Int. J. Contemp. Math. Sciences, Vol. 8, 2013, no. 3, 125 - 131
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Some Results on the Degree of a Vertex of a Graph with Respect to Any Vertexset

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Abstract

In this paper we have defined the degree of any vertex of a graph with respect to any vertexset. Based on this definition we have proved some results. Also we have obtained the relation between this degree and ordinary degree.

Mathematics Subject Classification: Primary 05C07

Keywords: Ordinary degree, Degree of any vertex of a graph with respect to a vertexset

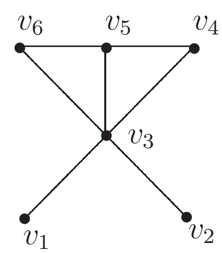
1. Introduction

For usual notations and terminologies we refer [1], [2]. Let $G(E, V)$ be a simple graph. The degree of a vertex $v_i \in V$ is denoted by $d(v_i)$ or $d_G(v_i)$ and is the number of vertices of V that are adjacent to v_i.
The new degree concepts in graphs has been defined by S S Kamath and R S Bhat [3]. In this paper we defined the degree of any vertex of a graph with respect to any vertexset.

2. Definition and Results

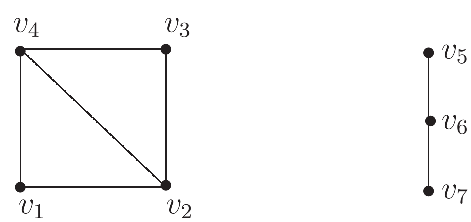
Degree of any vertex of a graph with respect to a vertexset :
Definition : Let $G = (V, E)$ be a simple graph and A be any vertex set. The degree of a vertex $v_i \in V$ of a graph G with respect to A is the number of vertices of A that are adjacent to v_i. This degree is denoted by $d_A(v_i)$.

Here A may or may not be the subset of V. However, if $A = \emptyset$ then $d_A(v_i) = 0$ and if $A = V$ then $d_A(v_i) = d(v_i) \quad \forall\, v_i \in V$.
***Example* 1 :**



Let $A = \{v_2, v_4, v_6\}$ then by our definition $d_A(v_1) = 0$, $d_A(v_2) = 0$, $d_A(v_3) = 3$, $d_A(v_4) = 0$, $d_A(v_5) = 2$, $d_A(v_6) = 0$.

***Example* 2 :**



Let $A = \{v_1, v_3, v_5, v_7\}$ then by our definition $d_A(v_1) = 0$, $d_A(v_2) = 2$, $d_A(v_3) = 0$, $d_A(v_4) = 2$, $d_A(v_5) = 0$, $d_A(v_6) = 2$, $d_A(v_7) = 0$.

The following results are straight forward from the definition.

1. For any vertexset A, $d_A(v_i) \leq d(v_i) \quad \forall\, v_i \in V$.

2. For any vertexset A, $d(v_i) = d_A(v_i) + d_{A'}(v_i) \quad \forall\, v_i \in V$ where A' is the complement of A.
   $\therefore\ 2q = \sum_{v_i \in V} d(v_i) = \sum_{v_i \in V} d_A(v_i) + \sum_{v_i \in V} d_{A'}(v_i)$ where q is the number of edges of a graph G.

3. $\sum_{v_i \in A} d_{A'}(v_i) = \sum_{v_i \in A'} d_A(v_i)$.

Now we prove some results concerning the new definition of degree of a vertex with respect to any vertexset.

***Theorem* 1 :** Let $G = (V,\, E)$ be any simple graph and A be any vertexset. Then $\sum\limits_{v_i \in V} d_A(v_i) = \sum\limits_{v_i \in A} d(v_i)$.

