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Homomorphism in Intuitionistic Fuzzy Subsemiring of a Semiring

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Abstract. In this paper, we made an attempt to study the algebraic nature of intuitionistic fuzzy subsemiring of a semiring under homomorphism and anti-homomorphism.

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INTRODUCTION: There are many concepts of universal algebras generalizing an associative ring (*R ; + ; .*). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b+c) = a. b+a. c and (b+c) .a = b. a+c. a for all a, b and c in R. A semiring R is said to be additively commutative if a+b = b+a for all a, b in R. A semiring R may have an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0+a = a = a+0 and a.0 = 0 = 0.a for all a in R. After the introduction of fuzzy sets by L.A. Zadeh [22], several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subset was introduced by K.T. Atanassov [3,4], as a generalization of the notion of fuzzy set. The notion of fuzzy subnearrings and ideals was introduced by S. Abou Zaid [18]. In this paper, we introduce the some theorems in intuitionistic fuzzy subsemiring of a semiring under homomorphism and anti-homomorphism.

1. PRELIMINARIES:

1.1 Definition: Let X be a non–empty set. A **fuzzy subset A** of X is a function $A : X \rightarrow [0, 1]$.

1.2 Definition: Let R be a semiring. A fuzzy subset A of R is said to be a **fuzzy subsemiring (FSSR)** of R if it satisfies the following conditions:

(i) $\mu_A(x + y) \geq \min\{\mu_A(x), \mu_A(y)\}$,

(ii) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R.

1.3 Definition [5]: An **intuitionistic fuzzy subset (IFS)** A in X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \,/\, x \in X\}$, where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $0 \le \mu_A(x) + \nu_A(x) \le 1$.

1.1 Example: Let X= {a, b,c} be a set. Then A= {⟨ a, 0.52, 0.34 ⟩, ⟨ b, 0.14, 0.71⟩, ⟨ c, 0.25, 0.34 ⟩} is an intuitionistic fuzzy subset of X.

1.4 Definition: Let R be a semiring. An intuitionistic fuzzy subset A of R is said to be an **intuitionistic fuzzy subsemiring (IFSSR)** of R if it satisfies the following conditions:

(i) $\mu_A(x + y) \ge \min\{\mu_A(x), \mu_A(y)\}$,

(ii) $\mu_A(xy) \ge \min\{ \mu_A(x), \mu_A(y) \}$,

(iii) $\nu_A(x + y) \le \max\{ \nu_A(x), \nu_A(y) \}$,

(iv) $\nu_A(xy) \le \max\{ \nu_A(x), \nu_A(y) \}$, for all x and y in R.

1.5 Definition: Let $(R, +, \cdot)$ and $(R^l, +, \cdot)$ be any two semirings. Let $f : R \to R^l$ be any function and A be an intuitionistic fuzzy subsemiring in R, V be an intuitionistic fuzzy subsemiring in $f(R)=R^l$, defined by $\mu_V(y)= \sup_{x\in f^{-1}(y)} \mu_A(x)$ and $\nu_V(y)= \inf_{x\in f^{-1}(y)} \nu_A(x)$, for all x in R and y in R^l. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.6 Definition: Let A be an intuitionistic fuzzy subset of X. For α, β in [0, 1], the level subset of A is the set $A_{(\alpha,\beta)} = \{ x\in X : \mu_A(x) \ge \alpha, \mu_A(x) \le \beta \}$.

2. PROPERTIES OF INTUITIONISTIC FUZZY SUBSEMIRING OF A SEMIRING R

2.1 Theorem: Let (R, +, .) and $(R^l, +, .)$ be any two semirings. The homomorphic image of an intuitionistic fuzzy subsemiring of R is an intuitionistic fuzzy subsemiring of R^l.

