The Influence of Hydrodynamics on Pollutant Dispersion in the River

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Abstract

Disposal of waste pollutants into the river, which progressively increased, has great potential as a cause of pollution. To specify the quality of the river has yet to connect the elements of hydrodynamics as a discharge, speed, and the structure of a river or pollutant parameters. Water quality modeling is generally still being partial, that is not yet incorporated the hydrodynamics, such as speed and discharge. This paper considers the influence of hydrodynamics on the spread of pollutant elements in the river, where the hydrodynamics elements used in this paper are the speed and depth of the river. The model constructed from continuum principle, then, it developed into the model hydrodynamics on spread of pollutants in the river. The model furthermore is solved using Alternating Direct Implicit (ADI) method. From the numerical simulation, it is obtained that the greater the depth of the river, make the speed of spread of pollutants higher. However, the speed of flow does not significantly influence the speed of the spread of pollutants.

Keywords: Hydrodynamics, Spread Pollutants, Alternating Direct Implicit

1. INTRODUCTION

The river is one of the natural water resources that should be kept from the influence of liquid waste or pollutants, which means that the river water quality should be maintained and secured from the causes of pollution, discharges / inputs from industrial waste, domestic waste, agricultural and other wastes into the river. Disposal of liquid industrial waste or non-industry, both the treated and untreated into the river has the potential effect to cause pollution to the river [5] and [6].
This is because each load of liquid waste is discharged into the river containing parameters of physical, chemical and biological water quality of rivers. This can alter or affect the value of dissolved oxygen in the river [8]. While the load of wastewater which is discharged into the river more and more increasing. Therefore, to maintain the river is required water quality monitoring efforts [4]. However, the implementation of river water quality monitoring often experience barriers significantly. It is characterized by frequent monitoring of the quality found in water that has been carried out by water institution, which almost always results below a predetermined threshold. Nieke [4] and Widodo et al [7] states that the research of Widodo [8] on "Application of finite difference methods on solving the flow equations and a one-dimensional pollutant transport", which is important in considering the level of water pollution is the dissolved oxygen levels, because the concentration of oxygen that can dissolve in water is determined by temperature, re-aeration, photosynthesis, respiration of animals and plants, parameters BOD, nitrification, salinity and some other substances.

This explanation is also corroborated by the statement of Rahardjo [1], stating that the mathematical model for hydrodynamic models are still largely separate streams, pollutant transport models and sediment transport models-base changes. Chapra [3] in the "Surface Water Quality Modeling" also states that in order to make a good and comprehension river model then the hydraulic (flow, velocity and dispersion) and geometric (depth, width and slope) of the river should considered.

Based on the description above, the purpose of this paper is to determine the influence of hydrodynamics on pollutant dispersion in the river. Hydrodynamic elements are used in this paper are the speed and depth of the river.

2. MATHEMATICAL MODELING OF POLLUTANTS DISPERSION IN THE RIVER

Like the river, to explain the pattern of spread of pollutants in the river, the required equation is the equation of continuity and momentum equations. Continuity equation, in the presence of pollutants that enter the mass means there is no source, so the continuity equation can be formulated with the pattern of spread of pollutants [6] and [9]:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0
\]  

(1)

While the momentum equation for the spread of pollutants in the river together with the momentum equations for the flow of river water. This is due to the spread of pollutants in rivers and streams are both experiencing momentum between the particles. The equation of momentum spread of pollutants can be shown by the following equation [6] and [9]:

\[
\left[ \frac{\partial \rho U}{\partial t} + \frac{\partial \rho u U}{\partial x} + \frac{\partial \rho v U}{\partial y} \right] - \left[ P_o + \rho gh \right] - \mu \frac{UA}{\Delta x \Delta y} = 0
\]  

(2)
Most of the flow velocity is in the middle of the river surface. Because of the momentum equation, namely equation (2) can be written towards the x-axis and y-axis direction.

By solving the derivative at each term, it is obtained momentum equation in the x-axis direction as follows:

\[
0 = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} - \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) + \left( \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} \right) - \frac{\mu}{\Delta x} \frac{u}{\Delta x} = 0
\]

Since \( U = u \), then

\[
0 = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} - \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) + \left( \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} \right) - \frac{\mu}{\Delta x} \frac{u}{\Delta x} = 0
\]

From the Equation (1) is obtained

\[
0 = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} - \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) + \left( \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} \right) - \frac{\mu}{\Delta x} \frac{u}{\Delta x} = 0 \tag{3}
\]

While the momentum equation in the y-axis direction is written as

\[
0 = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} - \rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) + \left( \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} \right) - \frac{\mu}{\Delta y} \frac{v}{\Delta y} = 0 \tag{4}
\]

Furthermore, The Equation (3) and equation (4) are solved numerically.

