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Prey Predator Model with Asymptotic Non-Homogeneous Predation

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Abstract

We have formulated and analyzed a prey-predator model in a two patch environment (patch 1 and patch 2). Each patch is supposed to be homogeneous. Patch 2 constitutes a reserved area of prey and no fishing is permitted in this zone whereas patch 1 is an open access fishery zone. The growth of prey in each patch is assumed to be logistic. The transmission function from zone 1 due to predation is considered as a modified function of general nature. Stability analyses along with the optimal harvest policy are also obtained. Numerical simulation has also been performed in support of analysis. For different values of predation parameter, the equilibrium level has been tabulated. It has been observed that the use of new transmission function lowers down the equilibrium level.

Mathematics Subject Classification: 92D30

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1. INTRODUCTION

The optimal management of renewable resources such as fishery and forestry is very important as they are directly linked to sustainable development. The growing need for more food and energy has led to increase in the exploitation of several biological resources. On the other hand, there is a global concern to protect the ecosystem at large.

Over the past three decades, mathematics has made a considerable impact as a tool to model and understand biological phenomena. Braza [7] analyzed a two predator, one prey model in which one predator interferes significantly with other. The analysis centers on bifurcation diagrams for various levels of interference in which harvesting is the primary bifurcation parameter. Kar. et. al. [16], in their paper, offer some mathematical analysis of the dynamics of a two prey, one predator system in the presence of a time delay. Singh et.al. [3] proposed a generalized mathematical model to study the depletion of resources by two kinds of populations, one is weaker and others stronger. The dynamics of resources is governed by generalized logistic equation whereas the population of interacting species follows the logistic law. Dubey et.al. [2] proposed and analyzed a mathematical model to study the dynamics of one prey, two predators system with ratio dependent predators growth rate.

Dubey et.al. [1] analyzed a dynamic model for a single species fishery which depends partially on a logistically growing resource in a two patch environment. They showed that both the equilibrium density of the fish population as well as the maximum sustainability yield increases as the resource biomass density increases. Further, Kar et.al. [14] modified the model proposed by Dubey et.al. [1] in the presence of predator, which seems to be more realistic. They discussed the local and global stability. The optimal harvesting policy has been discussed using Pontryagin Maximal Principal.

Taha et.al. [10] studied the effect of time delay and harvesting on the dynamics of the predator prey model with a time delay in the growth rate of the prey equation. A model of non-selective harvesting in a prey-predator fishery is given by Kar et.al. [12].In their further work [13], they described the regulation of a prey-predator fishery by taxation as the control instrument.

Kar [15] proposed and analyzed a non-linear mathematical model to study the dynamics of a fishery resource system in an aquatic environment that consists of two zones: a free fishing zone and a reserve zone where fishing is strictly prohibited. Biological equilibria of the system are obtained and criteria for local stability and global stability of the system derived. An optimal harvesting policy is also discussed using Pontryagin Maximal Principal.

2. DISCRIPTION OF THE MODEL

We study a prey-predator system in a two patch environment: one accessible to both prey and predators (patch 1) and the other one being a refuge for the prey (patch 2). Each patch is supposed to be homogeneous. The prey refuge (patch 2) constitutes a reserve area of prey and no fishing is permitted in the reserve zone while the unreserved zone area is an open access fishery zone. We supposed that the prey migrate between the two patches randomly .The growth of prey in each patch in absence of predator is assumed to be logistic. The transmission function from unreserved zone due to predation is considered as a modified function of general nature.

Following Kar [15], the mathematical formulation of the model takes the form

$$\begin{aligned}\frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right)-\sigma_1 x+\sigma_2 y-\frac{mx^2z}{a+x^2}-qEx,\\ \frac{dy}{dt} &= sy\left(1-\frac{y}{L}\right)+\sigma_1 x-\sigma_2 y, \qquad (2.1)\\ \frac{dz}{dt} &= -dz+\frac{m\alpha x^2 z}{a+x^2}.\end{aligned}$$

Where a is a positive constants and other symbols have the same meaning as defined in [15].

