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Fixed Point Theorems for Random Operators in Hilbert Space

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Abstract

Our main aim of this paper is introduced some new unique common random fixed point theorems of random operators in Hilbert Space by considering a sequence of measurable functions satisfying conditions A or B and C. Our results are motivated from [3, 5, 6, 7, 8].

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1. Introduction and preliminaries

In recent years, the study of random fixed points has attracted much attention; some of the recent literatures in random fixed point may be noted in [1, 2, 3]. In this paper we construct a sequence of measurable function and consider its convergence to the common unique random fixed point of two continuous random operators defined on a non- empty closed subset of a separable Hilbert space. For the purpose of obtaining the random fixed point of the two continuous random operators. We have introduced a rational inequality and used the parallelogram law.
Throughout this paper, (Ω, Σ) denotes a measurable space consisting of a set Ω and sigma algebra Σ of subset of Ω. H stands for a separable Hilbert space and C is nonempty closed subset of H.

1.1 Definition: A function f: $\Omega \to C$ is said to be measurable if f^{-1}(B$\cap$ C) $\in \Sigma$ for every Borel subset B of H.

1.2. Definition: A function F: $\Omega X C \to C$ is said to be a random operator if F(., x) : $\Omega \to\to C$ is measurable for every x $\in C$.

1.3. Definition: A measurable function g: $\Omega \to C$ is said to be a random fixed point of the random operator F: $\Omega X C \to$ C if F(t(g(t))) = g(t) for all t $\in \Omega$.

1.4. Definition: A random operator F: $\Omega X C \to$ C is said to be continuous if for fixed t $\in \Omega$, F(t, .) : $C \to$ C is continuous.

Condition A: Let S, T: C $\to$C be three mapping, when C is non empty subset of a Hilbert space H, is said to satisfy the condition A, if

$$\|E^r x - F^s y\|^2 \leq$$

Kmax

$$\left[c\,\|x-y\|^2,\ \left\{\|x-E^r x\|^2 + \|y-F^s y\|^2, \frac{1}{2}\|x-y\|^2 + \|x-F^s y\|^2 + \|y-E^r x\|^2\right\}\right]$$

$0 \leq K \leq 1/2$; $c \geq 0$ and r, s > 0

Condition B: At each pair of points x, y mapping E, F & C satisfies at least one of the following conditions

$$\|x-E^r x\|^2 + \|y-F^s y\|^2 \leq \alpha\|x-y\|^2,\ \ 1 \leq \alpha \leq 2.$$

$\|x - E^r x\|^2 + \|y - F^s y\|^2 \leq \beta\{\|x - F^s y\|^2 + \|y - E^r x\|^2 + \|x - y\|^2\}$,
$1/3 \leq \alpha \leq 2/5$
$\|x - E^r x\|^2 + \|Y - F^s x\|^2 + \|E^r x - F^s y\|^2 \leq \gamma\{\|x - F^s x\|^2 + \|y - E^r x\|^2\}$,
$1/2 \leq \alpha \leq 3$

Condition C: Two mapping S, T: C→ C, where C is a non –empty closed subset of a Hilbert space H, is said to satisfy condition (A) if

$$\|S_x - T_y\|^2 \leq \frac{a\|y - T_y\|^2[1 + \|x - S_x\|^2]}{1 + \|x - y\|^2} + b[\|x - S_x\|^2 + \|y - T_y\|^2] + C\|S_x - T_y\|^2$$

For each x, y in C, a, b, being positive real number such that $0 < a + b < 1/2$.

2. Main Results

2.1. Theorem: Let C be a non empty subset of Hilbert Space H. Let S and T be three continuous random operations defined on C such that for $\xi\epsilon\Omega$, T(ξ, .) C → C satisfying condition A or B. Then the sequence $\{g_n\}$ defined in (1) converges to the unique common random fixed point of E^r and F^s .

Proof A: $\{g_n\}$ is sequence of function defined in (1). For $\xi\epsilon\Omega$ and n=0, 1, 2, 3… .