### 3. NUMERICAL PROCEDURES

By considering that the momentum equation is in two dimensions, then the flow velocity along the x-axis is exemplified as \( u \) solved using x-momentum and flow velocity along the y-axis is exemplified as \( v \) solved using the y-momentum.

By looking again at the Equation (3), the two-dimensional momentum equation is applied to calculate the flow velocity along the axis x. The equation therefore must be modified in the following discrete equation:

\[
\rho \left( \frac{u_{i,j}^{n+1} - u_{i,j}^n}{\Delta t} \right) + \rho u_{i,j}^n \left( \frac{u_{i+1,j}^{n+1} - u_{i-1,j}^n}{2(\Delta x)} \right) + \rho v_{j}^n \left( \frac{u_{i,j+1}^{n+1} - u_{i,j-1}^n}{2(\Delta y)} \right) = \left[ P_0 + g \rho h \right] + \frac{\mu}{\Delta x} u_{i,j}^n \tag{5}
\]

By separating the flow velocity along the x-axis, the Equation (5) can be written as:

\[
- \frac{\rho u_{i,j}^n}{2(\Delta y)} u_{i,j}^{n+1} - \frac{\rho u_{i,j}^n}{2(\Delta x)} u_{i+1,j}^{n+1} + \left( \frac{\rho u_{i,j}^n}{2(\Delta x)} \right) u_{i,j}^{n+1} + \left( \frac{\rho u_{i,j}^n}{2(\Delta y)} \right) u_{i,j+1}^{n+1} + \left( \frac{\rho u_{i,j}^n}{2(\Delta y)} \right) u_{i,j+1}^{n+1} = P_0 + g \rho h + \frac{\mu}{\Delta x} u_{i,j}^n - \frac{1}{\Delta t} \rho u_{i,j}^n \tag{6}
\]

Equation (6) can be simplified to be:

\[
Au_{i,j}^{n+1} + Bu_{i+1,j}^{n+1} + Cu_{i,j+1}^{n+1} + Du_{i,j+1}^{n+1} + Eu_{i,j+1}^{n+1} = F
\]

where,
The Equation (6) can be made to calculate \( u_{i,j}^{n+1} \) at each grid point. To calculate the amount of flow velocity along the \( x \) axis of the pattern of these calculations is to apply the pattern of \( x \) momentum, thus forming the equation system:

\[
\begin{align*}
Cu_{i,1}^{n+1} + Du_{2,1}^{n+1} + Eu_{1,2}^{n+1} &= F_i \\
A_{1}u_{1,1}^{n+1} - Bu_{1,2}^{n+1} + Cu_{1,2}^{n+1} + Du_{2,2}^{n+1} + Eu_{2,2}^{n+1} &= F_2
\end{align*}
\]

It can be implemented in a general matrix form as follows:

\[
\begin{bmatrix}
C_1 & D_1 & \cdots & E_1 & \cdots & 0 & 0 \\
B_1 & C_2 & D_2 & 0 & E_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
A_1 & 0 & \cdots & 0 & \cdots & E_{56} & \cdots \\
\vdots & A_2 & 0 & \cdots & 0 & \cdots & \ddots \\
0 & \cdots & 0 & \cdots & D_{56} & \cdots & \vdots \\
0 & 0 & \cdots & A_{56} & \cdots & B_{56} & C_{64}
\end{bmatrix}
\begin{bmatrix}
u_{1,1}^{n+1} \\
u_{2,1}^{n+1} \\
\vdots \\
u_{8,1}^{n+1} \\
u_{1,2}^{n+1} \\
\vdots \\
u_{8,2}^{n+1} \\
u_{1,3}^{n+1} \\
\vdots \\
u_{8,3}^{n+1} \\
u_{1,4}^{n+1} \\
\vdots \\
u_{8,4}^{n+1} \\
u_{1,5}^{n+1} \\
\vdots \\
u_{8,5}^{n+1} \\
u_{1,6}^{n+1} \\
\vdots \\
u_{8,6}^{n+1} \\
u_{1,7}^{n+1} \\
\vdots \\
u_{8,7}^{n+1} \\
u_{1,8}^{n+1} \\
\vdots \\
u_{8,8}^{n+1}
\end{bmatrix}
= \begin{bmatrix}
P_0 + gh\rho + \frac{\mu}{\Delta x} u_{1,1}^{n} - \frac{1}{\Delta t} \rho u_{1,1}^{n} \\
P_0 + gh\rho + \frac{\mu}{\Delta x} u_{2,1}^{n} - \frac{1}{\Delta t} \rho u_{2,1}^{n} \\
\vdots \\
P_0 + gh\rho + \frac{\mu}{\Delta x} u_{8,1}^{n} - \frac{1}{\Delta t} \rho u_{8,1}^{n} \\
P_0 + gh\rho + \frac{\mu}{\Delta x} u_{1,2}^{n} - \frac{1}{\Delta t} \rho u_{1,2}^{n} \\
\vdots \\
P_0 + gh\rho + \frac{\mu}{\Delta x} u_{8,2}^{n} - \frac{1}{\Delta t} \rho u_{8,2}^{n} \\
P_0 + gh\rho + \frac{\mu}{\Delta x} u_{1,3}^{n} - \frac{1}{\Delta t} \rho u_{1,3}^{n} \\
\vdots \\
P_0 + gh\rho + \frac{\mu}{\Delta x} u_{8,3}^{n} - \frac{1}{\Delta t} \rho u_{8,3}^{n} \\
P_0 + gh\rho + \frac{\mu}{\Delta x} u_{1,4}^{n} - \frac{1}{\Delta t} \rho u_{1,4}^{n} \\
\vdots \\
P_0 + gh\rho + \frac{\mu}{\Delta x} u_{8,4}^{n} - \frac{1}{\Delta t} \rho u_{8,4}^{n} \\
P_0 + gh\rho + \frac{\mu}{\Delta x} u_{1,5}^{n} - \frac{1}{\Delta t} \rho u_{1,5}^{n} \\
\vdots \\
P_0 + gh\rho + \frac{\mu}{\Delta x} u_{8,5}^{n} - \frac{1}{\Delta t} \rho u_{8,5}^{n} \\
P_0 + gh\rho + \frac{\mu}{\Delta x} u_{1,6}^{n} - \frac{1}{\Delta t} \rho u_{1,6}^{n} \\
\vdots \\
P_0 + gh\rho + \frac{\mu}{\Delta x} u_{8,6}^{n} - \frac{1}{\Delta t} \rho u_{8,6}^{n} \\
P_0 + gh\rho + \frac{\mu}{\Delta x} u_{1,7}^{n} - \frac{1}{\Delta t} \rho u_{1,7}^{n} \\
\vdots \\
P_0 + gh\rho + \frac{\mu}{\Delta x} u_{8,7}^{n} - \frac{1}{\Delta t} \rho u_{8,7}^{n} \\
P_0 + gh\rho + \frac{\mu}{\Delta x} u_{1,8}^{n} - \frac{1}{\Delta t} \rho u_{1,8}^{n} \\
\vdots \\
P_0 + gh\rho + \frac{\mu}{\Delta x} u_{8,8}^{n} - \frac{1}{\Delta t} \rho u_{8,8}^{n}
\end{bmatrix}
\]

The Equation (7) then solved to obtain the flow velocity in the direction of the \( x \)-axis.

4. NUMERICAL SIMULATIONS

In numerical simulation, the concentration of BOD and COD are included in \( \rho \). It therefore \( \rho \) is a combination of the density of water and the concentration of BOD and COD. We further calculate dispersion pollutants in the river when the effect of velocity and depth included also when we put the initial value of the BOD is 1000.00287, COD is 1000.0129, atmospheric pressure \( P_0 \) is 1031, dynamic viscosity of water \( \mu \) is 1, the length and width of the river that used in the simulation is 15 respectively. The numerical results further are visualized as follow:
We consider three kinds of river depth, i.e. 1.77 m, 2.19 m and 3.63 m depth respectively. The depth is applied to see the effect of the velocity dispersion of BOD along the river with an initial velocity of 0.885 m / s. From the Figure 1, it is shown that when the depth of the river larger, the BOD rate more higher, especially at some distance at the original position. We further take numerical simulation when the initial velocity of the river is 0.885 m / s, 1.094 m / s and 2.149 m / s, respectively. This is conducted to see the effect of the velocity dispersion of BOD along the river with a depth of 1.77 m. Figure 2 shows that the initial velocity river has a little influence on the spreading of BOD in the river.

We further take the depth of the river of 1.20 m, 1.87 m and 2.92 m respectively. This is implied to see the effect of the dispersion of COD along the river with an initial velocity of 0.319 m / s. Figure 3 shows that when the depth of the river more larger, the COD rate higher, especially at some distance at the start location. When we put the initial velocity of the river is 0.242 m / s, 0.942 m / s and 1.041 m / s respectively and depth of 3.82 m, then the initial velocity has a little influence on the speed of COD. This is shown in Figure 4.

5. CONCLUSIONS

Based on our numerical results, it can be concluded that the elements of hydrodynamics, in case of the depth of the river included, the higher the depth of the river then the greater velocity dispersion of pollutants. However, the other
elements of hydrodynamics, i.e. the velocity of the river, did not significantly affect the dispersion of pollutants in the river. By applying several different speeds of the river, the change in velocity dispersion of pollutants in the downstream is very small.

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