: A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be non-negative trapezoidal fuzzy number iff $a \ge 0$.

Definition 7: Two trapezoidal fuzzy numbers $\tilde{A} = (a_1, b_1, c_1, d_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2)$ are said to be equal i.e., $\tilde{A} = \tilde{B}$ iff $a_1 = a_2, b_1 = b_2, c_1 = c_2, d_1 = d_2$.

2.2.2. (m,n,α,β) representation of trapezoidal fuzzy numbers with membership function

A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$, defined in the section 2.2.1, may also be represented as $\tilde{A} = (m, n, \alpha, \beta)$, where $m = b, n = c, \alpha = b - a, \beta = d - c$.

Definition 8 : A fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{if } -\infty < x \le m-\alpha \\ 1 + \dfrac{m-x}{\alpha}, & \text{if } m-\alpha < x \le m \\ 1, & \text{if } m < x \le n \\ \dfrac{n-x}{\beta} + 1, & \text{if } n < x \le n+\beta \\ 0, & \text{if } n+\beta < x < \infty. \end{cases} \qquad \text{where} \quad \alpha, \beta \ge 0.$$

Definition 9: A trapezoidal fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be zero trapezoidal fuzzy number iff $m = 0, n = 0, \alpha = 0, \beta = 0$.

Definition 10: A trapezoidal fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be non-negative trapezoidal fuzzy number iff $m - \alpha \ge 0$.

Definition 11: Two trapezoidal fuzzy numbers $\widetilde{A} = (m_1, n_1, \alpha_1, \beta_1)$ and $\widetilde{B} = (m_2, n_2, \alpha_2, \beta_2)$ are said to be equal i.e., $\widetilde{A} = \widetilde{B}$ iff $m_1 = m_2, n_1 = n_2, \alpha_1 = \alpha_2, \beta_1 = \beta_2$.

2.3. Arithmetic fuzzy operations

In this section addition and multiplication operations between two trapezoidal fuzzy numbers are presented.

2.3.1. Arithmetic operations between (a, b, c, d) type trapezoidal fuzzy numbers

Let $\widetilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\widetilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers, then

(i) $\widetilde{A}_1 \oplus \widetilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$,

(ii) $\widetilde{A}_1 \otimes \widetilde{A}_2 = (a', b', c', d')$, where

$$a' = \min(a_1a_2, b_1b_2, c_1c_2, d_1d_2),\ b' = b_1b_2,\ c' = c_1c_2,$$
$$d' = \max(a_1a_2, b_1b_2, c_1c_2, d_1d_2).$$

2.3.2. Arithmetic Operations between (m, n, α, β) type trapezoidal fuzzy numbers

Let $\widetilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)$ and $\widetilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)$ be two trapezoidal fuzzy numbers, then

(i) $\widetilde{A}_1 \oplus \widetilde{A}_2 = (m_1 + m_2, n_1 + n_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)$,

(ii) $\widetilde{A}_1 \otimes \widetilde{A}_2 = (m', n', \alpha', \beta')$, where $m' = m_1m_2$, $n' = n_1n_2$, $\alpha' = m' - \min(d')$, $\beta' = \max(d') - n'$, and

$$d' = \begin{pmatrix} m_1m_2 - m_1\alpha_2 - m_2\alpha_1 + \alpha_1\alpha_2, m_1m_2 + m_1\beta_2 - m_2\alpha_1 + \alpha_1\beta_1, \\ n_1n_2 - n_1\alpha_2 + n_2\beta_1 - \beta_1\alpha_2, n_1n_2 + n_1\beta_2 + n_2\beta_1 + \beta_1\beta_2 \end{pmatrix}.$$

2.4. Ranking function of fuzzy number

Ranking Function is the most important to compare the fuzzy numbers to our fuzzy approach.

$\Re: F(\mathbf{R}) \rightarrow \mathbf{R}$, where F(R) is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real space, where a natural order exists.

Let (a, b, c, d) be a trapezoidal fuzzy number. Then $\Re(a, b, c, d) = \dfrac{a + b + c + d}{4}$.

Also , Let (m, n, α, β) be a trapezoidal fuzzy number. Then
$\Re(m, n, \alpha, \beta) = \frac{m+n}{2} + \frac{\beta-\alpha}{4}$.

3. Linear Programming Formulation of Crisp Critical Path and Fuzzy Critical Path Problems

The fuzzy CPM is a fuzzy project network based method designed to assist in the planning, scheduling and control of the fuzzy project. Its objective is to construct the time scheduling for the fuzzy project. Two basic results provided by fuzzy CPM are the total duration time needed to complete the fuzzy project and the fuzzy critical path. One of the efficient approaches for finding the fuzzy critical paths and total duration time of fuzzy project networks is Linear programming. The fuzzy Linear programming formulation assumes that a unit flow enters the fuzzy project network at the start vertex and leaves at the terminal vertex.
In this section, the Linear Programming Formulation of Crisp Critical Path problems is reviewed and also the Linear formulation of Fuzzy Critical Path problems is proposed.

3.1. Linear Programming of Crisp Critical Path Problems

The linear programming model discussed in the book written by Taha[14] is reviewed in this section.
Consider a project network G=(V,E,T) consisting of a finite set V={1,2,…,n} of n nodes and E is the set of activities (i,j) , T is a function from E to **R** and $t_{ij} \in$ R is the time period of activity (i,j). The Linear Programming formulation of Crisp Critical Path Problem is defined as follows:

Maximize $\sum_{(i,j)\in E} t_{ij} x_{ij}$

subject to the constraints

$$\sum_{(i,j)\in E} x_{1j} = 1,$$

$$\sum_{(i,j)\in E} x_{ij} = \sum_{(j,k)\in E} x_{jk},\ i \neq 1, k \neq n,$$

$$\sum_{(i,n)\in E} x_{in} = 1,\ x_{ij} \text{ is a non-negative real number } \forall (i,j) \in E.$$

3.2. Linear Programming Formulation of Fuzzy Critical Path Problems

Suppose t_{ij} and $x_{ij}, \forall(i,j) \in E$ are vague and are represented by fuzzy numbers $\tilde{t}_{ij}$ and $\tilde{x}_{ij}, \forall(i,j) \in E$ respectively. Then the Fuzzy Critical Path Problems may be formulated into the following fuzzy linear programming problem:

Maximize $\sum_{(i,j)\in E} \tilde{t}_{ij} \otimes \tilde{x}_{ij}$

subject to

$$\sum_{(i,j)\in E} \tilde{x}_{1j} = \tilde{1},$$

$$\sum_{(i,j)\in E} \tilde{x}_{ij} = \sum_{(j,k)\in E} \tilde{x}_{jk}, \; i \neq 1, k \neq n,$$

$$\sum_{(i,n)\in A} \tilde{x}_{in} = \tilde{1}, \; (\tilde{1} = (1,1,1,1)),$$

$\tilde{x}_{ij}$ is a non-negative real number $\forall(i,j) \in E$.

4. Method to find fuzzy critical path and fuzzy completion time of a fuzzy project

The steps of the approach are as follows [9]:

Step 1: Represent all the parameters of linear programming formulation of Fuzzy Critical Path problems by a particular type of trapezoidal fuzzy number and formulate the given problem, as proposed in the section 3.2.

Step 2: Convert the fuzzy objective function and fuzzy constraints into the crisp objective function form by its ranking function.

Step 3: Find the critical path and completion fuzzy time for the obtained crisp linear Programming problem by using Tora software.

Step 4: Convert crisp solution to fuzzy solution using solution obtained from Step 3.

Step 5: Find the critical path and corresponding maximum total completion time in fuzzy sense from Step4.

4.1. Proposed method with (a,b,c,d) representation of trapezoidal fuzzy numbers

If all the parameters of linear programming formulation of Fuzzy Critical Path problems are represented by (a,b,c,d) type trapezoidal fuzzy numbers then the steps of the proposed method are as follows:

Step1: Suppose all the parameters $\tilde{t}_{ij}$ and $\tilde{x}_{ij}$ are represented by (a,b,c,d) type trapezoidal fuzzy numbers $(t_{ij}, t'_{ij}, t''_{ij}, t'''_{ij})$ and $(x_{ij}, y_{ij}, z_{ij}, \gamma_{ij})$ respectively then the Linear Programming formulation of Fuzzy Critical Path problems, proposed in the section 3.2., this can written as:

Maximize $\sum_{(i,j)\in E}(t_{ij}, t'_{ij}, t''_{ij}, t'''_{ij}) \otimes (x_{ij}, y_{ij}, z_{ij}, \gamma_{ij})$

subject to

$$\sum_{(i,j)\in E}(x_{1j}, y_{1j}, z_{1j}, \gamma_{1j}) = (1,1,1,1),$$

$$\sum_{(i,j)\in E}(x_{ij}, y_{ij}, z_{ij}, \gamma_{ij}) = \sum_{(j,k)\in E}(x_{jk}, y_{jk}, z_{jk}, \gamma_{jk}),\ i \neq 1, k \neq n,$$

$$\sum_{(i,n)\in E}(x_{in}, y_{in}, z_{in}, \gamma_{in}) = (1,1,1,1),$$

$(x_{ij}, y_{ij}, z_{ij}, \gamma_{ij})$ is a non-negative trapezoidal fuzzy number $\forall (i,j) \in E$.