Proof: Let $f: R \to R^l$ be a homomorphism. Then $f(x+y) = f(x) + f(y)$ and $f(xy)=f(x) f(y)$, for all x and y in R. Let V=f(A), where A is an intuitionistic fuzzy subsemiring of R. Now, for f(x), f(y) in R^l, $\mu_v(f(x)+f(y)) \ge \mu_A(x+y) \ge \min\{\mu_A(x), \mu_A(y)\}$, which implies that $\mu_v(f(x)+f(y)) \ge \min\{\mu_v(f(x)), \mu_v(f(y))\}$. Again, $\mu_v(f(x)f(y)) \ge \mu_A(xy) \ge \min\{\mu_A(x), \mu_A(y)\}$, which implies that $\mu_v(f(x)f(y)) \ge \min\{\mu_v(f(x)), \mu_v(f(y))\}$. Now, for f(x), f(y) in R^l, $\nu_v(f(x) + f(y)) \le \nu_A(x+ y) \le \max\{\nu_A(x), \nu_A(y)\}$, which implies that $\nu_v(f(x) + f(y)) \le \max\{\nu_v(f(x)),\nu_v(f(y))\}$. Again, $\nu_v(f(x)f(y)) \le \nu_A(xy) \le \max\{\nu_A(x), \nu_A(y)\}$, which implies that $\nu_v(f(x)f(y)) \le \max\{\nu_v(f(x)), \nu_v(f(y))\}$. Hence V is an intuitionistic fuzzy subsemiring of R^l.

2.2 Theorem: Let (R, +, .) and (R^{I}, +, .) be any two semirings. The homomorphic preimage of an intuitionistic fuzzy subsemiring of R^{I} is a intuitionistic fuzzy subsemiring of R.

Proof: Let f : R $\rightarrow$ R^{I} be a homomorphism. Then, f(x+y) = f(x) + f(y) and f(xy) = f(x) f(y), for all x and y in R. Let V = f(A), where V is an intuitionistic fuzzy subsemiring of R^{I}. Let x and y in R. Then, $\mu_A(x+y) = \mu_v(f(x)+ f(y)) \geq \min\{\mu_v(f(x)), \mu_v(f(y))\}= \min\{\mu_A(x), \mu_A(y)\}$, which implies that $\mu_A(x + y) \geq \min\{\mu_A(x), \mu_A(y)\}$. Again, $\mu_A(xy) = \mu_v(f(x)f(y)) \geq \min\{\mu_v(f(x)), \mu_v(f(y))\} = \min\{\mu_A(x), \mu_A(y)\}$, which implies that $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$. Then, $\nu_A(x+ y) = \nu_v(f(x + y)) = \nu_v(f(x) + f(y)) \leq \max\{\nu_v(f(x)), \nu_v(f(y))\} = \max\{\nu_A(x), \nu_A(y)\}$, which implies that $\nu_A(x+ y) \leq \max\{\nu_A(x), \nu_A(y)\}$. Again, $\nu_A(xy) = \nu_v(f(x)f(y)) \leq \max\{\nu_v(f(x)), \nu_v(f(y))\}= \max\{\nu_A(x), \nu_A(y)\}$, which implies that $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$. Hence A is an intuitionistic fuzzy subsemiring of R.

2.3 Theorem: Let (R, +, .) and (R^{I}, +, .) be any two semirings. The anti-homomorphic image of an intuitionistic fuzzy subsemiring of R is an intuitionistic fuzzy subsemiring of R^{I}.

Proof: Let f : R $\rightarrow$ R^{I} be an anti-homomorphism. Then f(x+y) = f(y) + f(x) and f(xy) = f(y) f(x), for all x, y $\in$ R. Let V = f(A), where A is an intuitionistic fuzzy subsemiring of R. Now, for f(x), f(y) in R^{I}, $\mu_v(f(x) +f(y)) \geq \mu_A(y+x) \geq \min\{\mu_A(y), \mu_A(x)\}=\min\{\mu_A(x),\mu_A(y)\}$, which implies that $\mu_v(f(x)+f(y))\geq\min\{\mu_v(f(x)), \mu_v(f(y))\}$. Again, $\mu_v(f(x)f(y)) \geq \mu_A(yx) \geq \min\{\mu_A(y), \mu_A(x)\}= \min\{\mu_A(x), \mu_A(y)\}$, which implies that $\mu_v(f(x)f(y)) \geq \min\{\mu_v(f(x)), \mu_v(f(y))\}$. Now, for f(x), f(y) in R^{I}, $\nu_v(f(x)+f(y)) \leq \nu_A(y+x) \leq \max\{\nu_A(y), \nu_A(x)\}= \max\{\nu_A(x), \nu_A(y)\}$, which implies that $\nu_v(f(x) +f(y)) \leq \max\{\nu_v(f(x)), \nu_v(f(y))\}$. Again, $\nu_v(f(x)f(y)) \leq \nu_A(yx) \leq \max\{\nu_A(y), \nu_A(x)\} = \max\{\nu_A(x),\nu_A(y)\}$, which implies that $\nu_v(f(x)f(y)) \leq \max\{\nu_v(f(x)), \nu_v(f(y))\}$. Hence V is an intuitionistic fuzzy subsemiring of R^{I}.

