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Fixed Point Result in Fuzzy Menger Space with EA Property

Rajesh Shrivastav [1], Vivek Patel [2] and Vanita Ben Dhagat [3]

1 Benajir Govt. College of Science, Bhopal(M.P.)

2 Laxmi Narayan College of Technology, Bhopal (M.P.), India

3 Jai Narayan College of Technology, Bhopal (M.P.), India
vanita1_dhagat@yahoo.co.in

Abstract

The object of this paper is to define JSR maps by using the notion of property (EA) and prove a common fixed point theorem for four maps in complete fuzzy menger space .

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1. INTRODUCTION

The notion of probabilistic metric space is introduced by Menger in 1942 [10] and the first result about the existence of a fixed point of a mapping which is defined on a Menger space is obtained by Sehgel and Barucha-Reid.

A number of fixed point theorems for single valued and multivalued mappings in menger probabilistic metric space have been considered by many authors [2],[3],[4],[5],[6],[7]. In 1998, Jungck [8] introduced the concept weakly compatible maps and proved many theorems in metric space. Hybrid fixed point theory for nonlinear single valued and multivalued maps is a new development in the domain of contraction type multivalued theory ([4],[7],[11],[12],[13],[14]). Jungck and Rhoades [8] introduced the weak compatibility to the setting of single valued and multivalued maps. Singh and Mishra introduced (IT)-commutativity for hybrid pair of single valued and multivalued maps which need not be weakly compatible.

Recently, Aamri and El Moutawakil [1] defined a property (EA) for self maps which contained the class of noncompatible maps. More recently, Kamran [9] extended the property (EA) for a hybrid pair of single valued and multivalued maps and generalized the (IT) commutativity for such pair.

The aim of this paper is to define a new property which contains the property (EA) for hybrid pair of single valued and multivalued maps and give some common fixed point theorems under hybrid contractive conditions in fuzzy meger space.

2. PRELIMINARIES

Let us define and recall some definitions:

Definition 2.1 A fuzzy probabilistic metric space (FPM space) is an ordered pair (X,F_α) consisting of a nonempty set X and a mapping F_α from XxX into the collections of all distribution functions $F_\alpha \in R$ for all $\alpha. \in [0,1]$. For $x, y \in X$ we denote the distribution function $F_\alpha(x,y)$ by $F_{\alpha(x,y)}$ and $F_{\alpha(x,y)}(u)$ is the value of $F_{\alpha(x,y)}$ at u in R.

The functions $F_{\alpha(x,y)}$ for all $\alpha. \in [0,1]$ assumed to satisfy the following conditions:

(a) $F_{\alpha(x,y)}(u) = 1 \ \forall \ u > 0$ iff $x = y$,
(b) $F_{\alpha(x,y)}(0) = 0 \ \forall \ x, y$ in X,
(c) $F_{\alpha(x,y)} = F_{\alpha(y,x)} \ \forall \ x, y$ in X,
(d) If $F_{\alpha(x,y)}(u) = 1$ and $F_{\alpha(y,z)}(v) = 1$ then $F_{\alpha(x,z)}(u+v) = 1 \ \forall \ x, y, z$ in X and $u, v > 0$

Definition 2.2 A commutative, associative and non-decreasing mapping $t: [0,1] \times [0,1] \to [0,1]$ is a t-norm if and only if $t(a,1)=a$ for all $a \in [0,1]$, $t(0,0)=0$ and $t(c,d) \geq t(a,b)$ for $c \geq a$, $d \geq b$.

Definition 2.3 A Fuzzy Menger space is a triplet (X,F_α,t), where (X,F_α) is a FPM-space,t is a t-norm and the generalized triangle inequality

$$F_{\alpha(x,z)}(u+v) \geq t\,(F_{\alpha(x,z)}(u), F_{\alpha(y,z)}(v)) \quad \text{holds for all x, y, z in X u, v > 0 and } \alpha. \in [0,1]$$

The concept of neighborhoods in Fuzzy Menger space is introduced as

Definition 2.4 Let (X,F_α,t) be a Fuzzy Menger space. If $x \in X$, $\varepsilon > 0$ and $\lambda \in (0,1)$, then (ε,λ) - neighborhood of x, called $U_x(\varepsilon,\lambda)$, is defined by

$$U_x(\varepsilon,\lambda) = \{y\in X: F_{\alpha(x,y)}(\varepsilon)>(1-\lambda)\}$$

An (ε,λ)-topology in X is the topology induced by the family $\{U_x(\varepsilon,\lambda): x \in X, \varepsilon > 0, \alpha.\in [0,1]$ and $\lambda\in(0,1)\}$ of neighborhood.