Step2: The fuzzy linear programming of Fuzzy Critical Path problems may be written as:

Maximize $\Re\left[\sum_{(i,j)\in E}(t_{ij}, t'_{ij}, t''_{ij}, t'''_{ij}) \otimes (x_{ij}, y_{ij}, z_{ij}, \gamma_{ij})\right]$

subject to the constraints

$$\sum_{(i,j)\in E}(x_{1j}, y_{1j}, z_{1j}, \gamma_{1j}) = (1,1,1,1),$$

$$\sum_{(i,j)\in E}(x_{ij}, y_{ij}, z_{ij}, \gamma_{ij}) = \sum_{(j,k)\in E}(x_{jk}, y_{jk}, z_{jk}, \gamma_{jk}),\ i \neq 1, k \neq n,$$

$$\sum_{(i,n)\in E}(x_{in}, y_{in}, z_{in}, \gamma_{in}) = (1,1,1,1),$$

$(x_{ij}, y_{ij}, z_{ij}, \gamma_{ij})$ is a non-negative trapezoidal fuzzy number $\forall (i,j) \in E$.

Now, the crisp linear programming problem becomes :

Maximize $\Re\left[\sum_{(i,j)\in E}(t_{ij}, t'_{ij}, t''_{ij}, t'''_{ij}) \otimes (x_{ij}, y_{ij}, z_{ij}, \gamma_{ij})\right]$

subject to the constraints

$$\sum_{(i,j)\in E} x_{1j} = 1,\ \sum_{j:(i,j)\in E} y_{1j} = 1,\ \sum_{j:(i,j)\in E} z_{1j} = 1,\ \sum_{j:(i,j)\in E} \gamma_{1j} = 1,$$
$$\sum_{i:(i,j)\in E} x_{ij} = \sum_{j:(j,k)\in E} x_{jk},\ i \neq 1, k \neq n,$$
$$\sum_{i:(i,j)\in E} y_{ij} = \sum_{j:(j,k)\in E} y_{jk},\ i \neq 1, k \neq n,$$
$$\sum_{i:(i,j)\in E} z_{ij} = \sum_{j:(j,k)\in E} z_{jk},\ i \neq 1, k \neq n,$$
$$\sum_{i:(i,j)\in E} \gamma_{ij} = \sum_{j:(j,k)\in E} \gamma_{jk},\ i \neq 1, k \neq n,$$
$$\sum_{i:(i,n)\in E} x_{in} = 1,\ \sum_{i:(i,n)\in E} y_{in} = 1,\ \sum_{i:(i,n)\in E} z_{in} = 1,\ \sum_{i:(i,n)\in E} \gamma_{in} = 1,$$
$$y_{ij} - x_{ij} \geq 0,\ z_{ij} - y_{ij} \geq 0,\ \gamma_{ij} - z_{ij} \geq 0,$$
$$x_{ij},\ y_{ij}, z_{ij}, \gamma_{ij} \geq 0\ \forall (i,j) \in E.$$

Step 3: Find the solution $x_{ij}, y_{ij}, z_{ij}, \gamma_{ij}$ by solving the Crisp Linear Programming problem, which is obtained in Step2.

Step 4: Find the fuzzy solution $\tilde{x}_{ij}$ by putting the values of x_{ij}, y_{ij}, z_{ij} and γ_{ij} in $\tilde{x}_{ij} = (x_{ij}, y_{ij}, z_{ij}, \gamma_{ij})$ and also calculate the maximum total completion fuzzy time by putting the values of $\tilde{x}_{ij}$ in $\sum_{(i,j)\in E} \tilde{t}_{ij} \otimes \tilde{x}_{ij}$.

Step 5: Find the fuzzy critical path by combining all the activities (i, j) such that $\tilde{x}_{ij} = (1,1,1,1)$.

4.2. Proposed method with (m, n, α, β) representation of trapezoidal fuzzy numbers

If all the parameters of Fuzzy Critical Path problems are represented by (m, n, α, β) type trapezoidal fuzzy numbers then the steps of the proposed method are as follows:

Step1: Suppose all the parameters $\tilde{t}_{ij}$ and $\tilde{x}_{ij}$ are represented by (m, n, α, β) type trapezoidal fuzzy numbers $(t'_{ij}, t''_{ij}, \lambda_{ij}, \delta_{ij})$ and $(y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij})$ respectively then the Linear Programming formulation of Fuzzy Critical Path problems, proposed in the section 3.2., this can written as:

Maximize $\sum_{(i,j)\in E} (t'_{ij}, t''_{ij}, \lambda_{ij}, \delta_{ij}) \otimes (y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij})$

subject to the constraints

$$\sum_{j:(i,j)\in E}(y_{1j}, z_{1j}, \alpha_{1j}, \beta_{1j}) = (1,1,0,0),$$
$$\sum_{i:(i,j)\in E}(y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij}) = \sum_{j:(j,k)\in E}(y_{jk}, z_{jk}, \alpha_{jk}, \beta_{jk}),\ i \neq 1, k \neq n,$$
$$\sum_{i:(i,n)\in E}(y_{in}, z_{in}, \alpha_{in}, \beta_{in}) = (1,1,0,0),$$
$(y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij})$ is a non-negative trapezoidal fuzzy number $\forall (i,j) \in E$.

Step2: Using the ranking function, presented in section 2.4., the Linear Programming of Fuzzy Critical Path problems may be written as:

Maximize $\Re\left[\sum_{(i,j)\in E}(t'_{ij}, t''_{ij}, \lambda_{ij}, \delta_{ij}) \otimes (y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij})\right]$

subject to the constraints

$$\sum_{j:(i,j)\in E}(y_{1j}, z_{1j}, \alpha_{1j}, \beta_{1j}) = (1,1,0,0),$$
$$\sum_{i:(i,j)\in E}(y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij}) = \sum_{j:(j,k)\in E}(y_{jk}, z_{jk}, \alpha_{jk}, \beta_{jk}),\ i \neq 1, k \neq n,$$
$$\sum_{i:(i,n)\in E}(y_{in}, z_{in}, \alpha_{in}, \beta_{in}) = (1,1,0,0),$$
$(y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij})$ is a non-negative trapezoidal fuzzy number $\forall (i,j) \in E$.

Now, the Crisp Linear Programming problem becomes :

Maximize $\Re\left[\sum_{(i,j)\in E}(t'_{ij}, t''_{ij}, \lambda_{ij}, \delta_{ij}) \otimes (y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij})\right]$

subject to the constraints

$$\sum_{j:(i,j)\in E} y_{1j} = 1,\ \sum_{j:(i,j)\in E} z_{1j} = 1,\ \sum_{j:(i,j)\in E} \alpha_{1j} = 0,\ \sum_{j:(i,j)\in E} \beta_{1j} = 0,$$

$$\sum_{i:(i,j)\in E} y_{ij} = \sum_{j:(j,k)\in E} y_{jk},\ i \neq 1, k \neq n,$$
$$\sum_{i:(i,j)\in E} z_{ij} = \sum_{j:(j,k)\in E} z_{jk},\ i \neq 1, k \neq n,$$
$$\sum_{i:(i,j)\in E} \alpha_{ij} = \sum_{j:(j,k)\in E} \alpha_{jk},\ i \neq 1, k \neq n,$$
$$\sum_{i:(i,j)\in E} \beta_{ij} = \sum_{j:(j,k)\in E} \beta_{jk},\ i \neq 1, k \neq n,$$
$$\sum_{i:(i,n)\in E} y_{in} = 1,\ \sum_{i:(i,n)\in E} z_{in} = 1,\ \sum_{i:(i,n)\in E} \alpha_{in} = 0,\ \sum_{i:(i,n)\in E} \beta_{in} = 0$$
$$z_{ij} - y_{ij} \geq 0, \alpha_{ij} - z_{ij} \geq 0,$$

$y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij} \geq 0 \ \forall (i,j) \in E$.

Step 3: Find the solution $y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij}$ by solving the Crisp Linear Programming problem, which is obtained in Step2.

Step 4: Find the fuzzy solution $\tilde{x}_{ij}$ by putting the values of y_{ij}, z_{ij} α_{ij} and β_{ij} in $\tilde{x}_{ij} = (y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij})$ .and also calculate the maximum total completion fuzzy time by putting the values of $\tilde{x}_{ij}$ in $\sum_{(i,j)\in E} \tilde{t}_{ij} \otimes \tilde{x}_{ij}$.