$\|g_{2n+1}(\xi) - g_{2n}(\xi)\|^2 = \|E^r(\xi, g_{2n}(\xi)) - F^s(\xi, g_{2n-1}(\xi))\|^2$
$\leq$Kmax[
$c\|g_{2n}(\xi) - g_{2n-1}(\xi)\|^2, \{\|g_{2n}(\xi) - E^r(\xi, g_{2n}(\xi))\|^2 + \|g_{2n}(\xi) - F^s(\xi, g_{2n-1}(\xi))\|^2\}$
$\frac{1}{3}\{\|g_{2n}(\xi) - g_{2n-1}(\xi)\|^2 + \|g_{2n}(\xi) - F^s(\xi, g_{2n-1}(\xi))\|^2 + \|g_{2n-1}(\xi) - E^r(\xi, g_{2n}(\xi))\|^2\}]$

=kmax[c$\|g_{2n}(\xi) - g_{2n-1}(\xi)\|^2$, $\{\|g_{2n}(\xi) - g_{2n+1}(\xi)\|^2 + \|g_{2n-1}(\xi) - g_{2n}(\xi)\}\|^2$,
$\frac{1}{3}\{\|g_{2n}(\xi) - g_{2n-1}(\xi)\|^2 + \|g_{2n}(\xi) - g_{2n}(\xi)\|^2 + \|g_{2n-1}(\xi) - g_{2n+1}(\xi)\|^2\}]$
= kmax[c$\|g_{2n}(\xi) - g_{2n-1}(\xi)\|^2$, $\{\|g_{2n}(\xi) - g_{2n+1}(\xi)\|^2 + \|g_{2n-1}(\xi) - g_{2n}(\xi)\|^2\}$,
$\frac{1}{3}\{\|g_{2n}(\xi) - g_{2n-1}(\xi)\|^2 + \|g_{2n-1}(\xi) - g_{2n}(\xi) + g_{2n}(\xi) - g_{2n+1}(\xi)\|^2\}]$
= kmax[c$\|g_{2n}(\xi) - g_{2n-1}(\xi)\|^2$, $\{\|g_{2n}(\xi) - g_{2n+1}(\xi)\|^2 + \|g_{2n-1}(\xi) - g_{2n}(\xi)\}\|^2$,
$\frac{1}{3}\{\|g_{2n}(\xi) - g_{2n-1}(\xi)\|^2 + 2\|g_{2n-1}(\xi) - g_{2n}(\xi)\|^2 + 2\|g_{2n}(\xi) - g_{2n+1}(\xi)\|^2]$
$-\|g_{2n-1}(\xi) - g_{2n}(\xi) - (g_{2n}(\xi) + g_{2n+1}(\xi))\|^2\}]$ (by parallelogram law)

$=k\max\,[c\|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2, \{\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2+\|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2\},$
$\frac{1}{3}\{3\|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2+2\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2$
$-\|g_{2n-1}(\xi)-g_{2n}(\xi)-(g_{2n}(\xi)+g_{2n+1}(\xi)\|^2\}]$
$=k[\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2+\|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2]$
$\|g_{2n+1}(\xi)-g_{2n}(\xi)\|^2 \leq \frac{k}{1-k}\|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2$

Also, $\|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2 \leq \frac{k}{1-k}\|g_{2n-2}(\xi)-g_{2n-1}(\xi)\|^2$

Hence in general $\|g_n(\xi)-g_{n+1}(\xi)\|^2 \leq \frac{k}{1-k}\|g_{n-1}(\xi)-g_n(\xi)\|^2$

Since $0<\frac{k}{1-k}<1$, therefore $\{g_n(\xi)\}$ be Cauchy sequence and hence convergent in H.

Therefore $g_n(\xi)\to g(\xi)$ as $n\to\infty$.Since C is closed, be a function. For all

