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Intuitionistic Fuzzy Completely π Generalized Semi Continuous Mappings in Topological Spaces

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Abstract

This paper introduces intuitionistic fuzzy completely π generalized semi continuous mappings, intuitionistic fuzzy π generalized semi homeomorphism and intuitionistic fuzzy iπ generalized semi homeomorphism in intuitionistic fuzzy topological spaces. Also they are related to the fundamental concepts of intuitionistic fuzzy continuous mappings and intuitionistic fuzzy open mappings.

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1. Introduction

The concept of fuzzy sets was introduced by Zadeh [14] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper we introduce intuitionistic fuzzy completely π generalized semi continuous mappings in intuitionistic fuzzy topological spaces, intuitionistic fuzzy π generalized semi homeomorphism and intuitionistic fuzzy iπ generalized semi homeomorphism. Also they are related to the fundamental concepts of intuitionistic fuzzy continuous mappings and intuitionistic fuzzy open mappings. We provide some characterizations of intuitionistic fuzzy π generalized semi homeomorphism.

2. Preliminaries

Definition 2.1: [1] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A=\{\langle x,\mu_A(x), \nu_A(x)\rangle/x\in X\}$where the values $\mu_A(x): X \to [0, 1]$ and $\nu_A(x): X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non -membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. The set of all intuitionistic fuzzy sets in X is denoted by IFS (X).

Definition 2.2 : [1] Let A and B be IFS's of the form
$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x) \rangle / x \in X\}$.Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in X\}$,

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle/ x \in X\}$,

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X\}$. For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle /x \in X\}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$. The intuitionistic fuzzy sets $0_\sim = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_\sim = \{ \langle x, 1, 0 \rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT in short) on a non empty set X is a family τ of IFS in X satisfying the following axioms:

(a) $0_\sim, 1_\sim \in \tau$

(b) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$

(c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i / i\in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set in X.

Definition 2.4 : [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

int(A) = $\cup$ { G / G is an IFOS in X and G $\subseteq$ A},

cl(A) = $\cap$ { K / K is an IFCS in X and A $\subseteq$ K }.

Note that for any IFS A in (X, τ), we have $cl(A^c) = [int(A)]^c$ and $int(A^c) = [cl(A)]^c$.

Definition 2.5: [5] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

(i) intuitionistic fuzzy semi open set (IFSOS) if A $\subseteq$ cl(int(A)),
(ii) intuitionistic fuzzy α-open set (IFαOS) if A $\subseteq$ int(cl(int(A))),
(iii) intuitionistic fuzzy regular open set (IFROS) if A = int(cl(A)).

Definition 2.6: [5] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an
(i) intuitionistic fuzzy semi closed set (IFSCS) if int(cl(A)) $\subseteq$ A,
(ii) intuitionistic fuzzy α-closed set (IFαCS) if cl(int(cl(A)) $\subseteq$ A,
(iii) intuitionistic fuzzy regular closed set (IFRCS) if A = cl(int((A).

Definition 2.7: [10] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy π- generalized semi closed set (IFπGSCS in short) if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFπOS in (X, τ).

Definition 2.8:[10] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy π generalized semi open mapping (IFπGS open mapping) if f(A) $\in$ IFπGSOS(X) for every IFOS A in X.

Definition 2.9:[10] A mapping f: (X, τ) $\rightarrow$ (Y,σ) is called an intuitionistic fuzzy π generalized semi closed mapping (IFπGS closed mapping) if f (A) is an IFπGSCS in (Y,σ) for every IFCS A of (X, τ).

Definition 2.10:[6] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be
(i) intuitionistic fuzzy continuous (IF continuous) if $f^{-1}(B) \in$ IFO(X) for every B $\in$ σ.
(ii) intuitionistic fuzzy semi continuous (IFS continuous) if $f^{-1}(B) \in$ IFSO(X) for every B $\in \sigma$.
(iii) intuitionistic fuzzygeneralized continuous(IFG continuous) if $f^{-1}(B) \in$ IFGC(X) for every IFCS B in Y.
(iv) intuitionistic fuzzy generalized semi continuous (IFGS continuous) if $f^{-1}(B)$

$\in$ IFGSC(X) for every IFCS B in Y.