Step 5: Find the fuzzy critical path by combining all the activities (i, j) such that $\tilde{x}_{ij} = (1,1,0,0)$.

5. A Novel representation of trapezoidal fuzzy numbers

In this section, a new representation of trapezoidal fuzzy numbers , Arithmetic operations and new ranking function is defined. Using this new representation, fuzzy linear programming model is generated and presented a method to find fuzzy critical path and maximum total fuzzy completion time.

Definition12. Let (a, b, c, d) be a trapezoidal fuzzy number then its New representation is $(x, y, \alpha, \beta)_{New}$, where $x = a, y = b, \alpha = b - a \geq 0$ and $\beta = d - c \geq 0$

Definition13. Let (m, n, α, β) be a trapezoidal fuzzy number then its New representation is $(x, y, \alpha, \beta)_{New}$, where $x = m - \alpha,\ y = n + \beta$.

Definition14. A trapezoidal fuzzy number $\tilde{A} = (x, y, \alpha, \beta)_{New}$ is said to be zero trapezoidal fuzzy number iff $x = 0, y = 0, \alpha = 0, ,\beta = 0$.

Definition 15. A trapezoidal fuzzy number $\tilde{A} = (x, y, \alpha, \beta)_{New}$ is said to be non-negative trapezoidal fuzzy number iff $x \geq 0, y \geq 0$.

Definition 16. Two trapezoidal fuzzy numbers $\tilde{A} = (x_1, y_1, \alpha_1, \beta_1)_{New}$ $\tilde{B} = (x_2, y_2, \alpha_2, \beta_2)_{New,}$ are said to be equal iff $x_1 = x_2, y_1 = y_2, \alpha_1 = \alpha_2, \beta_1 = \beta_2$.

5.1. Arithmetic operation between new trapezoidal fuzzy numbers

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers and $\tilde{A}_1 = (x_1, y_1, \alpha_1, \beta_1)_{New}$ $\tilde{A}_2 = (x_2, y_2, \alpha_2, \beta_2)_{New}$ be their new representation. Then the addition and multiplication operations are defined as follows :

(i) $\widetilde{A}_1 \oplus \widetilde{A}_2 = (x_1 + x_2, y_1 + y_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)$, and

(ii) $\widetilde{A}_1 \otimes \widetilde{A}_2 = (x_3, y_3, \alpha_3, \beta_3)$, where

$x_3 = \text{minimum}(a_1a_2, b_1b_2, c_1c_2, d_1d_2)$, $b' = b_1b_2$, $c' = c_1c_2$,

$d' = \text{maximum}(a_1a_2, b_1b_2, c_1c_2, d_1d_2)$.

5.2. Ranking function for new trapezoidal fuzzy numbers

Let $(x, y, \alpha, \beta)_{New}$ be a trapezoidal fuzzy number then ranking function

$$\Re(x, y, \alpha, \beta)_{New} = \frac{x+y}{2} + \frac{\beta - \alpha}{4}.$$

5.3. Proposed method with $(x,y,\alpha,\beta)_{New}$ representation of trapezoidal fuzzy numbers

In this all the parameters of Fuzzy Critical Path problems are represented by $(x, y, \alpha, \beta)_{New}$ trapezoidal fuzzy numbers then the steps of the proposed method are as follows:

Step 1: Suppose all the parameters $\widetilde{t}_{ij}$ and $\widetilde{x}_{ij}$ are represented by $(x, y, \alpha, \beta)_{New}$, trapezoidal fuzzy numbers $(t'_{ij}, t''_{ij}, \lambda_{ij}, \delta_{ij})$ and $(x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij})_{New}$, respectively then the Linear Programming formulation of Fuzzy Critical Path problems, proposed in the section 3.2., this can be written as:

Maximize $\sum_{(i,j)\in A} (t'_{ij}, t''_{ij}, \lambda_{ij}, \delta_{ij}) \otimes (x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij})$

subject to the constraints

$$\sum_{j:(i,j)\in A} (x_{1j}, y_{1j}, \alpha_{1j}, \beta_{1j}) = (1,1,0,0),$$

$$\sum_{i:(i,j)\in A} (x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij}) = \sum_{j:(j,k)\in A} (x_{jk}, y_{jk}, \alpha_{jk}, \beta_{jk}),\ i \neq 1, k \neq n,$$

$$\sum_{i:(i,n)\in A} (x_{in}, y_{in}, \alpha_{in}, \beta_{in}) = (1,1,0,0),$$

$(x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij})$ is a non-negative trapezoidal fuzzy number $\forall (i,j) \in A$.

Step 2: Using the ranking function presented in section 2.4., the Linear Programming of Fuzzy Critical Path problems may be written as:

Maximize $\Re\left[\sum_{(i,j)\in E} (t'_{ij}, t''_{ij}, \lambda_{ij}, \delta_{ij}) \otimes (x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij})\right]$

subject to the constraints

$$\sum_{j:(i,j)\in E}(x_{1j}, y_{1j}, \alpha_{1j}, \beta_{1j}) = (1,1,0,0),$$
$$\sum_{i:(i,j)\in E}(x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij}) = \sum_{j:(j,k)\in E}(x_{jk}, y_{jk}, \alpha_{jk}, \beta_{jk}),\ i \neq 1, k \neq n,$$
$\sum_{i:(i,n)\in E}(x_{in}, y_{in}, \alpha_{in}, \beta_{in}) = (1,1,0,0)$, $(x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij})$ is a non-negative trapezoidal fuzzy number $\forall (i,j) \in E$.

The Crisp Linear Programming problem becomes :

$$\text{Maximize } \Re\left[\sum_{(i,j)\in E}(t'_{ij}, t''_{ij}, \lambda_{ij}, \delta_{ij}) \otimes (x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij})\right]$$

subject to the constraints

$$\sum_{j:(i,j)\in E} x_{1j} = 1,\ \sum_{j:(i,j)\in E} y_{1j} = 1,\ \sum_{j:(i,j)\in E} \alpha_{1j} = 0,\ \sum_{j:(i,j)\in E} \beta_{1j} = 0,$$

$$\sum_{i:(i,j)\in E} x_{ij} = \sum_{j:(j,k)\in E} x_{jk},\ i \neq 1, k \neq n,$$
$$\sum_{i:(i,j)\in E} y_{ij} = \sum_{j:(j,k)\in E} y_{jk},\ i \neq 1, k \neq n,$$
$$\sum_{i:(i,j)\in E} \alpha_{ij} = \sum_{j:(j,k)\in E} \alpha_{jk},\ i \neq 1, k \neq n,$$
$$\sum_{i:(i,j)\in E} \beta_{ij} = \sum_{j:(j,k)\in E} \beta_{jk},\ i \neq 1, k \neq n,$$
$$\sum_{i:(i,n)\in E} x_{in} = 1,\ \sum_{i:(i,n)\in E} y_{in} = 1,\ \sum_{i:(i,n)\in E} \alpha_{in} = 0,\ \sum_{i:(i,n)\in E} \beta_{in} = 0$$
$$x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij} \geq 0\ \forall (i,j) \in E.$$

Step 3: Find the optimal solution $x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij}$ by solving the Crisp Linear Programming problem, which is obtained in Step2.

Step 4: Find the fuzzy optimal solution $\tilde{x}_{ij}$ by putting the values of $x_{ij}, y_{ij}, \alpha_{ij}$ and β_{ij} in $\tilde{x}_{ij} = (x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij})$ and also find the maximum total completion fuzzy time by putting the values of $\tilde{x}_{ij}$ in $\sum_{(i,j)\in E} \tilde{t}_{ij} \otimes \tilde{x}_{ij}$.

Step 5: Find the fuzzy critical path by combining all the activities (i,j) such that $\tilde{x}_{ij} = (1,1,0,0)$.

6. Numerical Example

To show the advantages of New representation over existing representation of fuzzy numbers, the same numerical example is solved by using all three representation of fuzzy numbers. The problem is to find the fuzzy critical path and maximum total completion fuzzy time of the project network, shown in Fig.1, in which the fuzzy time duration of each activity is represented by the following (a,b,c,d) type trapezoidal fuzzy numbers.

$\tilde{t}_{12}$=(2,2,3,4) , $\tilde{t}_{13}$=(2,3,3,6), $\tilde{t}_{15}$=(2,3,4,5), $\tilde{t}_{24}$=(2,2,4,5),

$\tilde{t}_{25}$=(2,2,5,8), $\tilde{t}_{34}$=(1,1,2,2), $\tilde{t}_{36}$=(7,8,11,15), $\tilde{t}_{45}$=(2,3,3,5), $\tilde{t}_{46}$=(3,3,4,6), and $\tilde{t}_{56}$=(1,1,1,2).