$\xi\in\Omega\,\|g(\xi)-F^s(\xi,(g(\xi))\|^2 \leq \|g(\xi)-g_{2n+1}(\xi)+g_{2n+1}(\xi)-F^s(\xi,(g(\xi))\|^2$
$\leq 2\|g(\xi)-g_{2n+1}(\xi)\|^2+2\|g_{2n+1}(\xi)-F^s(\xi,(g(\xi))\|^2$ by parallelogram law
$=2\|g(\xi)-g_{2n+1}(\xi)\|^2+2\|E^r(\xi,g_{2n}(\xi))-F^s(\xi,(g(\xi))\|^2$
$=2\|g(\xi)-g_{2n+1}(\xi)\|^2+2K\max[c\|g_{2n}(\xi)-g(\xi)\|^2,\{\|g_{2n}(\xi)-E^r(\xi,g_{2n}(\xi))\|^2,$
$+\|g(\xi)-F^s(\xi,g(\xi)\|^2\},\ \frac{1}{3}\{\|g_{2n}(\xi)-g(\xi)\|^2+\|g_{2n}(\xi)-F^s(\xi,g(\xi))\|^2$
$+\|g(\xi)-E^r(\xi,g_{2n}(\xi))\|^2\}]$
$=2\|g(\xi)-g_{2n+1}(\xi)\|^2+2K\max[c\|g_{2n}(\xi)-g(\xi)\|^2,\{\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2,$
$+\|g(\xi)-F^s(\xi,g(\xi)\|^2\},\ \frac{1}{3}\{\|g_{2n}(\xi)-g(\xi)\|^2+\|g_{2n}(\xi)-F^s(\xi,g(\xi))\|^2$
$+\|g(\xi)-g_{2n+1}(\xi)\|^2\}]$

As $n\to\infty$ $\|g(\xi)-F^s(\xi,g(\xi)\|=2k\,\|g(\xi)-F^s(\xi,g(\xi)\|$

Hence for all $F^s(\xi,g(\xi)=g(\xi)$. Similarly $E^r(g(\xi))=g(\xi)$

Now, if $G:\Omega\times C\to C$ Is a continuous random operation on a non empty subset of a separated Hilbert space H, them any measurable formula f: $\Omega\to C$ the function $g(\xi)$ $=G(\xi,f(\xi))$ is also measurable. Therefore the sequence of measurable function $\{g_n\}$ converges to measurable function of this fact along with $E^r(\xi,g(\xi)=g(\xi)$ $=F^s(\xi,g(\xi))$ shows that $g:\Omega\times C\to C$ common random fixed point of E^r and F^s .

Uniqueness:

Let h: Ω→C be another common random fixed point of E^r and F^s .

$\|g(\xi) - h(\xi)\|^2 = \|E^r(\xi, g(\xi)) - F^s(\xi, h(\xi))\|^2 \leq$

kmax[c $\|g(\xi) - h(\xi)\|^2$,{ $\|g(\xi) - E^r(\xi, g(\xi))\|^2$

+ $\|h(\xi) - F^s(\xi, h(\xi))\|^2$},$\frac{1}{3}${

$\|g(\xi) - h(\xi)\|^2 + \|g(\xi) - F^s h(\xi)\|^2 + \|h(\xi) - E^r(\xi, g(\xi))\|^2$]]

=kmax[c . $\|g(\xi) - h(\xi)\|^2$, $\|g(\xi) - h(\xi)\|^2$]

=k $\|g(\xi) - h(\xi)\|^2$

$g(\xi) = h(\xi)\ \forall \in \Omega$ (as k < 1/2)

Proof B: Suppose sequence $\{g_n\}$ satisfy (a) of condition B then

$\|g_{2n}(\xi) - E^r(\xi, g_{2n}(\xi))\|^2 + \|g_{2n+1}(\xi) - F^s(\xi, g_{2n-1}(\xi))\|^2 \leq \alpha\|g_{2n}(\xi) - g_{2n-1}(\xi))\|^2$

$\|g_{2n}(\xi) + g_{2n+1}(\xi))\|^2 + \|g_{2n+1}(\xi) - g_{2n+2}(\xi)\|^2 \leq \|g_{2n}(\xi) + g_{2n+1}(\xi))\|^2$

$\|g_{2n+1}(\xi) - g_{2n+2}(\xi))\|^2 \leq (\alpha - 1)\|g_{2n}(\xi) + g_{2n+1}(\xi))\|$

Similarly by (b), (c) we get

$\|g_{2n+1}(\xi) - g_{2n+2}(\xi))\|^2 \leq \frac{2\beta-1}{1-2\beta}\|g_{2n}(\xi) - g_{2n+1}(\xi)\|^2$

& $\|g_{2n+1}(\xi) - g_{2n+2}(\xi)\|^2 \leq \frac{2\gamma-1}{2-\gamma}\|g_{2n}(\xi) - g_{2n+1}(\xi)\|^2$

By above, we have

$\|g_{2n+1}(\xi) - g_{2n+2}(\xi)\|^2 \leq \lambda\|g_{2n}(\xi) - g_{2n+1}(\xi)\|^2$

Where $\lambda = \max\{(\alpha - 1), \frac{2\beta-1}{1-2\beta}, \frac{2\gamma-1}{2-\gamma}\} < 1.$

Repeating the same process as in **proof (A)**, we find $g(\xi)$ is unique common random fixed point of E^r *and* F^s for all r, s>0.