3. EXISTENCE OF THE EQUILIBRIA

Equilibria of model (2.1) can be obtained by equating right hand side to zero. This provides three equilibria $P_0(0,0,0), P_1(\bar{x},\bar{y},0), P_2(\hat{x},\hat{y},\hat{z})$. The equilibrium P_0 is trivial. In equilibrium point P_1, we have

$$a_1 x^3 + b_1 x^2 + c_1 x + d_1 = 0 \qquad (3.1)$$

Where,

$$a_1 = \frac{r^2 s}{K^2 L\sigma_2^2},$$

$$b_1 = -\frac{2rs(r-\sigma_1-qE)}{KL\sigma_2^2},$$

$$c_1 = \frac{s(r-\sigma_1-qE)^2}{L\sigma_2^2} - \frac{(s-\sigma_2)r}{K\sigma_2},$$
$$d_1 = \frac{(s-\sigma_2)}{\sigma_2}(r-\sigma_1-qE)-\sigma_1.$$

Equation (3.1) has unique positive solution $x=\overline{x}$ if the following inequalities hold:

$$\frac{s(r-\sigma_1-qE)^2}{L\sigma_2} < \frac{(s-\sigma_2)r}{K},$$
$$(s-\sigma_2)(r-\sigma_1-qE) < \sigma_1\sigma_2. \qquad (3.2)$$

And for $\overline{y}$ to be positive, we must have

$$\frac{K}{r}(r-\sigma_1-qE) > 0. \qquad (3.3)$$

Hence the equilibrium $P_1(\overline{x},\overline{y},0)$ exists under the above conditions.
Again $\hat{x},\hat{y}$ and $\hat{z}$ are positive solutions of

$$rx\left(1-\frac{x}{K}\right)-\sigma_1 x+\sigma_2 y-\frac{mx^2 z}{a+x^2}-qEx=0, \qquad (3.4a)$$
$$sy\left(1-\frac{y}{L}\right)+\sigma_1 x-\sigma_2 y=0, \qquad (3.4b)$$
$$-dz+\frac{m\alpha x^2 z}{a+x^2}=0. \qquad (3.4c)$$

From (3.4c) we get $\hat{x}=\sqrt{\frac{ad}{m\alpha-d}}$, which is positive if $m\alpha > d$.

Substituting the value of $\hat{x}$ in (3.4b), we get

$$\frac{s}{L}y^2-(s-\sigma_2)y-\sigma_1\hat{x}=0.$$

The above equation has atleast one positive solution $y=\hat{y}$. Substituting the value of $\hat{x}$, we get $\hat{z}$ as

$$\hat{z} = \left(\frac{a+\hat{x}^2}{m\hat{x}^2}\right)\left\{r\hat{x}\left(1-\frac{\hat{x}}{K}\right)-\sigma_1\hat{x}+\sigma_2\hat{y}-qE\hat{x}\right\}.$$

It may be noted that for $\hat{z}$ to be positive, we must have

$$\left\{(r-\sigma_1-qE)\hat{x}-\frac{r}{K}\hat{x}^2+\sigma_2\hat{y}\right\}>0.$$

4. DYNAMICAL BEHAVIOUR OF EQUILIBRIA

The dynamical behavior of equilibria can be studied by computing variational matrix corresponding to each equilibria. The variational matrix about $P_0(0,0,0)$ will provide the characteristic equation as

$$\lambda^2+a_3\lambda+b_3=0 \qquad (4.1)$$

where,
$$a_3=-\left(r+s-(\sigma_1+\sigma_2+qE)\right),$$
$$b_3=(r-qE)(s-\sigma_2)-\sigma_1 s.$$