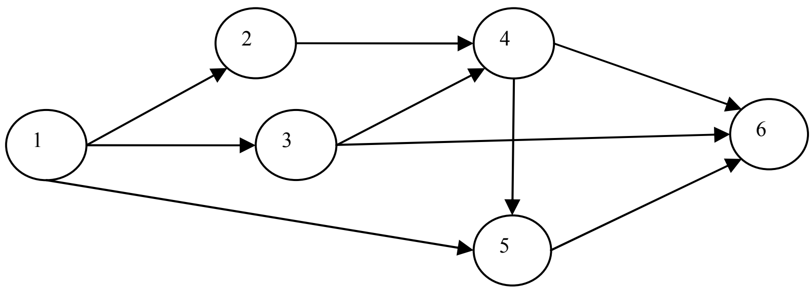

Fig.1: Project Network diagram

6.1. Fuzzy optimal solution using (a,b,c,d) representation of trapezoidal fuzzy numbers

Step1: Using the section 4.1, the given problem may be formulated as follows:

Maximize((2,2,3,4)$\otimes(x_{12},y_{12},z_{12},\gamma_{12})\oplus$(2,3,3,6)$\otimes(x_{13},y_{13},z_{13},\gamma_{13})\oplus$(2,3,4,5) $\otimes(x_{15},y_{15},z_{15},\gamma_{15})\oplus$(2,2,4,5)$\otimes(x_{24},y_{24},z_{24},\gamma_{24})\oplus$(2,2,5,8) $\otimes(x_{25},y_{25},z_{25},\gamma_{25})\oplus$ (1,1,2,2)$\otimes(x_{34},y_{34},z_{34},\gamma_{34})\oplus$(7,8,11,15)$\otimes(x_{36},y_{36},z_{36},\gamma_{36})\oplus$ (2,3,3,5)$\otimes(x_{45},y_{45},z_{45},\gamma_{45})\oplus$(3,3,4,6)$\otimes(x_{46},y_{46},z_{46},\gamma_{46})\oplus$(1,1,1,2) $\otimes(x_{56},y_{56},z_{56},\gamma_{56})$)

subject to the constraints

$(x_{12},y_{12},z_{12},\gamma_{12})\oplus(x_{13},y_{13},z_{13},\gamma_{13})\oplus(x_{15},y_{15},z_{15},\gamma_{15})=(1,1,1,1)$,

$(x_{24},y_{24},z_{24},\gamma_{24})\oplus(x_{25},y_{25},z_{25},\gamma_{25})=(x_{12},y_{12},z_{12},\gamma_{12})$,

$(x_{34},y_{34},z_{34},\gamma_{34})\oplus(x_{36},y_{36},z_{36},\gamma_{36})=(x_{13},y_{13},z_{13},\gamma_{13})$,

$(x_{45}, y_{45}, z_{45}, \gamma_{45}) \oplus (x_{46}, y_{46}, z_{46}, \gamma_{46}) = (x_{24}, y_{24}, z_{24}, \gamma_{24}) \oplus (x_{34}, y_{34}, z_{34}, \gamma_{34})$,
$(x_{56}, y_{56}, z_{56}, \gamma_{56}) = (x_{15}, y_{15}, z_{15}, \gamma_{15}) \oplus (x_{25}, y_{25}, z_{25}, \gamma_{25}) \oplus (x_{45}, y_{45}, z_{45}, \gamma_{45})$ and
$(x_{36}, y_{36}, z_{36}, \gamma_{36}) \oplus (x_{46}, y_{46}, z_{46}, \gamma_{46}) \oplus (x_{56}, y_{56}, z_{56}, \gamma_{56}) = (1,1,1,1)$.
where $(x_{12}, y_{12}, z_{12}, \gamma_{12})$, $(x_{13}, y_{13}, z_{13}, \gamma_{13})$,etc. are non negative trapezoidal fuzzy numbers.

Step2: Using ranking function to the Fuzzy Linear Programming problem, formulated in Step1, may be written as
Maximize
$\Re[(2,2,3,4) \otimes (x_{12}, y_{12}, z_{12}, \gamma_{12}) \oplus (2,3,3,6) \otimes (x_{13}, y_{13}, z_{13}, \gamma_{13}) \oplus (2,3,4,5)$
$\otimes (x_{15}, y_{15}, z_{15}, \gamma_{15}) \oplus (2,2,4,5) \otimes (x_{24}, y_{24}, z_{24}, \gamma_{24}) \oplus (2,2,5,8)$
$\otimes (x_{25}, y_{25}, z_{25}, \gamma_{25}) \oplus$
$(1,1,2,2) \otimes (x_{34}, y_{34}, z_{34}, \gamma_{34}) \oplus (7,8,11,15) \otimes (x_{36}, y_{36}, z_{36}, \gamma_{36}) \oplus$
$(2,3,3,5) \otimes (x_{45}, y_{45}, z_{45}, \gamma_{45}) \oplus (3,3,4,6) \otimes (x_{46}, y_{46}, z_{46}, \gamma_{46}) \oplus (1,1,1,2)$
$\otimes (x_{56}, y_{56}, z_{56}, \gamma_{56})]$
subject to the constraints
$(x_{12}, y_{12}, z_{12}, \gamma_{12}) \oplus (x_{13}, y_{13}, z_{13}, \gamma_{13}) \oplus (x_{15}, y_{15}, z_{15}, \gamma_{15}) = (1,1,1,1)$,
$(x_{24}, y_{24}, z_{24}, \gamma_{24}) \oplus (x_{25}, y_{25}, z_{25}, \gamma_{25}) = (x_{12}, y_{12}, z_{12}, \gamma_{12})$,
$(x_{34}, y_{34}, z_{34}, \gamma_{34}) \oplus (x_{36}, y_{36}, z_{36}, \gamma_{36}) = (x_{13}, y_{13}, z_{13}, \gamma_{13})$,
$(x_{45}, y_{45}, z_{45}, \gamma_{45}) \oplus (x_{46}, y_{46}, z_{46}, \gamma_{46}) = (x_{24}, y_{24}, z_{24}, \gamma_{24}) \oplus (x_{34}, y_{34}, z_{34}, \gamma_{34})$,
$(x_{56}, y_{56}, z_{56}, \gamma_{56}) = (x_{15}, y_{15}, z_{15}, \gamma_{15}) \oplus (x_{25}, y_{25}, z_{25}, \gamma_{25}) \oplus (x_{45}, y_{45}, z_{45}, \gamma_{45})$ and
$(x_{36}, y_{36}, z_{36}, \gamma_{36}) \oplus (x_{46}, y_{46}, z_{46}, \gamma_{46}) \oplus (x_{56}, y_{56}, z_{56}, \gamma_{56}) = (1,1,1,1)$.
$(x_{12}, y_{12}, z_{12}, \gamma_{12})$, $(x_{13}, y_{13}, z_{13}, \gamma_{13})$ etc. are non-negative trapezoidal fuzzy numbers.

The Crisp Linear Programming problem becomes :

Maximize$(0.5x_{12} + 0.5y_{12} + 0.75z_{12} + \gamma_{12} + 0.5x_{13} + 0.75y_{13} + 0.75z_{13} + 1.5\gamma_{13} + 0.5x_{15} + 0.75y_{15} + z_{15} + 1.25\gamma_{15} + 0.5x_{24} + 0.5y_{24} + z_{24} + 1.25\gamma_{24} + 0.5x_{25} + 0.5y_{25} + 1.25z_{25} + 2\gamma_{25} + 0.25x_{34} + 0.25y_{34} + 0.5z_{34} + 0.5\gamma_{34} + 1.75x_{36} + 2y_{36} + 2.75z_{36} + 3.75\gamma_{36} + 0.5x_{45} + 0.75y_{45} + 0.75z_{45} + 1.25\gamma_{45} + 0.75x_{46} + 0.75y_{46} + z_{46} + 1.5\gamma_{46} + 0.25x_{56} + 0.25y_{56} + 0.25z_{56} + 0.5\gamma_{56})$
subject to the constraints
$x_{12} + x_{13} + x_{15} = 1$, $y_{12} + y_{13} + y_{15} = 1$, $z_{12} + z_{13} + z_{15} = 1$, $\gamma_{12} + \gamma_{13} + \gamma_{15} = 1$,
$x_{24} + x_{25} = x_{12}$, $y_{24} + y_{25} = y_{12}$, $z_{24} + z_{25} = z_{12}$, $\gamma_{24} + \gamma_{25} = \gamma_{12}$,
$x_{34} + x_{36} = x_{13}$, $y_{34} + y_{36} = y_{13}$, $z_{34} + z_{36} = z_{13}$, $\gamma_{34} + \gamma_{36} = \gamma_{13}$,