Examples:

Let Ω =[0, 1] and Σ be the sigma algebra of Lebesgue's measurable subset of [0, 1]. Let H=R with $\|x - y\| = a^{|x-y|} - 1$, where a>1. Define random operators E^r and F^s : ΩxX→H as $E^r(\xi, x)$ = (1-ξ^2+2x)/3 and

$F^s(\xi, x)$ = (1-ξ^2+3x)/4. Also sequence of mapping $g_n : \Omega \to X$ is defined by $g_n(\xi) :=$ 1+(1/n)- ξ^2 for every $\xi \epsilon \Omega$ and n . Define measurable mapping $\xi : \Omega \to X$ as g(ξ) = 1-ξ^2 for every $\xi \epsilon \Omega$.

$\lim_{n\to\infty} \|E^r(\xi, g_n(\xi)) - g(\xi)\| = \lim_{n\to\infty} a^{|E^r(\xi, g_n(\xi)) - g(\xi)|} - 1 = \lim_{n\to\infty} a^{3/4n} - 1 = 0$

$\lim_{n\to\infty} \|F^s(\xi, g_n(\xi)) - g(\xi)\| = \lim_{n\to\infty} a^{|T(\omega, \xi n(\omega)) - \xi(\omega)|} - 1 = \lim_{n\to\infty} a^{2/3n} - 1 = 0$

Clearly E^r and F^s satisfy conditions A and B.

2.2. Theorem: Let C be a non-empty closed subset of a separable Hilbert space H. Let S and T be two continuous random operators defined on C such that for t $\in \Omega$, S(t, .), T(t,.): $C \to$ C satisfy condition (C). Then S and T have a common unique random fixed point in C

Proof C: We define a sequence of function $\{g_n\}$ as $g_0 : \Omega \to C$ is arbitrary measurable function for t$\in \Omega$ and n = 0, 1, 2, 3… .

$g_{2n+1}(t) = S(t, g_{2n}(t))$, $g_{2n+2}(t) = T(t, g_{2n+1}(t))$

(1)

If $g_{2n}(t) = g_{2n+1}(t) = g_{2n+2}(t)$ for t $\in \Omega$, for some n then we see that $g_{2n}(t)$ is a random fixed point of S and T. therefore we suppose that no two consecutive terms of sequence $\{g_n\}$ are equal. Now consider for t $\in \Omega$

$$\|g_{2n+1}(t) - g_{2n+2}(t)\|^2 = \|S(t, g_{2n}(t)) - T(t, g_{2n+1}(t))\|^2$$

$$\leq \frac{a\|g_{2n+1}(t) - T(t, g_{2n+1}(t))\|^2[1 + \|g_{2n}(t) - S(t, g_{2n}(t))\|^2]}{1 + \|g_{2n}(t) - g_{2n+1}(t)\|^2}$$

$$+b[\|g_{2n}(t) - S(t, g_{2n}(t))\|^2 + \|g_{2n+1}(t) - T(t, g_{2n+1}(t))\|^2]$$

$$+ c[\|S(t, g_{2n}(t)) - T(t, g_{2n+1}(t))\|^2]$$

$$= \frac{a\|g_{2n+1}(t) - g_{2n+2}(t))\|^2[1+\|g_{2n}(t) -, g_{2n+1}(t))\|^2]}{1+\|g_{2n}(t) - g_{2n+1}(t)\|^2}$$

$$+ b[\|g_{2n}(t) -, g_{2n+1}(t))\|^2 + \|g_{2n+1}(t) -, g_{2n+2}(t))\|^2]$$

$$+ c[\|(g_{2n+1}(t)) - g_{2n+2}(t))\|^2]$$

$$=(a+b+c)\|(g_{2n+1}(t)) - g_{2n+2}(t))\|^2 + b\|g_{2n}(t) -, g_{2n+1}(t))\|^2$$

$$\Rightarrow [1 - (a + b + c)]\|(g_{2n+1}(t)) - g_{2n+2}(t))\|^2 \leq b\|g_{2n}(t) -, g_{2n+1}(t))\|^2$$

$$\Rightarrow \|(g_{2n+1}(t)) - g_{2n+2}(t))\| \leq \frac{b}{[1 - (a + b + c)]}\|g_{2n}(t) -, g_{2n+1}(t))\|^2$$