Definition 2.11:[10] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be intuitionistic fuzzy π generalized semi continuous (IFπGS continuous) if $f^{-1}(B) \in$ IFπGSC(X) for every IFCS B in Y.

Definition 2.12: [10] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be intuitionistic fuzzy Almost continuous (IFA continuous) if $f^{-1}(B) \in$ IFC(X) for every IFRCS B in Y.

Definition 2.13: [10] A mapping f: (X, τ) $\to$ (Y,σ) is called an intuitionistic fuzzy Almost π generalized semi continuous (IFAπGS continuous) if $f^{-1}(B)$ is an IFπGSCS in (X, τ) for every IFRCS B of (Y, σ).

Definition 2.14: [4] The IFS $c(\alpha, \beta) = \langle x, c_\alpha, c_{1-\beta}\rangle$ where $\alpha \in (0, 1]$**,** $\beta \in [0, 1)$ and $\alpha + \beta \leq 1$ is called an intuitionistic fuzzy point (IFP) in X.

Note that an IFP $c(\alpha, \beta)$ is said to belong to an IFS $A = \langle x, \mu_{A,} \nu_A\rangle$ of X denoted by $c(\alpha, \beta) \in A$ if $\alpha \leq \mu_A$ and $\beta \geq \nu_{A.}$

Definition 2.15: [4] Let $c(\alpha, \beta)$ be an IFP of an IFTS (X, τ). An IFS A of X is called an intuitionistic fuzzy neighborhood (IFN) of $c(\alpha, \beta)$ if there exists an IFOS B in X such that $c(\alpha, \beta) \in B \subseteq A$.

Definition 2.16: [5] Two IFSs are said to be q-coincident ($A_q B$) if and only if there exists an element $x\in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.17: [5] Two IFSs are said to be not q-coincident ($A_q^c B$ in short) if and only if $A \subseteq B^c$.

Definition 2.18: [12] An IFS A is said to be an intuitionistic fuzzy dense (IFD for short) in another IFS B in an IFTS (X, τ), if cl(A) = B.

Definition 2.19 : [5] Let X and Y be two IFTSs. Let $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle/x\in X\}$ and $B =\{\langle y, \mu_B(y), \nu_B(y)\rangle/y\in Y\}$ be IFSs of X and Y respectively. Then A x B is an IFS of X x Y defined by $(A \times B)(x,y) = \langle (x,y), \min((\mu_A(x), \mu_B(y)), \max(\nu_A(x), \nu_B(y))\rangle$.

Definition 2.20: [5] Let $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$. The product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is defined $(f_1 \times f_2)(x_1,x_2) = (f_1(x_1),f_2(x_2))$ for every $(x_1,x_2) \in X_1 \times X_2$.

Definition 2.21:[11] Let f be a bijection mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be

(i) intuitionistic fuzzy homeomorphism(IF homeomorphism) if f and f^{-1} are IF continuous mappings.
(ii) intuitionistic fuzzy semi homeomorphism(IFS homeomorphism) if f and f^{-1} are IFS continuous mappings.
(iii) intuitionistic fuzzy generalized homeomorphism(IFG homeomorphism) if f and f^{-1} are IFG continuous mappings.
(iv) ntuitionisticfuzzy generalized semi homeomorphism(IFGS homeomorphism) if f and f^{-1} are IFGS continuous mappings.

Definition 2.22:[11] A mapping f: (X, τ) → (Y,σ) is called an intuitionistic fuzzy π - generalized semi closed mapping (IFπGS mapping) if f (A) is an IFπGSCS in (Y,σ) for every IFCS A of (X,τ).

Definition 2.23:[9] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then the map f is said to be an intuitionistic fuzzy π- generalized semi irresolute (IFπGS irresolute) if $f^{-1}(B) \in$ IFπGCS(X) for every IFπGCS B in Y.

Definition 2.24: [9] An IFTS (X, τ) is said to be an intuitionistic Fuzzy $\pi aT_{1/2}$ (in short IF$\pi aT_{1/2}$) space if every IFπGCS in X is an IFCS in X.