The roots of equation (4.1) have positive real part if $(r+s)<(\sigma_1+\sigma_2+qE)$. So under this condition, $P_0(0,0,0)$ will be unstable.
The characteristic equation about $P_1(\bar{x},\bar{y},0)$ is

$$\lambda^2+a_4\lambda+b_4=0 \qquad (4.2)$$

where,
$$a_4=-\left(r+s-\left(\sigma_1+\sigma_2+qE+\frac{2r\bar{x}}{K}+\frac{2s\bar{y}}{L}\right)\right)$$
$$b_4=\left(r-\frac{2r\bar{x}}{K}-qE\right)\left(s-\frac{2s\bar{y}}{L}-\sigma_2\right)-\sigma_1\left(s-\frac{2s\bar{y}}{L}\right).$$

The root of equation (4.2) has negative real part if

$$(r+s)>\left(\sigma_1+\sigma_2+qE+\frac{2r\bar{x}}{K}+\frac{2s\bar{y}}{L}\right).$$

So that $P_1(\bar{x},\bar{y},0)$ is locally asymptotically stable.

The characteristic equation about $P_2(\hat{x},\hat{y},\hat{z})$ is

$$\lambda^3 + a_5\lambda^2 + b_5\lambda + c_5 = 0 \qquad (4.3)$$

where

$$a_5 = \frac{r}{K}\hat{x} + \frac{s}{L}\hat{y} + \sigma_1\frac{\hat{x}}{\hat{y}} + \sigma_2\frac{\hat{y}}{\hat{x}} + \frac{m\hat{x}\hat{z}\left(a-\hat{x}^2\right)}{\left(a+\hat{x}^2\right)^2} - \frac{m\alpha\hat{x}^2}{\left(a+\hat{x}^2\right)} + d$$

$$b_5 = \left(\frac{r}{K}\hat{x} + \sigma_2\frac{\hat{y}}{\hat{x}} + \frac{m\hat{x}\hat{z}\left(a-\hat{x}^2\right)}{\left(a+\hat{x}^2\right)^2}\right)\left(\frac{s}{L}\hat{y} + \sigma_1\frac{\hat{x}}{\hat{y}} - \frac{m\alpha\hat{x}^2}{\left(a+\hat{x}^2\right)} + d\right)$$
$$+\left(\frac{s}{L}\hat{y} + \sigma_1\frac{\hat{x}}{\hat{y}}\right)\left(-\frac{m\alpha\hat{x}^2}{\left(a+\hat{x}^2\right)} + d\right) - \sigma_1\sigma_2 + \frac{2m^2 a\alpha\hat{x}^3\hat{z}}{\left(a+\hat{x}^2\right)^3},$$

$$c_5 = \left(\frac{r}{K}\hat{x} + \sigma_2\frac{\hat{y}}{\hat{x}} + \frac{m\hat{x}\hat{z}\left(a-\hat{x}^2\right)}{\left(a+\hat{x}^2\right)^2}\right)\left(\frac{s}{L}\hat{y} + \sigma_1\frac{\hat{x}}{\hat{y}}\right)\left(-\frac{m\alpha\hat{x}^2}{\left(a+\hat{x}^2\right)} + d\right)$$
$$-\sigma_1\sigma_2\left(-\frac{m\alpha\hat{x}^2}{\left(a+\hat{x}^2\right)} + d\right) + \frac{2m^2 a\alpha\hat{x}^3\hat{z}}{\left(a+\hat{x}^2\right)^3}\left(\frac{s}{L}\hat{y} + \sigma_1\frac{\hat{x}}{\hat{y}}\right).$$

Here $a_5 > 0, b_5 > 0$ and $c_5 > 0$ provided

$$\frac{r}{K}\hat{x} + \sigma_2\frac{\hat{y}}{\hat{x}} + \frac{ma\hat{x}\hat{z}}{\left(a+\hat{x}^2\right)^2} > \frac{m\hat{x}^3\hat{z}}{\left(a+\hat{x}^2\right)^2},$$