Remark: If t is continuous, then Fuzzy Menger space (X,F_α,t) is a Housdroff space in (ε,λ)-topology.

Let (X,F_α,t) be a complete Fuzzy Menger space and $A\subset X$. Then A is called a bounded set if
$$\lim_{u\to\infty} \inf_{x,y\in A} F_{\alpha(x,y)}(u) = 1$$

Definition 2.5 A sequence $\{x_n\}$ in (X,F_α,t) is said to be convergent to a point x in X if for every $\varepsilon>0$and $\lambda>0$, there exists an integer $N=N(\varepsilon,\lambda)$ such that $x_n \in U_x(\varepsilon,\lambda)$ for all $n \geq N$ or equivalently $F_\alpha(x_n, x; \varepsilon) > 1-\lambda$ for all $n \geq N$ and $\alpha\in[0,1]$.

Definition 2.6 A sequence $\{x_n\}$in (X,F_α,t) is said to be cauchy sequence if for every $\varepsilon > 0$ and $\lambda > 0$, there exists an integer $N=N(\varepsilon,\lambda)$ such that $F_\alpha(x_n,x_m; \varepsilon) > 1-\lambda\ \forall n, m \geq N$ for all $\alpha\in[0,1]$.

Definition 2.7 A Fuzzy Menger space (X,F_α,t) with the continuous t-norm is said to be complete if every Cauchy sequence in X converges to a point in X for all $\alpha\in[0,1]$.

Definition 2.8 Let (X,F_α,t) be a Fuzzy Menger space. Two mappings f, g :X→X are said to be weakly comptable if they commute at coincidence point for all $\alpha\in[0,1]$.

Lemma 1 Let $\{x_n\}$ be a sequence in a Fuzzy Menger space (X,F_α,t), where t is continuous and $t(p,p) \geq p$ for all $p\in[0,1]$, if there exists a constant k(0,1) such that $\forall p > 0$ and $n\in N$
$$T(F_\alpha(x_n,x_{n+1}; kp)) \geq t(F_\alpha(x_{n-1},x_n; p)),$$
for all $\alpha\in[0,1]$ then $\{x_n\}$ is cauchy sequence.

Lemma 2 If (X,d) is a metric space, then the metric d induces, a mapping F_α: XxX→L defined by $F_\alpha(p, q) = H_\alpha(x- d(p, q))$, $p, q \in R$ for all $\alpha\in[0,1]$. Further if $t: [0,1] \times [0,1]\to [0,1]$ is defined by $t(a,b) = \min\{a,b\}$, then (X,F_α,t) is a Fuzzy Menger space. It is complete if (X,d) is complete.

Definition 2.10: Let (X, F_α,t) be a Fuzzy Menger space. Maps s: X → X and T: X→CB(X)

(1) s is said to be T weakly commuting at $x\in X$ if $ssx \in Tsx$.

(2) are weakly compatible if the commute at their coincidence points,
i.e. if $sTx = Tsx$ whenever $sx\in Tx$.

(3) are (IT) commuting at $x\in X$ if $sTx \subset Tsx$ whenever $sx\in Tx$.

Definition 2.11: - Let (X, F_α,t) be a Fuzzy Menger space. Maps f, g: X→X are said to satisfy the property (EA) if there exists a sequence $\{x_n\}$ in x such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z \in X$$

Definition 2.12: -Maps f: X→X and T:X→CB(X) are said to satisfy the property (EA) if there exists a sequence $\{x_n\}$ in X, some z in X and A in CB(X) such that

$$\lim_{n\to\infty} fx_n = z \in A = \lim_{n\to\infty} Tx_n.$$

Definition 2.13: -Maps f,g,S,G: X→X. The pair (f,S) and (g,G) are said to satisfy the common property (EA) if there exist two sequences $\{x_n\}$,$\{y_n\}$ in

X and some z in X such that

$$\lim_{n\to\infty} Gy_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = z$$

Definition 2.14: -Maps f,g: X→X and S,G:X→CB(X). The maps pair (f,S) and (g,G) are said to satisfy the common property (EA) if there exist two sequences $\{x_n\}$,$\{y_n\}$ in X and some z in X, and A,B such that

$$\lim_{n\to\infty} Sx_n = A \text{ and } \lim_{n\to\infty} Gy_n = B, = \lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = z \in A\cap B.$$

Definition 2.15:- Let (X, F_α,t) be a Menger space. Let f and g be two self maps of a menger space. The pair {f,g} is said to be f-JSR mappings iff

$$\mu\ .t(F_\alpha\ (fgx_n, gx_n,p)) \geq \mu.\ t(\ F_\alpha(ffx_n,fx_n,p))$$

where μ = lim Sup or lim inf and $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some z∈X and for all t(p,p) > p.