***Proof* :** Consider, $\sum\limits_{v_i \in V} d_A(v_i)$

$$= \sum_{v_i \in A} d_A(v_i) + \sum_{v_i \in A'} d_A(v_i) \quad [\text{ Since } V = A \cup A' \,]$$

$$= \sum_{v_i \in A} d_A(v_i) + \sum_{v_i \in A} d_{A'}(v_i) \quad \left[\text{Since} \sum_{v_i \in A'} d_A(v_i) = \sum_{v_i \in A} d_{A'}(v_i)\right]$$

$$= \sum_{v_i \in A} [d_A(v_i) + d_{A'}(v_i)]$$

$$= \sum_{v_i \in A} d(v_i).$$

***Theorem* 2 :** Let $G = (V,\, E)$ be any simple graph. If A and B are any two vertex sets then $\sum\limits_{v_i \in V} d_{A \cup B}(v_i) = \sum\limits_{v_i \in V} d_A(v_i) + \sum\limits_{v_i \in V} d_B(v_i) - \sum\limits_{v_i \in V} d_{A \cap B}(v_i)$.

***Proof* :** Consider, $\sum\limits_{v_i \in V} d_{A \cup B}(v_i)$

$$= \sum_{v_i \in A \cup B} d(v_i) \quad [\text{ By } \textit{Theorem } 1 \,]$$

$$= \sum_{v_i \in A} d(v_i) + \sum_{v_i \in B} d(v_i) - \sum_{v_i \in A \cap B} d(v_i)$$

$$[\text{ Since } E(A \cup B) = E(A) + E(B) - E(A \cap B) \,]$$

$$= \sum_{v_i \in V} d_A(v_i) + \sum_{v_i \in V} d_B(v_i) - \sum_{v_i \in V} d_{A \cap B}(v_i).$$

***Note* :** If $A \cap B = \emptyset$ then $\sum\limits_{v_i \in V} d_{A \cup B}(v_i) = \sum\limits_{v_i \in V} d_A(v_i) + \sum\limits_{v_i \in V} d_B(v_i)$.

***Theorem* 3 :** Let $G = (V,\, E)$ be any simple graph. If A and B are any two vertex sets then $\sum\limits_{v_i \in V} d_{A-B}(v_i) = \sum\limits_{v_i \in V} d_A(v_i) - \sum\limits_{v_i \in V} d_{A \cap B}(v_i)$.

***Proof* :** Consider, $\sum\limits_{v_i \in V} d_{A-B}(v_i)$

$$= \sum_{v_i \in A-B}^{n} d(v_i)$$

$$= \sum_{v_i \in A} d(v_i) - \sum_{v_i \in A \cap B} d(v_i)$$

[Since $E(A - B) = E(A) - E(A \cap B)$]

$$= \sum_{v_i \in V} d_A(v_i) - \sum_{v_i \in V} d_{A \cap B}(v_i).$$

***Theorem* 4 :** Let $G = (V, E)$ be any simple graph. If A and B are any two vertex sets then $\sum_{v_i \in V} d_{A \triangle B}(v_i) = \sum_{v_i \in V} d_A(v_i) + \sum_{v_i \in V} d_B(v_i) - 2 \sum_{v_i \in V} d_{A \cap B}(v_i)$ where $A \triangle B$ is the symmetric difference of A and B.

Proof **:** Consider, $\sum_{v_i \in V} d_{A \triangle B}(v_i)$

$$= \sum_{v_i \in A \triangle B} d(v_i)$$

$$= \sum_{v_i \in A - B} d(v_i) + \sum_{v_i \in B - A} d(v_i)$$

$$= \sum_{v_i \in A} d(v_i) - \sum_{v_i \in A \cap B} d(v_i) + \sum_{v_i \in B} d(v_i) - \sum_{v_i \in B \cap A} d(v_i)$$

$$= \sum_{v_i \in A} d(v_i) + \sum_{v_i \in B} d(v_i) - 2 \sum_{v_i \in A \cap B} d(v_i)$$

$$= \sum_{v_i \in V} d_A(v_i) + \sum_{v_i \in V} d_B(v_i) - 2 \sum_{v_i \in V} d_{A \cap B}(v_i).$$

***Theorem* 5 :** Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be any two graphs with their orders m, n respectively. For any Vertexset A,

$$\sum_{v_i \in G_1 + G_2} d_A(v_i) = \sum_{v_i \in G_1} d_A(v_i) + \sum_{v_i \in G_2} d_A(v_i) + \sum_{v_i \in K_{m,n}} d_A(v_i).$$