Definition 2.25: [9] An IFTS (X, τ) is said to be an intuitionistic Fuzzy $\pi bT_{1/2}$ (in short IF$\pi bT_{1/2}$) space if every IFπGCS in X is an IFGCS in X.

3. INTUITIONISTIC FUZZY COMPLETELY π GENERALIZED SEMI CONTINUOUS MAPPINGS

In this section, we have introduced intuitionistic fuzzy completely π generalized semi continuous mappings and studied some of their properties.

Definition 3.1: A mapping f : (X, τ) → (Y, σ) is called an intuitionistic fuzzy completely π generalized semi continuous (IFcπGS continuous) mapping if $f^{-1}(B)$ is an IFRCS in (X, τ) for every IFπGSCS B of (Y, σ).

Theorem 3.2: Every IFcπGS continuous mapping is an IF continuous mapping but not conversely.

Proof: Assume that f : (X, τ) → (Y, σ) is an IFcπGS continuous mapping. Let B be an IFCS in Y. Then B is an IFπGSCS in Y. Since f is an IFcπGS continuous mapping, $f^{-1}(B)$ is an IFRCS in X. This implies $f^{-1}(B)$ is an IFCS in X. Hence the mapping f is an IF continuous mapping.

Example 3.3: Let X = { a, b }, Y = { u, v } and G_1 = ⟨ x, (0.3_a, 0.1_b), (0.5_a, 0.6_b) ⟩, G_2 = ⟨ x, (0_a, 0.1_b), (0.7_a, 0.7_b) ⟩ and G_3 = ⟨ y, (0_u, 0.1_v), (0.7_u, 0.7_v) ⟩. Then τ = { $0_\sim$, G_1, G_2, $1_\sim$ } and σ = { $0_\sim$, G_3, $1_\sim$ } are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF continuous mapping. But f is not an IFcπGS continuous mapping, since B = ⟨ y, (0.6_u, 0.7_v), (0_u, 0.1_v) ⟩ is an IFπGSCS in Y but f^{-1}(B) = ⟨ x, (0.6_a, 0.7_b), (0_a, 0.1_b) ⟩ is not an IFRCS in X.

Theorem 3.4: A mapping f : X → Y is an IFcπGS continuous mapping if and only if the inverse image of each IFπGSOS in Y is an IFROS in X.

Proof: Necessity: Let A be an IFπGSOS in Y. This implies A^c is an IFπGSCS in Y. Since f is an IFcπGS continuous, $f^{-1}(A^c)$ is IFRCS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFROS in X.

Sufficiency: Let A be an IFπGSCS in Y. This implies A^c is an IFπGSOS in Y. By hypothesis $f^{-1}(A^c)$ is an IFROS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFRCS in X. Hence f is an IFcπGS continuous mapping.

Theorem 3.5: If a mapping f: X → Y is an IFcπGS continuous mapping, then for every IFP c(α, β) ∈ X and for every IFN A of f(c(α, β)), there exists an IFROS B ⊆ X such that c(α, β) ∈ B ⊆ f^{-1}(A).

Proof: Let c(α, β) ∈ X and let A be an IFN of f(c(α, β)). Then there exists an IFOS C in Y such that f(c(α, β)) ∈ C ⊆ A. Since every IFOS is an IFπGSOS, C is an IFπGSOS in Y. Hence by hypothesis, f^{-1}(C) is an IFROS in X and c(α, β) ∈ f^{-1}(C). Now, let f^{-1}(C) = B. Therefore c(α, β) ∈ B ⊆ f^{-1}(A).

Theorem 3.6: If a mapping f: X → Y is an IFcπGS continuous mapping, then for every IFP c(α, β) ∈ X and for every IFN A of f(c(α, β)), there exists an IFROS B ⊆ X such that c(α, β) ∈ B and f(B) ⊆ A.

Proof: Let c(α, β) ∈ X and let A be an IFN of f(c(α, β)). Then there exists an IFOS C in Y such that f(c(α, β)) ∈ C ⊆ A. Since every IFOS is an IFπGSOS, C is an IFπGSOS in Y. Hence by hypothesis, f^{-1}(C) is an IFROS in X and c(α, β) ∈ f^{-1}(C). Now let f^{-1}(C) = B. Therefore c(α, β) ∈ B ⊆ f^{-1}(A). Thus f(B) ⊆ f(f^{-1}(A)) ⊆ A, which implies f(B) ⊆ A.