$x_{45} + x_{46} = x_{24} + x_{34}$, $y_{45} + y_{46} = y_{24} + y_{34}$, $z_{45} + z_{46} = z_{24} + z_{34}$,

$\gamma_{45} + \gamma_{46} = \gamma_{24} + \gamma_{34}$,

$x_{56} = x_{15} + x_{25} + x_{45}$, $y_{56} = y_{15} + y_{25} + y_{45}$, $z_{56} = z_{15} + z_{25} + z_{45}$,

$\gamma_{56} = \gamma_{15} + \gamma_{25} + \gamma_{45}$,

$x_{36} + x_{46} + x_{56} = 1$, $y_{36} + y_{46} + y_{56} = 1$, $z_{36} + z_{46} + z_{56} = 1$, $\gamma_{36} + \gamma_{46} + \gamma_{56} = 1$,

$y_{12} - x_{12} \geq 0, z_{12} - y_{12} \geq 0, \gamma_{12} - z_{12} \geq 0, y_{13} - x_{13} \geq 0, z_{13} - y_{13} \geq 0, \gamma_{13} - z_{13} \geq 0$

$y_{15} - x_{15} \geq 0, z_{15} - y_{15} \geq 0, \gamma_{15} - z_{15} \geq 0,$

$y_{24} - x_{24} \geq 0, z_{24} - y_{24} \geq 0, \gamma_{24} - z_{24} \geq 0,$

$y_{25} - x_{25} \geq 0, z_{25} - y_{25} \geq 0, \gamma_{25} - z_{25} \geq 0, y_{34} - x_{34} \geq 0, z_{34} - y_{34} \geq 0,$

$\gamma_{34} - z_{34} \geq 0$

$y_{36} - x_{36} \geq 0, z_{36} - y_{36} \geq 0, \gamma_{36} - z_{36} \geq 0,$

$y_{45} - x_{45} \geq 0, z_{45} - y_{45} \geq 0, \gamma_{45} - z_{45} \geq 0$

$y_{46} - x_{46} \geq 0, z_{46} - y_{46} \geq 0, \gamma_{46} - z_{46} \geq 0, y_{56} - x_{56} \geq 0, z_{56} - y_{56} \geq 0,$

$\gamma_{56} - z_{56} \geq 0$

$x_{12}, y_{12}, z_{12}, \gamma_{12}, x_{13}, y_{13}, z_{13}, \gamma_{13}, x_{14}, y_{14}, z_{14}, \gamma_{14}, x_{23}, y_{23}, z_{23}, \gamma_{23}, x_{25}, y_{25}, z_{25},$

$\gamma_{25}, x_{35}, y_{35}, z_{35}\ \gamma_{35}, x_{45}, y_{45}, z_{45}, \gamma_{45} \geq 0.$

Step 3: On solving Crisp Linear Programming Using TORA System, obtained in Step 2, solution is $x_{13}=y_{13}=z_{13}=\gamma_{13}=x_{36}=y_{36}=z_{36}=\gamma_{36}=1$ and the remaining all values

$x_{12}, y_{12}, z_{12}, \gamma_{12}, x_{24}, y_{24}, z_{24}, \gamma_{24}\ x_{25}, y_{25}, z_{25}, \gamma_{25}, x_{34}, y_{34}, z_{34}, \gamma_{34}, x_{45}, y_{45}, z_{45},$

$\gamma_{45}, x_{46}, y_{46}, z_{46}\ \gamma_{46}, x_{56}, y_{56}, z_{56}, \gamma_{56}$ are zero.

Step 4: Putting the values of x_{ij}, y_{ij}, z_{ij} and γ_{ij} in $\tilde{x}_{ij}=(x_{ij}, y_{ij}, z_{ij}, \gamma_{ij})$. The solution is

$\tilde{x}_{12}=(0,0,0,0), \tilde{x}_{13}=(1,1,1,1)$

$, \tilde{x}_{15}=(0,0,0,0), \tilde{x}_{24}=(0,0,0,0), \tilde{x}_{25}=(0,0,0,0), \tilde{x}_{34}=(0,0,0,0),$

$\tilde{x}_{36}=(1,1,1,1), \tilde{x}_{45}=(0,0,0,0),\ \tilde{x}_{46}=(0,0,0,0),\ \tilde{x}_{56}=(0,0,0,0).$

Step 5: Using the fuzzy solution, the fuzzy critical path is 1-3-6. Replacing the values of x_{ij}, y_{ij}, z_{ij} and γ_{ij} in Step1, the maximum total completion fuzzy time is (9,11,14,21).

Hence, in this problem, the fuzzy critical path is 1-3-6 and the corresponding maximum total completion fuzzy time is (9,11,14,21) respectively.

6.2. Fuzzy optimal solution using (m,n,α,β) representation of trapezoidal fuzzy numbers

Using the (m,n,α,β) representation of $\tilde{t}_{12}$=(2,2,3,4) , $\tilde{t}_{13}$=(2,3,3,6), $\tilde{t}_{15}$=(2,3,4,5), $\tilde{t}_{24}$=(2,2,4,5),
$\tilde{t}_{25}$=(2,2,5,8), $\tilde{t}_{34}$=(1,1,2,2), $\tilde{t}_{36}$=(7,8,11,15), $\tilde{t}_{45}$=(2,3,3,5), $\tilde{t}_{46}$=(3,3,4,6), $\tilde{t}_{56}$=(1,1,1,2). are $\tilde{t}_{12}$=(2,3,0.1) , $\tilde{t}_{13}$=(3,3,1,3), $\tilde{t}_{15}$=(3,4,1,1), $\tilde{t}_{24}$=(2,4,0,1), $\tilde{t}_{25}$=(2,5,0,3), $\tilde{t}_{34}$=(1,2,0,0), $\tilde{t}_{36}$=(8,11,1,4), $\tilde{t}_{45}$=(3,3,1,2), $\tilde{t}_{46}$=(3,4,0,2), and $\tilde{t}_{56}$=(1,1,0,1).

Step 1: Using the section 4.1, the given problem may be formulated as follows:
Maximize $[(2,3,0,1)\otimes(m_{12},n_{12},\alpha_{12},\beta_{12})\oplus(3,3,1,3)\otimes(m_{13},n_{13},\alpha_{13},\beta_{13})\oplus(3,4,1,1)$
$\otimes(m_{15},n_{15},\alpha_{15},\beta_{15})\oplus(2,4,0,1)\otimes(m_{24},n_{24},\alpha_{24},\beta_{24})\oplus(2,5,0,3)$
$\otimes(m_{25},n_{25},\alpha_{25},\beta_{25})\oplus(1,2,0,0)\otimes(m_{34},n_{34},\alpha_{34},\beta_{34})\oplus(8,11,1,4)$
$\otimes(m_{36},n_{36},\alpha_{36},\beta_{36})\oplus(3,3,1,2)\otimes(m_{45},n_{45},\alpha_{45},\beta_{45})\oplus(3,4,0,2)$
$\otimes(m_{46},n_{46},\alpha_{46},\beta_{46})\oplus(1,1,0,1)\otimes(m_{56},n_{56},\alpha_{56},\beta_{56})]$
subject to the constraints
$(m_{12},n_{12},\alpha_{12},\beta_{12})\oplus(m_{13},n_{13},\alpha_{13},\beta_{13})\oplus(m_{15},n_{15},\alpha_{15},\beta_{15})=(1,1,0,0)$,
$(m_{24},n_{24},\alpha_{24},\beta_{24})\oplus(m_{25},n_{25},\alpha_{25},\beta_{25})=(m_{12},n_{12},\alpha_{12},\beta_{12})$,
$(m_{34},n_{34},\alpha_{34},\beta_{34})\oplus(m_{36},n_{36},\alpha_{36},\beta_{36})=(m_{13},n_{13},\alpha_{13},\beta_{13})$,
$(m_{45},n_{45},\alpha_{45},\beta_{45})\oplus(m_{46},n_{46},\alpha_{46},\beta_{46})=(m_{24},n_{24},\alpha_{24},\beta_{24})+(m_{34},n_{34},\alpha_{34},\beta_{34})$
$(m_{56},n_{56},\alpha_{56},\beta_{56})=(m_{15},n_{15},\alpha_{15},\beta_{15})\oplus(m_{25},n_{25},\alpha_{25},\beta_{25})\oplus(m_{45},n_{45},\alpha_{45},\beta_{45})$
and
$(m_{36},n_{36},\alpha_{36},\beta_{36})\oplus(m_{46},n_{46},\alpha_{46},\beta_{46})\oplus(m_{56},n_{56},\alpha_{56},\beta_{56})=(1,1,0,0)$.
$(m_{12},n_{12},\alpha_{12},\beta_{12})$, $(m_{13},n_{13},\alpha_{13},\beta_{13})$ etc. are non-negative trapezoidal fuzzy numbers.