Example Let X = [0,1] with d(x,y) =|x-y| and f,g are two self mapping on X defined by fx =2/(x+2), gx = 1/(x+1) for x ∈Now the sequence $\{x_n\}$ in X is defined as $x_n = 1/n$,n∈N. Then we have $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = 1$

$|fgx_n-gx_n| \to 1/3$ and $|ffx_n-fx_n| \to 2/3$ as n→∞. Clearly we have

$$|fgx_n-Tx_n| < |ffx_n-fx_n|.$$

Thus pair {f,g} is f-JSR mapping. But this pair is neither compatible nor weakly compatible or other non commuting mapping S. Hence pair of JSR mapping is more general then others.

Let f: X→X self map of a menger space (X, F_α,t) and S: X → CB(X) be multivalued map. The pair {f,S} is said to be hybrid S-JSR mappings for all Δ(p,p) > p iff

$$\mu\ .t(F_\alpha\ (Sfx_n,fx_n,p)) \geq \mu\ .t(F_\alpha(SSx_n,Sx_n,p))$$

where μ = lim Sup or lim inf and $\{x_n\}$ is a sequence in X such that

$\lim_{n\to\infty} fx_n = z \in A = \lim_{n\to\infty} Sx_n$.

Let $\phi : R \to R^+$ be continuous and satisfying the conditions

(i) ϕ is nonincreasing on R,

(ii) $\phi(t) > t$, for each $t \in (0,\infty)$.

3. MAIN RESULTS

Theorem 3.1: Let (X, F_α,t) be a Menger space. Let f,g: X→X and S,G: X→ CB(X) such that

(I) (f,S) and (g,G) satisfy the common property (EA),

(II) f(X) and g(X) are closed,

(III) Pair (f,S) is S-JSR maps and pair (g,G) is G-JSR maps,

(IV) $t(H_\alpha(Sx,Gy,kp))$

$$\geq\phi[\min\{t(F_\alpha(fx,gy,p)),t(F_\alpha(fx,Sx,p)),t(F_\alpha(gy,Gy,p)),t(F_\alpha(fx,Gy,p)),t(F_\alpha(Sx,gy,p))\}]$$

Then f, g, S and G have a common fixed point in X.

Proof: By (I) there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X and u∈X, A, B in CB(X) such that

$\lim_{n\to\infty} Sx_n = A$ and $\lim_{n\to\infty} Gy_n = B$, and $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = u \in A\cap B$.

Since f(X) and g(X) are closed, we have u = fv and u = gr for some v,r∈X.

Now by (IV) we get

$t(H_\alpha (Sx_n,Gr,kp)) \geq\phi[\min\{t(F_\alpha(fx_n,gr,p)),t(F_\alpha(fx_n,Sx_n ,p)),t(F_\alpha(gr,Gr,p)),$

$t(F_\alpha (fx_n,Gr ,p)),\ t(F_\alpha (Sx_n,gr,p))\}]$

On taking limit n→∞, we obtain

$t(H_\alpha (A,Gr,kp))$

$\geq \phi[\min\{\ t(F_\alpha(fv,gr,p)),\ t(F_\alpha(fv,A,p)),\ t(F_\alpha (gr,Gr,p)),\ t(F_\alpha (fv,Gr ,p)),\ t(F_\alpha (A,gr,p))\}]$

$\geq \phi\ t((F_\alpha (gr,Gr,p)))$

$> t(F_\alpha(gr,Gr,p)) > F_\alpha(gr,Gr,p)$

Since gr =fv ∈ A and $F_\alpha(gr,Gr,p) \geq H_\alpha(A,Gr,p) > F_\alpha(gr,Gr,p)$.

Hence gr∈Gr

Similarly

$t((H_\alpha(Sv,Gy_n,kp)) \geq\phi[\min\{t(F_\alpha(fv,gy_n,p)),\ t(F_\alpha(fv,Sv,p)),\ t(F_\alpha(gy_n,Gy_n,p)),$

$t(F_\alpha(fv,Gy_n,p)), t(F_\alpha(Sv,gy_n,p))\}]$

$t(H_\alpha(Sv,B,kp)) \geq \phi[\min\{t(F_\alpha(fv,gr,p)), t(F_\alpha(fv,Sv,p)), t(F_\alpha(gr,B,p)), t(F_\alpha(fv,B,p)), t(F_\alpha(Sv,gr,p))\}]$

$\geq \phi (t(F_\alpha(fv,Sv,p)))$

$> t(F_\alpha(fv,Sv,p)) > F_\alpha(fv,Sv,p)$

Since $fv = gr \in A$ and

$t(F_\alpha(fv,Sv,p)) \geq t(H_\alpha(Sv,B,p)) > t(F_\alpha(fv,Sv,p))$

We get $fv \in Sv$.