Theorem 3.7: A mapping f: X → Y is an IFcπGS continuous mapping then int(cl(f^{-1}(int(B)))) ⊆ f^{-1}(B) for every IFS B in Y.

Proof: Let B ⊆ Y be an IFS. Then, int(B) is an IFOS in Y and hence an IFπGSOS in Y. By hypothesis, f^{-1}(int(B)) is an IFROS in X. Hence

$int(cl(f^{-1}(int(B)))) = f^{-1}(int(B)) \subseteq f^{-1}(B)$.

Theorem 3.8: A mapping f: X → Y is an IFcπGS continuous mapping then the following are equivalent.

(i) for any IFπGSOS A in Y and for any IFP $c(\alpha, \beta) \in X$, if $f(c(\alpha, \beta))\ _q A$ then $c(\alpha, \beta)\ _q\ int(f^{-1}(A))$.

(ii) for any IFπGSOS A in Y and for any $c(\alpha, \beta) \in X$, if $f(c(\alpha, \beta))\ _q\ A$ then there exists an IFOS B such that $c(\alpha, \beta)\ _q\ B$ and $f(B) \subseteq A$.

Proof: (i) ⇒ (ii) Let $A \subseteq Y$ be an IFπGSOS and let $c(\alpha, \beta) \in X$. Let $f(c(\alpha, \beta))\ _q\ A$. Then $c(\alpha, \beta)\ _q\ f^{-1}(A)$. (i) implies that $c(\alpha, \beta)\ _q\ int(f^{-1}(A))$, where $int(f^{-1}(A))$ is an IFOS in X. Let $B = int(f^{-1}(A))$. Since $int(f^{-1}(A)) \subseteq f^{-1}(A)$, $B \subseteq f^{-1}(A)$. Then, $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.(ii) ⇒ (i) Let $A \subseteq Y$ be an IFπGSOS and let $c(\alpha, \beta) \in X$. Suppose $f(c(\alpha, \beta))\ _q\ A$, then by (ii) there exists an IFOS B in X such that $c(\alpha, \beta)\ _q\ B$ and $f(B) \subseteq A$. Now $B \subseteq f^{-1}(f(B)) \subseteq f^{-1}(A)$. That is $B = int(B) \subseteq int(f^{-1}(A))$. Therefore, $c(\alpha, \beta)\ _q\ B$ implies $c(\alpha, \beta)\ _q\ int(f^{-1}(A))$.

Theorem 3.9: For any two IFcπGS continuous mappings f_1 and f_2 : $(X, \tau) \rightarrow (Y,\sigma)$, the function $(f_1,f_2) : (X, \tau) \rightarrow (Y \times Y , \sigma \times \sigma)$ is also an IFcπGS continuous mapping where $(f_1,f_2)(x) = ((f_1(x) ,f_2(x))$ for every $x \in X$.

Proof : Let $A \times B$ be an IFπGSOS in $Y \times Y$. Then $(f_1,f_2)^{-1} (A \times B)(x) = (A \times B) (f_1(x), f_2(x)) = \langle (x, \min(\mu_A(f_1(x)), \mu_B(f_2(x))), \max (\nu_A(f_1(x)), \nu_B(f_2(x))) \rangle = \langle (x, \min(f_1^{-1} (\mu_A(x)), f_2^{-1}(\mu_B (x))), \max(f_1^{-1} (\nu_A(x)), f_2^{-1}(\nu_B(x))) \rangle = f_1^{-1}(A) \cap f_2^{-1}(B) (x)$. Since f_1 and f_2 are IFcπGS continuous mappings,$f_1^{-1}(A)$ and $f_2^{-1}(B)$ are IFROSs in X. Since intersection of IFROSs is an IFROS, $f_1^{-1}(A) \cap f_2^{-1}(B)$ is an IFROS in X. Hence (f_1,f_2) is an IFcπGS continuous mappings.

Theorem 3.10: Let f: X → Y be a mapping. Then the following are equivalent.

(i) f is an IFcπGS continuous mapping.

(ii) $f^{-1}(B)$ is an IFROS in X for every IFπGSOS B in Y.