Step 2: Using ranking formula to the Fuzzy Linear Programming problem, formulated in Step1, may be written as
Maximize $\Re((2,3,0,1)\otimes(m_{12},n_{12},\alpha_{12},\beta_{12})\oplus(3,3,1,3)\otimes(m_{13},n_{13},\alpha_{13},\beta_{13})\oplus(3,4,1,1)$
$\otimes(m_{15},n_{15},\alpha_{15},\beta_{15})\oplus(2,4,0,1)\otimes(m_{24},n_{24},\alpha_{24},\beta_{24})\oplus(2,5,0,3)$
$\otimes(m_{25},n_{25},\alpha_{25},\beta_{25})\oplus(1,2,0,0)\otimes(m_{34},n_{34},\alpha_{34},\beta_{34})\oplus(8,11,1,4)$

$\otimes(m_{36},n_{36},\alpha_{36},\beta_{36})\oplus(3,3,1,2)\otimes(m_{45},n_{45},\alpha_{45},\beta_{45})\oplus(3,4,0,2)$
$\otimes(m_{46},n_{46},\alpha_{46},\beta_{46})\oplus(1,1,0,1)\otimes(m_{56},n_{56},\alpha_{56},\beta_{56}))$
subject to the constraints
$(m_{12},n_{12},\alpha_{12},\beta_{12})\oplus(m_{13},n_{13},\alpha_{13},\beta_{13})\oplus(m_{15},n_{15},\alpha_{15},\beta_{15})=(1,1,0,0)$,
$(m_{24},n_{24},\alpha_{24},\beta_{24})\oplus(m_{25},n_{25},\alpha_{25},\beta_{25})=(m_{12},n_{12},\alpha_{12},\beta_{12})$,
$(m_{34},n_{34},\alpha_{34},\beta_{34})\oplus(m_{36},n_{36},\alpha_{36},\beta_{36})=(m_{13},n_{13},\alpha_{13},\beta_{13})$,
$(m_{45},n_{45},\alpha_{45},\beta_{45})\oplus(m_{46},n_{46},\alpha_{46},\beta_{46})=(m_{24},n_{24},\alpha_{24},\beta_{24})+(m_{34},n_{34},\alpha_{34},\beta_{34})$
$(m_{56},n_{56},\alpha_{56},\beta_{56})=(m_{15},n_{15},\alpha_{15},\beta_{15})\oplus(m_{25},n_{25},\alpha_{25},\beta_{25})\oplus(m_{45},n_{45},\alpha_{45},\beta_{45})$
and
$(m_{36},n_{36},\alpha_{36},\beta_{36})\oplus(m_{46},n_{46},\alpha_{46},\beta_{46})\oplus(m_{56},n_{56},\alpha_{56},\beta_{56})=(1,1,0,0)$.
$(m_{13},n_{13},\alpha_{13},\beta_{13})$, $(m_{12},n_{12},\alpha_{12},\beta_{12})$ etc. are non-negative trapezoidal fuzzy numbers.

The Crisp Linear Programming problem becomes :

Maximize($m_{12}+n_{12}$ -0 α_{12} +0.25 β_{12} +1.5 m_{13} +1.5 n_{13} -
0.25 α_{13} +0.75 β_{13} +1.5 m_{15} + n_{15} -0.25 α_{15} +0.25 β_{15} + m_{24} +2 n_{24} -
0 α_{24} +0.25 β_{24} + m_{25} +2.5 n_{25} -0 α_{25} +0.75 β_{25} +0.5 m_{34} + n_{34} -
0 α_{34} +0 β_{34} +4 m_{36} +5.5 n_{36} -0.25 α_{36} + β_{36} +1.5 m_{45} +1.5 n_{45} -
0.25 α_{45} +0.5 β_{45} +1.5 m_{46} +2 n_{46} -0 α_{46} +0.5 β_{46} +0.5 m_{56} +0.5 n_{56} -0 α_{56} +0.25 β_{56}).

subject to the constraints
$m_{12}+m_{13}+m_{15}=1$, $n_{12}+n_{13}+n_{15}=1$, $\alpha_{12}+\alpha_{13}+\alpha_{15}=0$, $\beta_{12}+\beta_{13}+\beta_{15}=0$,
$m_{24}+m_{25}=m_{12}$, $n_{24}+n_{25}=n_{12}$, $\alpha_{24}+\alpha_{25}=\alpha_{12}$, $\beta_{24}+\beta_{25}=\beta_{12}$,
$m_{34}+m_{36}=m_{13}$, $n_{34}+n_{36}=n_{13}$, $\alpha_{34}+\alpha_{36}=\alpha_{13}$, $\beta_{34}+\beta_{36}=\beta_{14}$,
$m_{45}+m_{46}=m_{24}+m_{34}$, $n_{45}+n_{46}=n_{24}+n_{34}$, $\alpha_{45}+\alpha_{46}=\alpha_{24}+\alpha_{34}$,
$\beta_{45}+\beta_{46}=\beta_{24}+\beta_{34}$
$m_{56}=m_{15}+m_{25}+m_{45}$, $n_{56}=n_{15}+n_{25}+n_{45}$, $\alpha_{56}=\alpha_{15}+\alpha_{25}+\alpha_{45}$,
$\beta_{56}=\beta_{15}+\beta_{25}+\beta_{45}$,
$m_{36}+m_{46}+m_{56}=1$, $n_{36}+n_{46}+n_{56}=1$, $\alpha_{36}+\alpha_{46}+\alpha_{56}=0$, $\beta_{36}+\beta_{46}+\beta_{56}=0$,
$n_{12}-m_{12}\geq 0$, $\alpha_{12}-n_{12}\geq 0$, $n_{13}-m_{13}\geq 0$, $\alpha_{13}-n_{13}\geq 0$,
$n_{15}-m_{15}\geq 0$, $\alpha_{15}-n_{15}\geq 0$,
$n_{24}-m_{24}\geq 0$, $\alpha_{24}-n_{24}\geq 0$,
$n_{25}-m_{25}\geq 0$, $\alpha_{25}-n_{25}\geq 0$, $n_{34}-m_{34}\geq 0$, $\alpha_{34}-n_{34}\geq 0$,
$n_{36}-m_{36}\geq 0$, $\alpha_{36}-n_{36}\geq 0$, $n_{45}-m_{45}\geq 0$, $\alpha_{45}-n_{45}\geq 0$, $n_{46}-m_{46}\geq 0$,
$\alpha_{46}-n_{46}\geq 0$

$n_{56} - m_{56} \geq 0, \alpha_{56} - n_{56} \geq 0$.

$m_{12}, n_{12}, \alpha_{12}, \beta_{12}, m_{13}, n_{13}, \alpha_{13}, \beta_{13}, m_{15}, n_{15}, \alpha_{15}, \beta_{15}, m_{24}, n_{24}, \alpha_{24}, \beta_{24}, m_{25}, n_{25}, \alpha_{25}, \beta_{25}, m_{34}, n_{34}, \alpha_{34}, \beta_{34}, m_{36}, n_{36}, \alpha_{36}, \beta_{36}, m_{45}, n_{45}, \alpha_{45}, \beta_{45}, m_{46}, n_{46}, \alpha_{46}, \beta_{46}, m_{56}, n_{56}, \alpha_{56}, \beta_{56} \geq 0$.

Step 3: On solving Crisp Linear Programming Using TORA System, obtained in Step2, an optimal solution is $m_{13}=n_{13}=\alpha_{13}=\beta_{13}=m_{36}=n_{36}=\alpha_{36}=\beta_{36}=1$ and the remaining all values
$m_{12}, n_{12}, \alpha_{12}, \beta_{12}, m_{15}, n_{15}, \alpha_{15}, \beta_{15}, m_{24}, n_{24}, \alpha_{24}, \beta_{24}, m_{25}, n_{25}, \alpha_{25}, \beta_{25}, m_{34}, n_{34}, \alpha_{34}, \beta_{34}$ $m_{45}, n_{45}, \alpha_{45}, \beta_{45}$ $m_{46}, n_{46}, \alpha_{46}, \beta_{46}, m_{56}, n_{56}, \alpha_{56}, \beta_{56}$ are zero.

Step 4: Putting the values of $x_{ij}, y_{ij}, \alpha_{ij}$ and β_{ij} in $\tilde{x}_{ij}=(m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})$. The solution is
$\tilde{x}_{12}$=(0,0,0,0), $\tilde{x}_{13}$=(1,1,1,1),
$\tilde{x}_{15}$=(0,0,0,0), $\tilde{x}_{24}$=(0,0,0,0), $\tilde{x}_{25}$=(0,0,0,0), $\tilde{x}_{34}$=(0,0,0,0),
$\tilde{x}_{36}$=(1,1,1,1), $\tilde{x}_{45}$=(0,0,0,0), $\tilde{x}_{46}$=(0,0,0,0), $\tilde{x}_{56}$=(0,0,0,0).