2.4 Theorem: Let (R, +, .) and (R^{I}, +, .) be any two semirings. The anti-homomorphic preimage of an intuitionistic fuzzy subsemiring of R^{I} is an intuitionistic fuzzy subsemiring of R.

Proof: Let f : R $\rightarrow$ R^{I} be an anti-homomorphism. Then, f(x+y) = f(y) + f(x) and f(xy) = f(y) f(x), for all x and y in R. Let V = f(A), where V is an intuitionistic fuzzy subsemiring of R^{I}. Then $\mu_A(x+y) \geq \min\{\mu_v(f(y)), \mu_v(f(x))\} = \min\{\mu_v(f(x)), \mu_v(f(y))\} = \min\{\mu_A(x), \mu_A(y)\}$, which implies that $\mu_A(x + y) \geq \min\{\mu_A(x), \mu_A(y)\}$. Again, $\mu_A(xy) \geq \min\{\mu_v(f(y)), \mu_v(f(x))\} = \min\{\mu_v(f(x)),\mu_v(f(y))\} = \min\{\mu_A(x), \mu_A(y)\}$, which implies that $\mu_A(xy) \geq \min\{\mu_A(x),\mu_A(y)\}$. Then $\nu_A(x+ y) \leq \max\{\nu_v(f(y)),\nu_v(f(x))\} = \max\{\nu_v(f(x)),\nu_v(f(y))\} = \max\{\nu_A(x),\nu_A(y)\}$, which implies that $\nu_A(x+y) \leq \max\{\nu_A(x), \nu_A(y)\}$. Again, $\nu_A(xy) \leq \max\{\nu_v(f(y)), \nu_v(f(x))\} = \max\{\nu_v(f(x)), \nu_v(f(y))\} = \max\{\nu_A(x), \nu_A(y)\}$, which implies that $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$. Hence A is an intuitionistic fuzzy subsemiring of R.

2.5 Theorem: Let A be an intuitionistic fuzzy subsemiring of a semiring (R, +, .). Then for α and β in [0,1], $A_{(\alpha,\beta)}$ is a subsemiring of R.

Proof: For all x and y in $A_{(\alpha,\beta)}$, we have, $\mu_A(x) \geq \alpha$ and $\nu_A(x) \leq \beta$ and $\mu_A(y) \geq \alpha$ and $\nu_A(y) \leq \beta$. Now, $\mu_A(x + y) \geq \min\{\mu_A(x), \mu_A(y)\} \geq \min\{\alpha, \alpha\} = \alpha$, which implies that, $\mu_A(x + y) \geq \alpha$. Now, $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\} \geq \min\{\alpha, \alpha\} = \alpha$, which implies that, $\mu_A(xy) \geq \alpha$. And also, $\nu_A(x+y) \leq \max\{\nu_A(x), \nu_A(y)\} \leq \max\{\beta, \beta\} = \beta$, which implies that, $\nu_A(x+y) \leq \beta$. And also, $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\} \leq \max\{\beta, \beta\} = \beta$, which implies that, $\nu_A(xy) \leq \beta$. Therefore, $\mu_A(x+y) \geq \alpha$, $\mu_A(xy) \geq \alpha$ and $\nu_A(x+y) \leq \beta$, $\nu_A(xy) \leq \beta$, we get x + y and xy in $A_{(\alpha,\beta)}$. Hence $A_{(\alpha,\beta)}$ is a subsemiring of R.

2.6 Theorem : Let (R, +, •) and (R^{l}, +, •) be any two semirings. If f : R→ R^{l} is a homomorphism, then the homomorphic image of a level subsemiring of an intuitionistic fuzzy subsemiring of R is a level subsemiring of an intuitionistic fuzzy subsemiring of R^{l}.