(iii) for every IFP $c(\alpha, \beta) \in X$ and for every IFπGSOS B in Y such that $f(c(\alpha, \beta)) \in B$ there exists an IFROS A in X such that $c(\alpha, \beta) \in A$ and $f(A) \subseteq B$.

Proof : (i) ⇒ (ii) is obviously.

(ii) ⇒ (iii) Let $c(\alpha, \beta) \in X$ and $B \subseteq Y$ such that $f(c(\alpha, \beta)) \in B$. This implies $c(\alpha, \beta) \in f^{-1}(B)$. Since B is an IF$\pi$GSOS B in Y, by hypothesis $f^{-1}(B)$ is an IFROS in X. Let $A = f^{-1}(B)$. Then $c(\alpha, \beta) \subseteq f^{-1}(f(c(\alpha, \beta))) \in f^{-1}(B) = A$. Therefore $c(\alpha, \beta) \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$. This implies $f(A) \subseteq B$.(iii) ⇒ (i) Let $B \subseteq Y$ be an IFπGSOS. Let $c(\alpha, \beta) \in X$ and $f(c(\alpha, \beta)) \in B$. By hypothesis, there exists an IFROS C in X such that $c(\alpha, \beta) \in C$ and $f(C) \subseteq B$. This implies $C \subseteq f^{-1}(f(C)) \subseteq$

$f^{-1}(B)$. Therefore, $c(\alpha, \beta) \in C \subseteq f^{-1}(B)$. That is $f^{-1}(B) = \cup_{c(\alpha, \beta) \in f^{-1}(B)} c(\alpha, \beta) \subseteq \cup_{c(\alpha, \beta) \in f^{-1}(B)} C \subseteq f^{-1}(B)$. This implies $f^{-1}(B) = \cup_{c(\alpha, \beta) \in f^{-1}(B)} C$. Since union of IFROSs is IFROS, $f^{-1}(B)$ is an IFROS in X. Hence f is an IFcπGS continuous mapping.

Theorem 3.11: If f : $(X, \tau) \to (Y, \sigma)$ is an IFAπGS continuous mapping and g : $(Y, \sigma) \to (Z, \eta)$ is an IFcπGS continuous mapping, then $g \circ f : (X, \tau) \to (Z, \eta)$ is an IFπGS continuous mapping.

Proof: Let A be an IFCS in Z. Then A is an IFπGSCS in Z. Since g is an IFcπGS continuous mapping, $g^{-1}(A)$ is an IFRCS in Y. Since f is an IFAπGS continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFπGSCS in X. Hence $g \circ f$ is an IFπGS continuous mapping.

Theorem 3.12: If f : $(X, \tau) \to (Y, \sigma)$ is an IFA continuous mapping and g : $(Y, \sigma) \to (Z, \eta)$ is an IFcπGS continuous mapping, then $g \circ f : (X, \tau) \to (Z, \eta)$ is an IF continuous mapping.

Proof: Let A be an IFCS in Z. Then A is an IFπGSCS in Z. Since g is an IFcπGS continuous mapping, $g^{-1}(A)$ is an IFRCS in Y. Since f is an IFA continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFCS in X. Hence $g \circ f$ is an IF continuous mapping.

4. INTUITIONISTIC FUZZY π GENERALIZED SEMI HOMEOMORPHISMS

In this section we introduce intuitionistic fuzzy π generalized semi homeomorphism and studied some of its properties.

Definition 4.1: A bijection mapping f: $(X, \tau) \to (Y,\sigma)$ is called an intuitionistic fuzzy π generalized semi homeomorphism (briefly, IFπGS homeomorphism) if f and f^{-1} are IFπGS continuous mappings.

We denote the family of all IFπGS-homeomorphism of a topological space (X, τ) onto itself by IFπGS-h(X, τ).

Example 4.2: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.2_a, 0.2_b), (0.6_a, 0.7_b) \rangle$, $G_2 = \langle y, (0.4_u, 0.7_v), (0.4_u, 0.2_v) \rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a bijection mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFπGS continuous mapping and f^{-1} is also an IFπGS continuous mapping. Therefore f is an IFπGS homeomorphism.

