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Some Results on Subtractive Ideals in Semirings

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Abstract

In this article some results on ϕ-prime and subtractive ideals in semirings are investigared.

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1 Introduction

This paper is concerned with generalizing some results of ring theory ro semiring theory. Throughout this paper a semiring will be defined as follows: A semiring is a set R together with two binary operations called addition "+" and multiplication "." such that $(R, +)$ is a commutative semigroup and $(R, .)$ is semigroup; connecting the two algebraic structures are the distrubutive laws: $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for all $a, b, c \in R$.

We shall assume that $(R, +, .)$ has an absorbing zero 0, that is $a + 0 = 0 + a = a$ and $a.0 = 0.a = 0$ holds for all $a \in R$. A subset I of a semiring R is called an ideal of R if for $a, b \in I$, and $r \in R$, $a + b \in I$ and $ra \in I$. Throughout this paper we let the semiring R commutative with identity 1. Let R be a semiring, a subtractive ideal I is a ideal of R such that if $x, x + y \in I$, then $y \in I$ (So $\{0_R\}$ is a subtractive ideal of R). A prime ideal of R is a proper ideal P of R in which $a \in P$ or $b \in P$ whenever $ab \in P$. The collection of all ideal of R denoted by $I(R)$ and the collection of all proper ideal of R denoted by $I^*(R)$. The ideal I is said to be ϕ- prime ideal if $ab \in I - \phi(I)$, then $a \in I$ or $b \in I$ where ϕ is the function $\phi : I(R) \longrightarrow I(R) \cup \{\phi\}$.

2 Main Results

Remarks. We can consider $\phi(I) \subseteq I$ because we have $I - \phi(I) = I - (\phi(I) \cap I)$.

Lemma 1.2. Every prime ideal is ϕ- prime ideal.

Definition 1.2. For two functions $\psi_1, \psi_2 : I(R) \longrightarrow I(R) \cup \{\phi\}$ we set $\psi_1 \leq \psi_2$ if $\psi_1(I) \subseteq \psi_2(I)$ for all $I \in I(R)$.

Definition 2.2. Let R be a semiring.

Define the following functions $\phi_\alpha : I(R) \longrightarrow I(R) \cup \{\phi\}$:

$$\begin{array}{ll} \phi_\phi & \phi(J) = \phi \\ \phi_0 & \phi(J) = 0 \\ \phi_2 & \phi(J) = J^2 \\ \phi_n & \phi(J) = J^n \\ \phi_w & \phi_w(J) = \cap J^n \\ \phi_1 & \phi(J) = J \end{array}$$

observe that $\phi_\phi \leq \phi_0 \leq \phi_w \leq \cdots \leq \phi_{n+1} \leq \phi_n \leq \cdots \leq \phi_2 \leq \phi_1$.

Leema 2.2. Let R be a semiring and I be a proper ideal of R also $\psi_1, \psi_2 : I(R) \longrightarrow I(R) \cup \{\phi\}$ be two functions such that $\psi_1 \leq \psi_2$. If I is a ψ_1- prime ideal, then I is a ψ_2- prime ideal.

Proof. Suppose that $ab \in I - \psi_2(I)$ where $a, b \in R$. since $\psi_1(I) \subseteq \psi_2(I)$, $I - \psi_2(I) \subseteq I - \psi_1(I)$. Hence $ab \in I - \psi_1(I)$ so, $a \in I$ or $b \in I$.

In the following proposition, $R_{n\times n}$ and $\phi_{n\times n}(I_{n\times n})$ denote the collection of Matrixs $n \times n$ with entry respectively in R and $\phi(I)$.

Proposition 1.2. Let R be a semiring and I be a proper ideal of R also $I_{n\times n}$ be a $\phi_{n\times n}$- prime ideal of $R_{n\times n}$. Then I is a ϕ- prime ideal of R.

Proof. Suppose that $ab \in I - \phi(I)$ where $a, b \in R$, then we have $aE_{11}bE_{11} \subseteq I_{n\times n} - \phi_{n\times n}(I_{n\times n})$, hence $aE_{11} \subseteq I_{n\times n}$ or $bE_{11} \subseteq I_{n\times n}$, so $a \in I$ or $b \in I$.

Lemma 3.2. Let I be a subtractive ϕ- prime ideal of semiring R. Then $I^2 \subseteq \phi(I)$ of I is a prime ideal.

Proof. Let $I^2 \nsubseteq \phi(I)$ and $ab \in I$ for some $a, b \in R$. If $ab \notin \phi(I)$, then $a \in I$ or $b \in I$ (I is a ϕ- prime ideal). So, assume that $ab \in \phi(I)$. If $aI \nsubseteq \phi(I)$, then there exists $x \in I$ such that $ax \notin \phi(I)$. Hence $a(x + b) \in I - \phi(I)$ and then we have $a \in I$ or $b + x \in I$. Thus $a \in I$ or $b \in I$, since I is subtractive ideal.

Therefore let $aI \subseteq \phi(I)$. Now we have two case first assume $bI \nsubseteq \phi(I)$. Thus, there exists $y \in I$ such that $by \notin \phi(I)$ and hence we have $(a + y)b \in I - \phi(I)$, therefore $a \in I$ or $b \in I$ since I is subtractive ideal. Second case assume that $bI \subseteq \phi(I)$. because $I^2 \nsubseteq \phi(I)$ therefore there exists $r, s \in I$ such that $rs \notin \phi(I)$. Hence $(a + r)(b + s) \in I - \phi(I)$, so, $a + r \in I$ or $b + s \in I$. Thus $a \in I$ or $b \in I$ because I is subtractive ideal.

Definition 3.2. The semiring R is said to be ϕ- semiprime if for evry $n \in N$ and ideal I of R, $I^2 \subseteq \phi(I)$ implies that $I \subseteq \phi(I)$.

Corollary 1.2. Let R be a ϕ- semiprime semiring and I a subtractive ideal of R. Then I is a ϕ- prime ideal if and only if $I = \phi(I)$ or I is a prime ideal.

Proof. Apply Lemma 3.2 and Definition 3.2.

Corollary 2.2. Let I be a subtractive φ- prime ideal of R and $\varphi \leq \varphi_3$. Then I is a φ_w- prime ideal.

Proof. If I is a prime ideal then we have no things for prove because for every φ, I is φ- prime. Now, suppose that $I^2 \subseteq \phi(I)$, then we have $I^2 \subseteq \varphi(I) \subseteq \varphi_3(I) = I^3$ because $\varphi \leq \varphi_3$. Therefore $\varphi(I) = I^n$ for every $n \in N$, hence I is φ_n- prime for every $n \geq 2$, thus I is φ_w- prime.

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