Proof: Let f : R→ R^{l} be a homomorphism. That is, f(x+y) = f(x)+f(y), for all x and y in R and f(xy) = f(x)f(y), for all x and y in R. Let V = f(A), where A is an intuitionistic fuzzy subsemiring of R. Clearly V is an intuitionistic fuzzy subsemiring of R^{l}. If x and y in R, then f(x) and f(y) in R^{l}. Let $A_{(\alpha,\beta)}$ be a level subsemiring of A. Suppose x and y in $A_{(\alpha,\beta)}$, then x+y and xy in $A_{(\alpha,\beta)}$.Now, $\mu_V(f(x)) \geq \mu_A(x) \geq \alpha$, implies that $\mu_V(f(x)) \geq \alpha$; $\mu_V(f(y)) \geq \mu_A(y) \geq \alpha$, implies that $\mu_V(f(y)) \geq \alpha$ and $\mu_V(f(x)+f(y)) \geq \mu_A(x+y) \geq \alpha$, which implies that $\mu_V(f(x) + f(y)) \geq \alpha$, for all f(x) and f(y) in R^{l}. And $\mu_V(f(x)f(y)) \geq \mu_A(xy) \geq \alpha$,which implies that $\mu_V(f(x) f(y)) \geq \alpha$, for all f(x) and f(y) in R^{l}. And, $\nu_V(f(x)) \leq \nu_A(x) \leq \beta$, implies that $\nu_V(f(x)) \leq \beta$; $\nu_V(f(y)) \leq \nu_A(y) \leq \beta$, implies that $\nu_V(f(y)) \leq \beta$, then $\nu_V(f(x) + f(y)) \leq \nu_A(x+y) \leq \beta$, which implies that $\nu_V(f(x)+ f(y)) \leq \beta$, for all f(x) and f(y) in R^{l}. Then $\nu_V(f(x)f(y)) \leq \nu_A(xy) \leq \beta$, which implies that $\nu_V(f(x)f(y)) \leq \beta$, for all f(x) and f(y) in R^{l}. Hence f ($A_{(\alpha,\beta)}$) is a level subsemiring of an intuitionistic fuzzy subsemiring V of a semiring R^{l}.

2.7 Theorem: Let (R, +, •) and (R^{l}, +, •) be any two semirings. If f : R→ R^{l} is a homomorphism, then the homomorphic pre-image of a level subsemiring of an intuitionistic fuzzy subsemiring of R^{l} is a level subsemiring of an intuitionistic fuzzy subsemiring of R.

Proof: Let f: R→R^{l} be a homomorphism. That is, f(x + y) = f(x) + f(y), for all x and y in R and f(xy) = f(x)f(y), for all x and y in R. Let V = f(A), where V is an intuitionistic fuzzy subsemiring of R^{l}. Clearly A is an intuitionistic fuzzy subsemiring of R. Let x and y in R. Let f($A_{(\alpha,\beta)}$) be a level subsemiring of V. Suppose f(x) and f(y) in f($A_{(\alpha,\beta)}$), then f(x)+ f(y) and f(x)f(y) in f($A_{(\alpha,\beta)}$). Now, $\mu_A(x) = \mu_V(f(x)) \geq \alpha$, implies that $\mu_A(x) \geq \alpha$; $\mu_A(y) = \mu_V(f(y)) \geq \alpha$, implies that $\mu_A(y) \geq \alpha$, we have $\mu_A(x+y) \geq \alpha$, which implies that $\mu_A(x+y) \geq \alpha$, for all x and y in

R. And $\mu_A(xy) \geq \alpha$, which implies that $\mu_A(xy) \geq \alpha$, for all x and y in R. And, $\nu_A(x) = \nu_V(f(x)) \leq \beta$, implies that $\nu_A(x) \leq \beta$; $\nu_A(y) = \nu_V(f(y)) \leq \beta$, implies that $\nu_A(y) \leq \beta$, we have $\nu_A(x+y) = \nu_V(f(x)+f(y)) \leq \beta$, which implies that $\nu_A(x+y) \leq \beta$, for all x and y in R. And $\nu_A(xy) = \nu_V(f(x)f(y)) \leq \beta$, which implies that $\nu_A(xy) \leq \beta$, for all x and y in R. Hence $A_{(\alpha,\beta)}$ is a level subsemiring of an intuitionistic fuzzy subsemiring A of R.

2.8 Theorem: Let (R, +, •) and (R^l, +, •) be any two semirings. If f : R→R^l is an anti-homomorphism, then the anti-homomorphic image of a level subsemiring of an intuitionistic fuzzy subsemiring of R is a level subsemiring of an intuitionistic fuzzy subsemiring of R^l.