$$d > \frac{m\alpha\hat{x}^2}{\left(a+\hat{x}^2\right)},$$

$$\left(\frac{r}{K}\hat{x} + \sigma_2\frac{\hat{y}}{\hat{x}} + \frac{m\hat{x}\hat{z}\left(a-\hat{x}^2\right)}{\left(a+\hat{x}^2\right)^2}\right)\left(\frac{s}{L}\hat{y} + \sigma_1\frac{\hat{x}}{\hat{y}} + d - \frac{m\alpha\hat{x}^2}{\left(a+\hat{x}^2\right)}\right)$$
$$+\left(\frac{s}{L}\hat{y} + \sigma_1\frac{\hat{x}}{\hat{y}}\right)\left(d - \frac{m\alpha\hat{x}^2}{\left(a+\hat{x}^2\right)}\right) + \frac{2m^2 a\alpha\hat{x}^3\hat{z}}{\left(a+\hat{x}^2\right)^3} > \sigma_1\sigma_2,$$

$$\left(\frac{r}{K}\hat{x}+\sigma_2\frac{\hat{y}}{\hat{x}}+\frac{m\hat{x}\hat{z}\left(a-\hat{x}^2\right)}{\left(a+\hat{x}^2\right)^2}\right)\left(\frac{s}{L}\hat{y}+\sigma_1\frac{\hat{x}}{\hat{y}}\right)$$

$$\left(d-\frac{m\alpha\hat{x}^2}{\left(a+\hat{x}^2\right)}\right)+\frac{2m^2a\alpha\hat{x}^3\hat{z}}{\left(a+\hat{x}^2\right)^3}\left(\frac{s}{L}\hat{y}+\sigma_1\frac{\hat{x}}{\hat{y}}\right)>\sigma_1\sigma_2\left(d-\frac{m\alpha\hat{x}^2}{\left(a+\hat{x}^2\right)}\right).$$

Performing simple calculations it can easily be verified that $a_5b_5>c_5$ under the above conditions. Thus by Routh Hurwitz criterion, all Eigen values of (4.3) will have negative real part. Hence $P_2\left(\hat{x},\hat{y},\hat{z}\right)$ is asymptotically stable.
In the following lemma we show that all the solutions of model (2.1) are positive and uniformly bounded.

Lemma: *The set* $\Omega=\left\{(x,y,z)\in\mathbf{R}_3^+:\omega=x+y+\frac{1}{\alpha}z<\mu\upsilon\right\}$ *is a region of attraction for all solutions initiating in the interior of the positive octant, where* $\upsilon>d$ *is a positive constant and*

$$\mu=\frac{K}{2r}\left(r+\upsilon-qE\right)^2+\frac{L}{2s}\left(s+\upsilon\right)^2.$$

Proof: Let $\omega(t)=x(t)+y(t)+\frac{1}{\alpha}z(t)$ and $\upsilon>0$ be a constant.
Then we have,

$$\frac{d\omega}{dt}+\upsilon\omega=-\frac{r}{K}x^2+\left(r+\upsilon-qE\right)x-\frac{s}{L}y^2+\left(s+\upsilon\right)y-\left(\frac{d}{\alpha}-\frac{\upsilon}{\alpha}\right)z$$

$$\leq\frac{K}{2r}\left(r+\upsilon-qE\right)^2+\frac{L}{2s}\left(s+\upsilon\right)^2=\mu\,(\textbf{Say})$$

Thus $t\to\infty,\omega\leq\frac{\mu}{\upsilon}$. This proves the lemma.

Theorem: *The equilibrium point* P_1 *is globally asymptotically stable.*

Proof: Let us consider the Lyapunov function

$$V=\left(x-\overline{x}-\overline{x}In\frac{x}{\overline{x}}\right)+K_1\left(y-\overline{y}-\overline{y}In\frac{y}{\overline{y}}\right).$$

Differentiating V w.r.t. t, we get,

$$\frac{dV}{dt} = \frac{x-\bar{x}}{x}\frac{dx}{dt} + K_1\frac{y-\bar{y}}{y}\frac{dy}{dt}.$$

Choose $K_1 = \dfrac{\bar{y}\sigma_2}{\bar{x}\sigma_1}$,

$$\frac{dV}{dt} = -\frac{r}{K}\left(x-\bar{x}\right)^2 - \frac{\bar{y}}{\bar{x}\sigma_1 L}\left(y-\bar{y}\right)^2 - \frac{\sigma_2}{x\bar{x}y}\left(x\bar{y}-\bar{x}y\right)^2 < 0.$$

Therefore $P_1\left(\bar{x},\bar{y},0\right)$ is globally asymptotically stable.