Now as pair (f,S) is S-JSR maps therefore $fp \in Sp$

and similarly as pair (g,G) is G-JSR maps therefore $gu \in Gu$

$t(F_\alpha(fx_n,gu,p)) \geq t(H_\alpha(Sx_n,Gu,kp))$

$\geq \phi[\min\{t(F_\alpha(fx_n,gu,p)),t(F_\alpha(fx_n,Sx_n,p)),t(F_\alpha(gu,Gu,p)), t(F_\alpha(fx_n,Gu,p)), t(F_\alpha(Sx_n,gu,p))\}]$

On taking limit $n\to\infty$, we obtain

$t(F_\alpha(u,gu,p)) \geq \phi[\min\{t(F_\alpha(u,gu,p)),t(F_\alpha(u,A,p)), t(F_\alpha(gu,Gu,p)), t(F_\alpha(u,Gu,p)), t(F_\alpha(A,gu,p))\}]$

$\geq \phi[\min\{t(F_\alpha(u,gu,p)),t(F_\alpha(u,A,p)), t(F_\alpha(gu,Gu,p)), t(F_\alpha(u,Gu,p)),t(F_\alpha(A,u,p/2)+ F_\alpha(u,gu,p/2))\}]$

By triangular inequality and as $u \in A\cap B$, we obtain

$t(Fu,gu,p) \geq t(Fu,gu,p)$

$\Rightarrow gu = u$.

Again

$t(F_\alpha(fu,gx_n,p)) \geq t(H_\alpha(Su, Gx_n,p))$

$\geq \phi[\min\{t(F_\alpha(fu,gx_n,p)), t(F_\alpha(fu,Su,p)), t(F_\alpha(gx_n,Gx_n,p)), t(F_\alpha(fu,Gx_n,p)), t(F_\alpha(Su,gx_n,p))\}]$.

On taking limit $n\to\infty$, we obtain

$t(F_\alpha(fu,u,p)) \geq \phi[\min\{t(F_\alpha(fu,u,p)), t(F_\alpha(fu,Su,p)), t(F_\alpha(u,Gu,p)), t(F_\alpha(fu,B,p)), t(F_\alpha(Su,u,p))\}]$

$t(F_\alpha(fu,u,p)) \geq \phi[\min\{t(F_\alpha(fu,u,p)), t(F_\alpha(fu,Su,p)), t(F_\alpha(u,Gu,p)), t (F_\alpha(fu,u,p/2)+ F_\alpha(u,B,p/2)), t(F_\alpha(Su,u,p))\}]$.

By triangular inequality and as $u \in A\cap B$, we obtain

$F_\alpha(fu,u,p) \geq F_\alpha(fu,u,p)$

$\Rightarrow fu = u$.

Hence $u = fu \in Su$ and $u=gu \in Su$.

Example: Let $X = [1,\infty)$ with usual metric .Define $S: X\to X$ as $Sx = 2+x/3$ and $T: CB(X) \to X$ as $Tx = [1,2+x]$. Consider the sequence $\{x_n\} = \{3+1/n\}$. Then all conditions are satisfies of the theorem and hence 3 is the common fixed point for all $\alpha \in [0,1]$.

Theorem 3.2: Let (X, d) be the metric space. Let $f, g: X \to X$ and S_i, G_j are sequences of functions from X into CB(X) such that

(I) (f, S_i) and (g, G_j) satisfy the common property (EA),

(II) f(X) and g(X) are closed,

(III) pair (f, S_i) is S_i-JSR maps and pair (g, G_j) is G_j-JSR maps,

(IV) $t(H_\alpha(S_ix,G_jy,kp)) \geq \phi[\min\{t(F_\alpha(fx,gy,p)), t(F_\alpha(fx,S_ix,p)), t(F(gy,G_jy,p)), t(F_\alpha(fx,G_jy,p)), t(F_\alpha(S_ix,gy,p))\}]$

Then f, g, S_i and G_j have a common fixed point in X.

Proof: Same as theorem 2.1 for each sequences S_i and G_j.

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