Step 5: Using the fuzzy solution, the fuzzy critical path is 1-3-6. Replacing the values of $x_{ij}, y_{ij}, \alpha_{ij}$ and β_{ij} in Step1, the maximum total completion fuzzy time is (11,14,2,7).
Hence, in this problem, the fuzzy critical path is 1-3-6 and the corresponding maximum total completion fuzzy time is (11,14,2,7) respectively.

6.3. Fuzzy optimal solution using (x, y, α, β) representation of trapezoidal fuzzy numbers

Using the (x, y, α, β) representation of $\tilde{t}_{12}$=(2,2,3,4), $\tilde{t}_{13}$=(2,3,3,6), $\tilde{t}_{15}$=(
2,3,4,5), $\tilde{t}_{24}$=(2,2,4,5),
$\tilde{t}_{25}$=(2,2,5,8), $\tilde{t}_{34}$=(1,1,2,2), $\tilde{t}_{36}$=(7,8,11,15), $\tilde{t}_{45}$=(2,3,3,5), $\tilde{t}_{46}$=(3,3,4,6),
$\tilde{t}_{56}$=(1,1,1,2) are
$\tilde{t}_{12}$=(2,4,0.1), $\tilde{t}_{13}$=(2,6,1,3), $\tilde{t}_{15}$=(
2,5,1,1), $\tilde{t}_{24}$=(2,5,0,1), $\tilde{t}_{25}$=(2,8,0,3), $\tilde{t}_{34}$=(1,2,0,0),
$\tilde{t}_{36}$=(7,15,1,4), $\tilde{t}_{45}$=(2,5,1,2), $\tilde{t}_{46}$=(3,6,0,2), and $\tilde{t}_{56}$=(1,2,0,1).

Step1: Using the section 4.1, the given problem may be formulated as follows:
Maximize((2,4,0,1) $\otimes (x_{12}, y_{12}, \alpha_{12}, \beta_{12}) \oplus$ (2,6,1,3) $\otimes (x_{13}, y_{13}, \alpha_{13}, \beta_{13}) \oplus$ (
2,5,1,1)

$\otimes(x_{15}, y_{15}, \alpha_{15}, \beta_{15}) \oplus (2,5,0,1) \otimes (x_{24}, y_{24}, \alpha_{24}, \beta_{24}) \oplus (2,8,0,3)$
$\otimes(x_{25}, y_{25}, \alpha_{25}, \beta_{25}) \oplus (1,2,0,0) \otimes (x_{34}, y_{34}, \alpha_{34}, \beta_{34}) \oplus (7,15,1,4)$
$\otimes(x_{36}, y_{36}, \alpha_{36}, \beta_{36}) \oplus (2,5,1,2) \otimes (x_{45}, y_{45}, \alpha_{45}, \beta_{45}) \oplus (3,6,0,2)$
$\otimes(x_{46}, y_{46}, \alpha_{46}, \beta_{46}) \oplus (1,2,0,1) \otimes (x_{56}, y_{56}, \alpha_{56}, \beta_{56}))$
subject to the constraints
$(x_{12}, y_{12}, \alpha_{12}, \beta_{12}) \oplus (x_{13}, y_{13}, \alpha_{13}, \beta_{13}) \oplus (x_{15}, y_{15}, \alpha_{15}, \beta_{15}) = (1,1,0,0)$,
$(x_{24}, y_{24}, \alpha_{24}, \beta_{24}) \oplus (x_{25}, y_{25}, \alpha_{25}, \beta_{25}) = (x_{12}, y_{12}, \alpha_{12}, \beta_{12})$,
$(x_{34}, y_{34}, \alpha_{34}, \beta_{34}) \oplus (x_{36}, y_{36}, \alpha_{36}, \beta_{36}) = (x_{13}, y_{13}, \alpha_{13}, \beta_{13})$,
$(x_{45}, y_{45}, \alpha_{45}, \beta_{45}) + (x_{46}, y_{46}, \alpha_{46}, \beta_{46}) = (x_{24}, y_{24}, \alpha_{24}, \beta_{24}) \oplus (x_{34}, y_{34}, \alpha_{34}, \beta_{34})$,
$(x_{56}, y_{56}, \alpha_{56}, \beta_{56}) = (x_{15}, y_{15}, \alpha_{15}, \beta_{15}) + (x_{25}, y_{25}, \alpha_{25}, \beta_{25}) \oplus (x_{45}, y_{45}, \alpha_{45}, \beta_{45})$
and
$(x_{25}, y_{25}, \alpha_{25}, \beta_{25}) \oplus (x_{35}, y_{35}, \alpha_{35}, \beta_{35}) \oplus (x_{45}, y_{45}, \alpha_{45}, \beta_{45}) = (1,1,0,0)$.
$(x_{12}, y_{12}, \alpha_{12}, \beta_{12}), (x_{13}, y_{13}, \alpha_{13}, \beta_{13})$ etc. are non-negative trapezoidal fuzzy numbers.

Step2: Using ranking function to the Fuzzy Linear Programming problem, formulated in Step1, may be written as
Maximize $\Re$ $[(2,4,0,1) \otimes (x_{12}, y_{12}, \alpha_{12}, \beta_{12}) \oplus (2,6,1,3) \otimes (x_{13}, y_{13}, \alpha_{13}, \beta_{13}) \oplus ($ 2,5,1,1)
$\otimes(x_{15}, y_{15}, \alpha_{15}, \beta_{15}) \oplus (2,5,0,1) \otimes (x_{24}, y_{24}, \alpha_{24}, \beta_{24}) \oplus (2,8,0,3)$
$\otimes(x_{25}, y_{25}, \alpha_{25}, \beta_{25}) \oplus (1,2,0,0) \otimes (x_{34}, y_{34}, \alpha_{34}, \beta_{34}) \oplus (7,15,1,4)$
$\otimes(x_{36}, y_{36}, \alpha_{36}, \beta_{36}) \oplus (2,5,1,2) \otimes (x_{45}, y_{45}, \alpha_{45}, \beta_{45}) \oplus (3,6,0,2)$
$\otimes(x_{46}, y_{46}, \alpha_{46}, \beta_{46}) \oplus (1,2,0,1) \otimes (x_{56}, y_{56}, \alpha_{56}, \beta_{56})]$
subject to the constraints
$(x_{12}, y_{12}, \alpha_{12}, \beta_{12}) \oplus (x_{13}, y_{13}, \alpha_{13}, \beta_{13}) \oplus (x_{15}, y_{15}, \alpha_{15}, \beta_{15}) = (1,1,0,0)$,
$(x_{24}, y_{24}, \alpha_{24}, \beta_{24}) \oplus (x_{25}, y_{25}, \alpha_{25}, \beta_{25}) = (x_{12}, y_{12}, \alpha_{12}, \beta_{12})$,
$(x_{34}, y_{34}, \alpha_{34}, \beta_{34}) \oplus (x_{36}, y_{36}, \alpha_{36}, \beta_{36}) = (x_{13}, y_{13}, \alpha_{13}, \beta_{13})$,
$(x_{45}, y_{45}, \alpha_{45}, \beta_{45}) + (x_{46}, y_{46}, \alpha_{46}, \beta_{46}) = (x_{24}, y_{24}, \alpha_{24}, \beta_{24}) \oplus (x_{34}, y_{34}, \alpha_{34}, \beta_{34})$,
$(x_{56}, y_{56}, \alpha_{56}, \beta_{56}) = (x_{15}, y_{15}, \alpha_{15}, \beta_{15}) + (x_{25}, y_{25}, \alpha_{25}, \beta_{25}) \oplus (x_{45}, y_{45}, \alpha_{45}, \beta_{45})$
and
$(x_{25}, y_{25}, \alpha_{25}, \beta_{25}) \oplus (x_{35}, y_{35}, \alpha_{35}, \beta_{35}) \oplus (x_{45}, y_{45}, \alpha_{45}, \beta_{45}) = (1,1,0,0)$.
$(x_{12}, y_{12}, \alpha_{12}, \beta_{12}), (x_{13}, y_{13}, \alpha_{13}, \beta_{13})$ etc. are non-negative trapezoidal fuzzy numbers.