Proof **:** Consider, $\sum_{v_i \in G_1 + G_2} d_A(v_i)$

$$= \sum_{v_i \in A} d_{G_1 + G_2}(v_i)$$

$$= \sum_{v_i \in A} [d_{G_1}(v_i) + d_{G_2}(v_i) + d_{K_{m,n}}(v_i)]$$

[Since $E(G_1 + G_2) = E(G_1) + E(G_2) + E(K_{m,n})$]

$$= \sum_{v_i \in A} d_{G_1}(v_i) + \sum_{v_i \in A} d_{G_2}(v_i) + \sum_{v_i \in A} d_{K_{m,n}}(v_i)$$

$$= \sum_{v_i \in G_1} d_A(v_i) + \sum_{v_i \in G_2} d_A(v_i) + \sum_{v_i \in K_{m,n}} d_A(v_i).$$

***Theorem* 6 :** Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be any two graphs. For any

Vertexset A of $G_1 \times G_2$, $\sum_{(v_i, v_j) \in G_1 \times G_2} d_A(v_i, v_j) = \sum_{v_i \in G_1} d_{A_1}(v_i) + \sum_{v_j \in G_2} d_{A_2}(v_j)$

where $A_1 = \{v_i / (v_i, v_j) \in A\}$ and $A_2 = \{v_j / (v_i, v_j) \in A\}$.

Proof **:** Consider, $\sum_{(v_i, v_j) \in G_1 \times G_2} d_A(v_i, v_j)$

$$= \sum_{(v_i, v_j) \in A} d_{G_1 \times G_2}(v_i, v_j)$$

$$= \sum_{(v_i, v_j) \in A} [d_{G_1}(v_i) + d_{G_2}(v_j)]$$

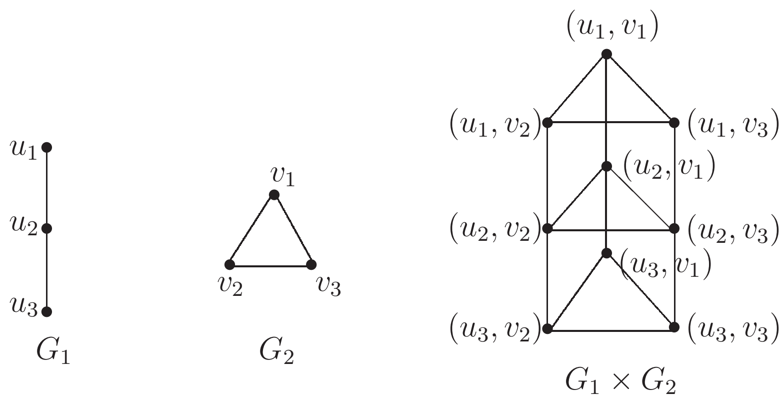
[Since $d_{G_1 \times G_2}(v_i, v_j) = d_{G_1}(v_i) + d_{G_2}(v_j) \ \forall \ (v_i, v_j) \in G_1 \times G_2$]

$$= \sum_{(v_i, v_j) \in A} d_{G_1}(v_i) + \sum_{(v_i, v_j) \in A} d_{G_2}(v_j)$$

$$= \sum_{v_i \in A_1} d_{G_1}(v_i) + \sum_{v_j \in A_2} d_{G_2}(v_j)$$

$$= \sum_{v_i \in G_1} d_{A_1}(v_i) + \sum_{v_j \in G_2} d_{A_2}(v_j).$$

***Example* 3 :**



(**i**) If $A = \{(u_1,\, v_3),\, (u_2,\, v_1),\, (u_3,\, v_3)\}$ then

$$\text{LHS} = \sum_{(v_i,v_j)\in G_1\times G_2} d_A(v_i, v_j) = 3 + 4 + 3 = 10 \text{ and}$$

$$\text{RHS} = \sum_{v_i\in G_1} d_{A_1}(v_i) + \sum_{v_j\in G_2} d_{A_2}(v_j) = (1 + 2 + 1) + (2 + 2 + 2) = 10.$$

$\therefore$ LHS = RHS.