$\Rightarrow \|(g_{2n+1}(t)) - g_{2n+2}(t))\| \leq K\|g_{2n}(t) -, g_{2n+1}(t))\|^2$ where $K=[\frac{b}{[1-(a+b+c)]}]^{\frac{1}{2}} \leq \frac{1}{2}$

In general

$\Rightarrow \|g_n(t) -, g_{n+1}(t))\| \leq K\|g_{n-1}(t) -, g_n(t))\|$

$\Rightarrow \|g_n(t) -, g_{n+1}(t))\| \leq K^n\|g_0(t) -, g_1(t))\|$ for all $t \in \Omega$

(2)

Now, we shall prove that for $t \in \Omega$, $\{g_n(t)\}$ is a Cauchy sequence. For this for every positive integer *p* we have, for $t \in \Omega$

$$\Rightarrow \|g_n(t) - g_{n+p}(t)\| = \|g_n(t) - g_{n+1}(t) + g_{n+1}(t) - \dots\dots\dots + g_{n+p-1}(t) - g_{n+p}(t)\|$$
$$\leq \|g_n(t) - g_{n+1}(t)\|$$
$$+\|g_{n+1}(t) -, g_{n+2}(t))\| + \dots\dots + \|g_{n+p-1}(t) - g_{n+p}(t)\|$$
$$\leq [K^n + K^{n+1} + \dots + K^{n+p-1}]\|g_0(t) -, g_1(t))\| \quad \text{by (2)}$$
$$= K^n[1 + k + K^2 \dots + K^{p-1}]\|g_0(t) - g_1(t))\|$$
$$\leq \frac{K^n}{(1-k)}\|g_0(t) - g_1(t))\| \quad \text{for all } t \in \Omega$$

as $n \to \infty$, $\|g_n(t) - g_{n+p}(t)\| \to 0$, it follows that for $t \in \Omega$, $\{g_n(t)\}$ is a Cauchy sequence and hence is convergent in Hilbert space H.

For $t \in \Omega$, let

$\{g_n(t)\} \to g(t)$ as $n \to \infty$

(3)

Since C is closed, g is a function from C to C.

Existence of random fixed point: For $t \in \Omega$,

$$\|g(t) - T(t, g(t))\|^2 = \|g(t) - g_{2n+1}(t) + g_{2n+1}(t) - \dots\dots\dots T(t, g(t))\|$$
$$\leq 2\|g(t) - g_{2n+1}(t)\|^2 + 2\|g_{2n+1}(t) - T(t, g(t))\|^2$$

[by parlallelogram law $\|x+y\|^2 \leq 2\|x\|^2 + 2\|y\|^2$]

$$= 2\|g(t) - g_{2n+1}(t)\|^2 + 2\|S(t, g_{2n}(t)) - T(t, g(t))\|^2$$
$$\leq 2\|g(t) - g_{2n+1}(t)\|^2 + \frac{2a\|g(t) - T(t, g(t))\|^2[1 + \|g_{2n}(t) - S(t, g_{2n}(t))\|^2]}{1 + \|g_{2n}(t) - g(t)\|^2}$$
$$+2b\left[\|g_{2n}(t) - S(t, g_{2n}(t))\|^2 + \|g(t) - T(t, g(t))\|^2\right]$$

$$+2c[\|S(t, g_{2n}(t)) - T(t, g(t))\|^2]$$

$$=2\|g(t) - g_{2n+1}(t)\|^2 + \frac{2a\|g(t)-T(t,g(t))\|^2[1+\|g_{2n}(t)-g_{2n+1}(t)\|^2]}{1+\|g_{2n}(t)-g(t)\|^2}$$

$$+2b[\|g_{2n}(t) - g_{2n+1}(t)\|^2 + \|g(t) - T(t, g(t))\|^2]$$

$$+2c[\|g_{2n+1}(t) - T(t, g(t))\|^2]$$

As $\{g_{2n+1}(t)\}$ and $\{g_{2n+2}(t)\}$ are subsequences of $\{g_n(t)\}$ as $n \to \infty$, $\{g_{2n+1}(t)\} \to g(t)$ and $\{g_{2n+2}(t)\} \to g(t)$