Theorem 4.3: Every IF homeomorphism is an IFπGS homeomorphism.

Proof: Let f: (X, τ) → (Y, σ) be an IF homeomorphism. Then f and f^{-1} are IF continuous and f is bijection. As every IF continuous mappings is IFπGS continuous mappings, we have f and f^{-1} are IFπGS continuous mappings. Therefore f is IFπGS homeomorphism.

The converse of the above Theorem need not be true, as seen from the following example.

Example 4.4: Let X = {a, b}, Y = {u, v} and G_1 = ⟨ x, (0.3_a, 0.2_b), (0.6_a, 0.7_b) ⟩, G_2 = ⟨ y, (0.5_u, 0.4_v), (0.4_u, 0.2_v) ⟩. Then τ = {0~, G_1, 1~} and σ = { 0~, G_2, 1~} are IFTs on X and Y respectively. Define a bijection mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFπGS homeomorphism but not an IF homeomorphism since f and f^{-1} are not an IF continuous mappings.

Theorem 4.5: Let f: (X, τ) → (Y, σ) be an IFπGS homeomorphism, then f is an IF homeomorphism if X and Y are IF$\pi a T_{1/2}$ space.

Proof: Let B be an IFCS in Y. Then f^{-1}(B) is an IFπGSCS in X, by hypothesis. Since X is an IF$\pi a T_{1/2}$ space, f^{-1}(B) is an IFCS in X. Hence f is an IF continuous mapping. By hypothesis f^{-1}: (Y, σ) → (X, τ) is a IFπGS continuous mapping. Let A be an IFCS in X. Then $(f^{-1})^{-1}$(A) = f(A) is an IFπGSCS in Y, by hypothesis. Since Y is an IF$\pi a T_{1/2}$ space, f(A) is an IFCS in Y. Hence f^{-1} is an IF continuous mapping. Therefore the mapping f is an IF homeomorphism.

Theorem 4.6: Let f: (X, τ) → (Y, σ) be an IFπGS homeomorphism, then f is an IFG homeomorphism if X and Y are IF$\pi b T_{1/2}$ space.

Proof: Let B be an IFCS in Y. Then f^{-1}(B) is an IFπGSCS in X, by hypothesis. Since X is an IF$\pi b T_{1/2}$ space, f^{-1}(B) is an IFGCS in X. Hence f is an IFG continuous mapping. By hypothesis f^{-1}: (Y, σ) → (X, τ) is a IFπGS continuous mapping. Let A be an IFCS in X. Then $(f^{-1})^{-1}$(A) = f(A) is an IFπGSCS in Y, by hypothesis. Since Y is an IF$\pi b T_{1/2}$ space, f(A) is an IFGCS in X. Hence f^{-1} is an IFG continuous mapping. Therefore the mapping f is an IFG homeomorphism.

Theorem 4.7: Let f: (X, τ) → (Y, σ) be a bijective mapping. If f is an IFπGS continuous mapping, then the following are equivalent.

(i) f is an IFπGS closed mapping
(ii) f is an IFπGS open mapping
(iii) f is an IFπGS homeomorphism.

Proof: (i) →(ii): Let f: (X, τ) → (Y, σ) be a bijective mapping and let f is an IFπGS closed mapping. This implies f^{-1}: (Y, σ) → (X, τ) is IFπGS continuous mapping. That is every IFOS in X is an IFπGSOS in Y. Hence f^{-1} is an IFπGS open mapping.

Proof: (ii) →(iii): Let f: (X, τ) → (Y, σ) be a bijective mapping and let f is an IFπGS open mapping. This implies f^{-1}: (Y, σ) → (X, τ) is IFπGS continuous mapping. Hence f and f^{-1} are IFπGS continuous mappings. That is f is an IFπGS homeomorphism.

(iii) →(i): Let f is an IFπGS homeomorphism. That is f and f^{-1} are IFπGS continuous mappings. Since every IFCS in X is an IFπGSCS in Y, f is an IFπGS closed mapping.

Remark 4.8: The composition of two IFπGS homeomorphisms need not be an IFπGS homeomorphism in general.