(**ii**) If $A = \{(u_1,\, v_2),\, (u_2,\, v_3)\ ,(u_3,\, v_1),(u_3,\, v_2)\}$ then

$$\text{LHS} = \sum_{(v_i,v_j)\in G_1\times G_2} d_A(v_i, v_j) = 3 + 4 + 3 + 3 = 13 \text{ and}$$

$$\text{RHS} = \sum_{v_i\in G_1} d_{A_1}(v_i) + \sum_{v_j\in G_2} d_{A_2}(v_j) = (1 + 2 + 1 + 1) + (2 + 2 + 2 + 2) = 13.$$

$\therefore$ LHS = RHS.

***Theorem* 7 :** Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be any two graphs with their orders m, n respectively. For any Vertexset A of $G_1[G_2]$

$$\sum_{(v_i,v_j)\in G_1[G_2]} d_A(v_i, v_j) = n \sum_{v_i\in G_1} d_{A_1}(v_i) + \sum_{v_j\in G_2} d_{A_2}(v_j)$$

where $A_1 = \{v_i/(v_i,\ v_j) \in A\}$ and $A_2 = \{v_j/(v_i,\ v_j) \in A\}$.

***Proof* :** Consider, $\sum_{(v_i,\, v_j)\in G_1[G_2]} d_A(v_i, v_j)$

$$= \sum_{(v_i,\, v_j)\in A} d_{G_1[G_2]}(v_i,\, v_j)$$

$$= \sum_{(v_i,\, v_j)\in A} [n\, d_{G_1}(v_i) + d_{G_2}(v_j)]$$

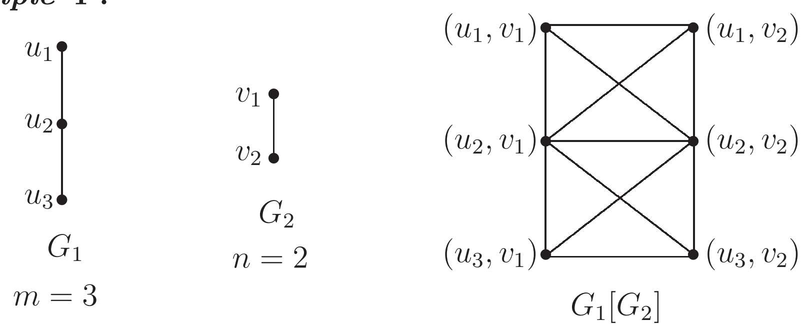
[Since $d_{G_1[G_2]}(v_i,\ v_j) = n\ d_{G_1}(v_i) + d_{G_2}(v_j)\ \ \forall\, (v_i, v_j) \in\ G_1[G_2]$]

$$= n \sum_{(v_i,\, v_j)\in A} d_{G_1}(v_i) + \sum_{(v_i,\, v_j)\in A} d_{G_2}(v_j)$$

$$= n \sum_{v_i \in A_1} d_{G_1}(v_i) + \sum_{v_j \in A_2} d_{G_2}(v_j)$$
$$= n \sum_{v_i \in G_1} d_{A_1}(v_i) + \sum_{v_j \in G_2} d_{A_2}(v_j).$$

Similarly, $\sum_{(v_i,\, v_j) \in G_2[G_1]} d_A(v_i, v_j) = m \sum_{v_i \in G_1} d_{A_1}(v_i) + \sum_{v_j \in G_2} d_{A_2}(v_j).$

***Example* 4 :**



If $A = \{(u_1, v_2), (u_2, v_1), (u_3, v_2)\}$ then $A_1 = \{u_1, u_2, u_3\}$ and $A_2 = \{v_2, v_1, v_2\}$
LHS $= \sum_{(v_i, v_j) \in G_1[G_2]} d_A(v_i, v_j) = 3 + 5 + 3 = 11.$
RHS $= n \sum_{v_i \in G_1} d_{A_1}(v_i) + \sum_{v_j \in G_2} d_{A_2}(v_j) = 2(1 + 2 + 1) + (1 + 1 + 1) = 11.$
$\therefore$ LHS = RHS.

References

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Received: October, 2012