Proof: Let f : R→ R^l be an anti-homomorphism. That is, f(x+y) = f(y)+f(x), for all x and y in R and f(xy) = f(y)f(x), for all x and y in R. Let V = f(A), where A is an intuitionistic fuzzy subsemiring of R. Clearly V is an intuitionistic fuzzy subsemiring of R^l. If x and y in R, then f(x) and f(y) in R^l. Let $A_{(\alpha,\beta)}$ be a level subsemiring of A. Suppose x and y in $A_{(\alpha,\beta)}$, then y+x and yx in $A_{(\alpha,\beta)}$. Now, $\mu_V(f(x)) \geq \mu_A(x) \geq \alpha$, implies that $\mu_V(f(x)) \geq \alpha$; $\mu_V(f(y)) \geq \mu_A(y) \geq \alpha$, implies that $\mu_V(f(y)) \geq \alpha$ and $\mu_V(f(x) + f(y)) \geq \mu_A(y+x) \geq \alpha$, which implies that $\mu_V(f(x) + f(y)) \geq \alpha$, for all f(x) and f(y) in R^l. And $\mu_V(f(x)f(y)) \geq \mu_A(yx) \geq \alpha$, which implies that $\mu_V(f(x)f(y)) \geq \alpha$, for all f(x) and f(y) in R^l. And, $\nu_V(f(x)) \leq \nu_A(x) \leq \beta$, implies that $\nu_V(f(x)) \leq \beta$; $\nu_V(f(y)) \leq \nu_A(y) \leq \beta$, implies that $\nu_V(f(y)) \leq \beta$ and $\nu_V(f(x) + f(y)) \leq \nu_A(y+x) \leq \beta$, which implies that $\nu_V(f(x)+ f(y)) \leq \beta$, for all f(x) and f(y) in R^l. And $\nu_V(f(x)f(y)) \leq \nu_A(yx) \leq \beta$, which implies that $\nu_V(f(x) f(y)) \leq \beta$, for all f(x) and f(y) in R^l. Hence f ($A_{(\alpha,\beta)}$) is a level subsemiring of an intuitionistic fuzzy subsemiring V of a semiring R^l.

2.9 Theorem: Let (R, +, •) and (R^l, +, •) be any two semirings. If f : R →R^l is an anti-homomorphism, then the anti-homomorphic pre-image of a level subsemiring of an intuitionistic fuzzy subsemiring of R^l is a level subsemiring of an intuitionistic fuzzy subsemiring of R.

Proof: Let f: R→R^l be an anti-homomorphism. That is, f(x + y) = f(y) + f(x), for all x and y in R and f(xy) = f(y)f(x), for all x and y in R. Let V = f(A), where V is an intuitionistic fuzzy subsemiring of R^l. Clearly A is an intuitionistic fuzzy subsemiring of R. Let x and y in R. Let f($A_{(\alpha,\beta)}$) be a level subsemiring of V. Suppose f(x) and f(y) in f($A_{(\alpha,\beta)}$), then f(y)+ f(x) and f(y)f(x) in f($A_{(\alpha,\beta)}$). Now, $\mu_A(x) = \mu_V(f(x)) \geq \alpha$, implies that $\mu_A(x) \geq \alpha$; $\mu_A(y) = \mu_V(f(y)) \geq \alpha$, implies that $\mu_A(y) \geq \alpha$, we have $\mu_A(x+y) = \mu_V(f(y)+f(x)) \geq \alpha$, which implies that $\mu_A(x+y) \geq \alpha$, for all x and y in R. And $\mu_A(xy) = \mu_V(f(y)f(x)) \geq \alpha$, which implies that $\mu_A(xy) \geq \alpha$, for all x and y in R. And, $\nu_A(x) = \nu_V(f(x)) \leq \beta$, implies that $\nu_A(x) \leq \beta$; $\nu_A(y) = \nu_V(f(y)) \leq \beta$, implies that $\nu_A(y) \leq \beta$, we have $\nu_A(x+y) = \nu_V(f(y) + f(x)) \leq \beta$, which implies that $\nu_A(x+y) \leq \beta$, for all x and y in R. And $\nu_A(xy) = \nu_V(f(y) f(x)) \leq \beta$, which implies that $\nu_A(xy) \leq \beta$, for all x and y in R. Hence $A_{(\alpha,\beta)}$ is a level subsemiring of an intuitionistic fuzzy subsemiring A of R.

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