The Crisp Linear Programming problem becomes :

Maximize (x_{12} +2 y_{12} - 0 α_{12} +0.25 β_{12} + x_{13} +3 y_{13} -0.25 α_{13} +0.75 β_{13} + x_{15} +2.5 y_{15} - 0.25 α_{15} +0.25 β_{15} + x_{24} +2.5 y_{24} -0 α_{24} +0.25 β_{24} + x_{25} +4 y_{25} - 0 α_{25} +0.75 β_{25} +0.5 x_{34} + y_{34} -0 α_{34} +0 β_{34} +3.5 x_{36} +7.5 y_{36} - 0.25 α_{36} + β_{36} + x_{45} +2.5 y_{45} -0.25 α_{45} +0.5 β_{45} +1.5 x_{46} +3 y_{46} - 0 α_{46} +0.5 β_{46} +0.5 x_{56} + y_{56} -0 α_{56} +0.25 β_{56}).

subject to the constraints

$x_{12} + x_{13} + x_{15} = 1$, $y_{12} + y_{13} + y_{15} = 1$, $\alpha_{12} + \alpha_{13} + \beta_{15} = 0$, $\beta_{12} + \beta_{13} + \beta_{15} = 0$

$x_{24} + x_{25} = x_{12}$, $y_{24} + y_{25} = y_{12}$, $\alpha_{24} + \alpha_{25} = \alpha_{12}$, $\beta_{24} + \beta_{25} = \beta_{12}$

$x_{34} + x_{36} = x_{13}$, $y_{34} + y_{36} = y_{13}$, $\alpha_{34} + \alpha_{36} = \alpha_{13}$, $\beta_{34} + \beta_{36} = \beta_{13}$

$x_{45} + x_{46} = x_{24} + x_{34}$, $y_{45} + y_{46} = y_{24} + y_{34}$, $\alpha_{45} + \alpha_{46} = \alpha_{24} + \alpha_{34}$,

$\beta_{45} + \beta_{46} = \beta_{24} + \beta_{34}$

$x_{56} = x_{15} + x_{25} + x_{45}$, $y_{56} = y_{15} + y_{25} + y_{56}$, $\alpha_{56} = \alpha_{15} + \alpha_{25} + \alpha_{45}$,

$\beta_{56} = \beta_{15} + \beta_{25} + \beta_{45}$

$x_{36} + x_{46} + x_{56} = 1$, $y_{36} + y_{46} + y_{56} = 1$, $\alpha_{36} + \alpha_{46} + \alpha_{56} = 0$, $\beta_{36} + \beta_{46} + \beta_{56} = 0$

$x_{12}, y_{12}, \alpha_{12}, \beta_{12}, x_{13}, y_{13}, \alpha_{13}, \beta_{13}, x_{15}, y_{15}, \alpha_{15}, \beta_{15}, x_{24}, y_{24}, \alpha_{24}, \beta_{24}, x_{25}, y_{25}, \alpha_{25}, \beta_{25}, x_{34}, y_{34}, \alpha_{34}, \beta_{34}, x_{36}, y_{36}, \alpha_{36}, \beta_{36}, x_{45}, y_{45}, \alpha_{45}, \beta_{45}, x_{46}, y_{46}, \alpha_{46}, \beta_{46}, x_{56}, y_{56}, \alpha_{56}, \beta_{56} \geq 0$.

Step 3: On solving Crisp Linear Programming Using TORA System, obtained in Step2, an optimal solution is

$x_{13} = y_{13} = \alpha_{13} = \beta_{13} = x_{36} = y_{36} = \alpha_{36} = \beta_{36} = 1$ and the remaining all values $x_{12}, y_{12}, \alpha_{12}, \beta_{12}, x_{15}, y_{15}, \alpha_{15}, \beta_{15}, x_{24}, y_{24}, \alpha_{24}, \beta_{24}, x_{25}, y_{25}, \alpha_{25}, \beta_{25}, x_{34}, y_{34}, \alpha_{34}, \beta_{34}, x_{45}, y_{45}, \alpha_{45}, \beta_{45}, x_{46}, y_{46}, \alpha_{46}, \beta_{46}, x_{56}, y_{56}, \alpha_{56}, \beta_{56}$ are zero.

Step 4: Putting the values of $x_{ij}, y_{ij}, \alpha_{ij}$ and β_{ij} in $\tilde{x}_{ij} = (x_{ij}, y_{ij}, \alpha_{ij}, \beta_{ij})$. The solution is

$\tilde{x}_{12}$ =(0,0,0,0), $\tilde{x}_{13}$ =(1,1,1,1),

$\tilde{x}_{15}$ =(0,0,0,0), $\tilde{x}_{24}$ =(0,0,0,0), $\tilde{x}_{25}$ =(0,0,0,0), $\tilde{x}_{34}$ =(0,0,0,0),

$\tilde{x}_{36}$ =(1,1,1,1), $\tilde{x}_{45}$ =(0,0,0,0), $\tilde{x}_{46}$ =(0,0,0,0), $\tilde{x}_{56}$ =(0,0,0,0).

Step 5: Using the fuzzy optimal solution, the fuzzy critical path is 1-3-6. Replacing the values of $x_{ij}, y_{ij}, \alpha_{ij}$ and β_{ij} in Step1, the maximum total completion fuzzy time is (9,21,2,7). Hence, the fuzzy critical path is 1-3-6 and the corresponding maximum total completion fuzzy time is (9,21,2,7) respectively.

7. Comparision of proposed method with existing method

The results of the numerical example obtained from the sections 6.1 , 6.2,and 6.3 are presented in table I. From table I, we can easily seen that number of constraints in crisp linear programming problem represented by new representation of trapezoidal fuzzy numbers are less compare to crisp linear programming problem represented by existing representations of trapezoidal fuzzy numbers. Hence, it is better to use crisp linear programming problem represented by new representation of trapezoidal fuzzy numbers.

Table I: Results for existing method and proposed representation of trapezoidal fuzzy numbers

Representation of Trapezoidal fuzzy number	Number of constraints in Fuzzy Linear programming problem	Number of constraints in Crisp Linear Programming problem	Fuzzy critical path	Maximum total completion fuzzy time
(a,b,c,d)	16	(4×16)+(3×10)=48+30=78	$1\to3\to6$	(9,11,14,21)
(m,n,α,β)	16	(4×16)+(2×10)=48+20=68	$1\to3\to6$	(11,14,2,7)
$(x,y,\alpha,\beta)_{New}$	16	(4×16)=48	$1\to3\to6$	(9,21,2,7)

8. Conclusion

A new method has been proposed to find the fuzzy critical path and fuzzy completion time of a fuzzy project. Also a new representation of trapezoidal fuzzy numbers is proposed and it is shown that it is better to use the proposed representation of trapezoidal fuzzy numbers instead of existing representations to find the fuzzy critical path and fuzzy completion time of fuzzy critical path problems.

References

[1] Zadeh, L.A.(1965). Fuzzy Sets, Information and Control, 8(3), 338-353.

[2] Chanas, S. and Kamburowski, J. (1981). The use of fuzzy variables in PERT, Fuzzy Sets and Systems, 5(1),11-19.

[3] Gazdik, I.(1983). Fuzzy network planning-FNET, IEEE Transactions on Reliability, 32(3),304-313.

[4] Kaufmann, A. and Gupta, M.M.(1988). Fuzzy Mathematical Models in Engineering and Management Sciences. Elsevier, Amsterdam.

[5] Mc Cahon, C.S. and Lee, E.S. (1988). project network analysis with fuzzy activity times, Computer and Mathematics with Applications, 15(10), 829-838.

[6] Nasution, S.H.(1994). Fuzzy Critical Path Method, IEEE Transactions on systems, Man and Cybernetics, 24(1),48-57.

[7] Yao, J.S. and Lin, F.T.(2000). Fuzzy Critical Path method based on signed distance ranking of fuzzy numbers, IEEE Transactions on Systems, Man and Cybernetics- Part A: Systems and Humans, 30(1),76-82.

[8] Dubois, D., Fargier, H. and Galvagnon, V.(2003) on latest starting times and float in activity networks with ill-known durations, European Journal of Operational Research, 147(2), 266-280.

[9] Kumar , A and Kaur ,P (2010). A New method for fuzzy critical path analysis in project networks with a new representation of triangular fuzzy numbers, Applications and Applied Mathematics, 5(10), 1442-1466.

[10] Kaufmann, A. and Gupta, M.M.(1985). Introduction to Fuzzy Arithmetics: Theory and Applications, Van Nostrand Reinhold, New York.

[11] Chen, S.P.(2007). Analysis of critical paths in a project network with fuzzy activity times, European Journal of Operational Research, 183(1), 442-459.

[12] Chen, S.P. and Hsueh, Y.J. (2008). A simple approach to fuzzy critical path analysis in project networks, Applied Mathematical Modelling, 32(7), 1289-1297.

[13] Yakhchali, S.H. and Ghodsypour, S.H. (2010). Computing latest starting times of activities in interval-valued networks with minimal time lags, European Journal of Operational Research, 200(3), 874-880.

[14] Taha, H.A.(2003). Operations Research: An Introduction, Prentice-Hall, New Jersey.

Received: October, 2012