Example 4.9: Let X = {a, b}, Y = {c, d} and Z = {u, v}. Let G_1 = ⟨x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)⟩, G_2 = ⟨ y, (0.6_c, 0.1_d), (0.4_c, 0.3_d) ⟩ and G_3 = ⟨ z, (0.4_u, 0.4_v), (0.6_u, 0.2_v) ⟩. Then τ = {$0_\sim$, G_1, $1_\sim$}, σ = { $0_\sim$, G_2, $1_\sim$} and Ω = { $0_\sim$, G_3, $1_\sim$} are IFTs on X,Y and Z respectively. Define a bijection mapping f: (X, τ) → (Y, σ) by f(a) = c , f(b) = d and g: (Y, σ) → (Z, Ω) by f(c) = u , f(d) = v. Then f and f^{-1} are IFπGS continuous mappings. Also g and g^{-1} are IFπGS continuous mappings. Hence f and g are IFπGS homeomorphisms. But the composition g o f: X → Z is not an IFαG homeomorphism since g o f is not an IFπGS continuous mapping.

5. INTUITIONISTIC FUZZY Iπ GENERALIZED SEMI HOMEOMORPHISMS

Definition 5.1: A bijection mapping f: (X, τ) → (Y,σ) is called an intuitionistic fuzzy iπ generalized semi homeomorphism (IFiπGS homeomorphism in short) if f and f^{-1} are IFπGS irresolute mappings.

We denote the family of all IFiπGS -homeomorphism of a topological space (X, τ) onto itself by IFiπGS-h(X, τ).

Theorem 5.2: Every IFiπGS homeomorphism is an IFπGS homeomorphism but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFiπGS homeomorphism. Let B be IFCS in Y. This implies B is an IFπGSCS in Y. By hypothesis f^{-1}(B) is an IFπGSCS in X. Hence f is an IFπGS continuous mapping. Similarly we can prove f^{-1} is an IFπGS continuous mapping. Hence f and f^{-1} are IFπGS continuous mappings. This implies the mapping f is an IFπGS homeomorphism.

Example 5.3: Let X = {a, b}, Y = {u, v} and G_1 = ⟨ x, (0.4_a, 0.3_b), (0.6_a, 0.7_b) ⟩, G_2 = ⟨ y, (0.2_u, 0.1_v), (0.4_u, 0.5_v) ⟩. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a bijection mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFπGS homeomorphism. Let us consider an IFS G = ⟨ y, (0.3_u, 0.2_v), (0.7_u, 0.7_v) ⟩ in Y. Clearly G is an IFπGSCS in Y. But $f^{-1}(G)$ is not an IFπGSCS in X. That is f is not an IFπGS irresolute mapping. Hence f is not an IFiπGS homeomorphism.

Theorem 5.4: If f: X → Y is an IFiπGS homeomorphism, then $scl(f^{-1}(B)) = f^{-1}(scl(B))$ for every IFS B in Y.

Proof: Since f is an IFiπGS homeomorphism, f is an IFπGS irresolute mapping. Consider an IFS B in Y. Clearly scl(B) is an IFπGSCS in Y. This implies scl(B) is an IFπGSCS in Y. By hypothesis $f^{-1}(scl(B))$ is an IFπGSCS in X. Since $f^{-1}(B) \subseteq f^{-1}(scl(B))$, $scl(f^{-1}(B)) \subseteq scl(f^{-1}(scl(B))) = f^{-1}(scl(B)))$. This implies $scl(f^{-1}(B)) \subseteq f^{-1}(scl(B))$.Since f is an IFi$\pi$GS homeomorphism, f^{-1}: Y → X is an IFπGS irresolute mapping. Consider an IFS $f^{-1}(B)$ in X. Clearly $scl(f^{-1}(B))$ is an IFπGSCS in X. Hence $scl(f^{-1}(B))$ is an IFπGSCS in X. This implies $(f^{-1})^{-1}(scl(f^{-1}(B))) = f(scl(f^{-1}(B)))$ is an IFπGSCS in Y. Clearly $B = (f^{-1})^{-1}(f^{-1}(B)) \subseteq (f^{-1})^{-1}(scl(f^{-1}(B))) = f(scl(f^{-1}(B)))$. Therefore $scl(B) \subseteq scl(f(scl(f^{-1}(B)))) = f(scl(f^{-1}(B))$, since f^{-1} is an IFπGS irresolute mapping. Hence $f^{-1}(scl(B)) \subseteq f^{-1}(f(scl(f^{-1}(B)) = scl(f^{-1}(B))$. That is $f^{-1}(scl(B)) \subseteq scl(f^{-1}(B))$. This implies $scl(f^{-1}(B)) = f^{-1}(scl(B))$.