Therefore,

$$\Rightarrow \|g(t) - T(t,g(t))\|^2 \leq 2\|g(t) - g(t)\|^2 + \frac{2a\|g(t)-T(t,g(t))\|^2[1+\|g(t)-g(t)\|^2]}{1+\|g(t)-g(t)\|^2}$$

$$+2b[\|g(t) - g(t)\|^2 + \|g(t) - T(t, g(t))\|^2] + 2C[\|g(t) - T(t, g(t))\|^2]$$

$$\Rightarrow [1 - 2(a + b + c)]\|g(t) - T(t, g(t))\|^2 \leq 0$$

$$\Rightarrow \|g(t) - T(t, g(t))\|^2 = 0 \quad \text{(as } 2(a+b+c)<1$$

$$\Rightarrow T(t,g(t)) = g(t) \quad \forall t \in \Omega \tag{4}$$

In an exactly similar way we can prove that for all $t \in \Omega$,

$$S(t,g(t)) = g(t) \tag{5}$$

Again if $A: \Omega X C \to$ is a continuous random operator on a non empty subset C of a separable Hilbert space H, then for any measurable function $f: \Omega \to C$, the function $h(t) = A(t, f(t))$ is also measurable [1]

It follows from the construction of $\{g_n\}$ (by (1)) and the above consideration that $\{g_n\}$ is a sequence of measurable function. From (3), it follow that g is also a measurable function. This fact along with (4) and (5) shows that $g: \Omega \to C$ is common random fixed point of S and T.

Uniqueness:

Let h: $\Omega \to C$ be another random fixed point common to S and T that is for t $\in \Omega$.
S(t, h(t)) = h(t)
T(t, h(t)) = h(t)

Then for t $\in \Omega$.

$$\Rightarrow \quad \|g(t) - h(t)\|^2 = \left\|S(t, g(t) - T(t, h(t))\right\|^2$$

$$\leq \frac{a\left\|h(t) - T(t, h(t))\right\|^2 [1 + \|g(t) - S(t, g(t))\|^2]}{1 + \|g(t) - h(t)\|^2}$$

$$+b\left[\left\|g(t) - S(t, g(t))\right\|^2 + \left\|h(t) - T(t, h(t))\right\|^2\right]$$

$$+c\left[\left\|S(t, g(t)) - T(t, h(t))\right\|\right]^2$$

$$= \frac{a\|h(t) - h(t)\|^2 [1 + \|g(t) - g(t)\|^2]}{1 + \|g(t) - h(t)\|^2}$$

$$+b[\|g(t) - g(t)\|^2 + \|h(t) - h(t)\|^2]$$

$$+c[\|g(t) - h(t)\|]^2$$

$$\Rightarrow (1 - c)\|g(t) - h(t)\|^2 \leq 0$$

$$\Rightarrow g(t) = h(t) \text{ for all } t \in \Omega.$$

This complete proof of theorem.

References

[1] Choudhary, B. S.: A common unique fixed point theorem for two random operators in Hilbert space, IJMMS 32(3) (2002), 177 – 82.

[2] Choudhury, B. S. and Ray, M.: Convergence of an iteration leading to a solution of a random operator equation, J, Appl. Math. Stochastic Anal., 12(2) (1999), 161 – 168.

[3] Choudhury, B. S. and Upadhay, A.: An iteration leading solution and fixed point of operators, Soochow J. Math. 25(4) (1999), 395 – 400.

[4] Choudhury, B. S.: Convergence of random iteration scheme to random fixed point, J, Appl. Math. Stochastic Anal., 8(2) (1995), 139 – 142.

[5] Fisher, B.: Common fixed point and constant mapping satisfying a rational inequality, Math Sem Kobe Univ., 6(1978), 29 – 35.

[6] Himmelberg, C. J.: Measurable relations, Fund Math, 87 (1975), 53 – 72.

[7] Pagey, S. S., Shrivastava, S. and Nair, S.: Common fixed point theorem for rational inequality in a quasi 2- metric space, J. Pure Math, 22 (2005), 99 – 104.

[8] Rhoades, B. E.: Iteration to obtain random solution and fixed points of operators in uniformly convex Banach spaces, Soochow Journal of Mathematics 27(4) (2001), 401 – 404.

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