Theorem 5.5: If f: X → Y is an IFiπGS homeomorphism, then scl(f(B)) = f(scl(B)) for every IFS B in X.

Proof: Since f is an IFiπGS homeomorphism, f^{-1} is an IFπGS homeomorphism. Let us consider an IFS B in X. By theorem(5.4) scl(f(B)) = f(scl(B)) for every IFS B in X.

Theorem 5.6: If f : X → Y is a IFiπGS homeomorphism, then f (sint(B)) = sint(f(B)) for every IFS B in X.

Proof : For any IFS B in X, $sint(B) = (scl(B^c))^c$, Thus by utilizing Theorem 5.5, we obtain $f(sint(B)) = f(scl(B^c))^c) = (f(scl(B^c)))^c =(scl(f(B^c)))^c = (scl(f(B))^c)^c = sintf(B)$.

Theorem 5.7: If f : X → Y is a IFiπGS homeomorphism, then f^{-1} (sint(B)) = $sint(f^{-1}(B))$ for every IFS B in X.

Proof : Since f^{-1}: Y → X is also a IFiπGS homeomorphism , the proof follows from theorem 5.6.

Proposition 5.8: If f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, γ) are IFiπGS homeomorphisms then their composition g ∘ f : (X, τ) → (Z, γ) is also IFiπGS homeomorphisms.

Proof: Let f: X → Y and g: Y → Z be any two IFiπGS homeomorphisms. Let A be an IFπGSCS in Z. Then by hypothesis, $g^{-1}(A)$ is an IFπGSCS in Y. Then by hypothesis, $f^{-1}(g^{-1}(A))$ is an IFπGSCS in X. Hence $(g \text{ o } f)^{-1}$ is an IFπGS irresolute mapping. Now let B be an IFπGSCS in X. Then by hypothesis, f(B) is an IFπGSCS in Y. Then by hypothesis g(f(B)) is an IFπGSCS in Z. This implies g o f is an IFπGS irresolute mapping. Hence g o f is an IFiπGS homeomorphism. That is the composition of two IFiπGS homeomorphisms is an IFπGS homeomorphism in general.

Proposition 5.9: The set IFiπGS-h(X, τ) is a group under the composition of maps.

Proof : Define a binary operation * : IFiπGS-h(X, τ) X IFiπGS-h(X, τ) → IFiπGS-h(X, τ) by f * g = g ∘ f for all f∘ g ∈ IFiπGS-h(X, τ) and ∘ is the usual operation of composition of maps. Then g ∘ f ∈ IFiπGS-h(X, τ). We know that the composition of maps is associative and the identity map I : (X, τ) → (X, τ) belonging to IFiπGS-h(X, τ) servers as the identity element. If f∈ IFiπGS-h(X, τ), then f^{-1}∈ IFiπGS-h(X, τ) such that $f \circ f^{-1} = f^{-1} \circ f = I$ and so inverse exists for each element of IFiπGS-h(X, τ). Therefore, (IFiπGS- h(X, τ) , ∘) is a group under the operation of composition of maps.

Theorem 5.10 : Let f: X → Y be a IFiπGS homeomorphism. Then f induces an isomorphism form the group IFiπGS-h(X, τ) onto the group IFiπGS-h(Y, σ).

Proof : Using the map f, we define a map θ_f : IFiπGS-h(X, τ) → IFiπGS-h(Y, σ), by θ_f (h): $f \circ h \circ f^{-1}$ for every h ∈ IFiπGS-h(X, τ). Then θ_f is a bijection. Further for all h_1,h_2 ∈ IFiπGS-h(X, τ), $\theta_f (h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = \theta_f (h_1) \circ \theta_f (h_2)$. Therefore, θ_f is a homeomorphism and so it is an isomorphism induced by f.

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