5. OPTIMAL HARVESTING POLICY

Our objective is to maximize the present value J of continuous time stream of revenue given by

$$J = \int_0^\infty e^{-\delta t}\left(pqx\left(t\right)-c\right)E\left(t\right)dt \qquad (5.1)$$

where δ is instantaneous rate of annual discount. Thus our objective is to maximize J subject to state equation (2.1) and to the control constraints

$$0 \le E\left(t\right) \le E_{\max} \qquad (5.2)$$

To solve this optimization problem, we utilize the Pontryagin Maximal Principle. The associated Hamiltonian is given by

$$\begin{aligned} H &= e^{-\delta t}\left(pqx-c\right)E + \lambda_1\left(t\right)\left(rx - \frac{r}{K}x^2 - \sigma_1 x + \sigma_2 y - \frac{mx^2 z}{a+x^2} - qEx\right) \\ &\quad + \lambda_2\left(t\right)\left(sy - \frac{s}{L}y^2 + \sigma_1 x - \sigma_2 y\right) \\ &\quad + \lambda_3\left(t\right)\left(-dz + \frac{m\alpha x^2 z}{a+x^2}\right). \end{aligned} \qquad (5.3)$$

where $\lambda_1, \lambda_2, \lambda_3$ are adjoint variables and

$$\sigma(t) = e^{-\delta t}\left(pqx - c\right) - \lambda_1 qx$$

is called switching function.

Since H is linear in control variable E, the optimal control will be a combination of bang-bang control and singular control. The optimal control $E(t)$ which maximizes H must satisfy the following conditions:

$$E = E_{\max}, \text{ when } \sigma(t) > 0, \text{ i.e. when } \lambda_1 e^{\delta t} < p - \frac{c}{qx} \tag{5.4a}$$

$$E = 0, \text{ when } \sigma(t) < 0, \text{ i.e. when } \lambda_1 e^{\delta t} > p - \frac{c}{qx} \tag{5.4b}$$

$\lambda_1 e^{\delta t}$ is the usual shadow price and $p - \frac{c}{qx}$ is the net economic revenue on the unit harvest. This shows that $E = E_{\max}$ or zero according to the shadow price is less then or greater than the net economic revenue on a unit harvest. Economically, condition $(5.4a)$ implies that if the profit after paying all the expenses is positive then, it is beneficial to harvest up to the limit of available effort. Condition $(5.4b)$ implies that when the shadow price exceeds the fisherman's net economic revenue on the unit harvest, then the fisherman will not exert any effort.

When $\sigma(t) = 0$ i.e. the shadow price equals the net economic revenue on the unit harvest, then the Hamiltonian H becomes independent of the control variable $E(t)$ i.e. $\frac{\partial H}{\partial E} = 0$. This is the necessary condition for the singular control $\hat{E}(t)$ to be optimal over the control set $0 < \hat{E} < E_{\max}$.

The optimal harvest policy is

$$E(t) = \begin{cases} E_{\max}, \sigma(t) > 0 \\ 0 \quad\ , \sigma(t) < 0 \\ \hat{E} \quad\ , \sigma(t) = 0 \end{cases} \tag{5.5}$$

When $\sigma(t) = 0$ it follows that

$$\lambda_1 qx = e^{-\delta t}\left(pqx - c\right) = e^{-\delta t}\,\frac{\partial \Pi}{\partial E}. \tag{5.6}$$

This implies that the user's cost of harvest per unit effort equals the discounted value of the future marginal profit of the effort at the steady state.

Now, in order to find the path of singular control we utilize the Pontriagin Maximal Principle, and the adjoint variables $\lambda_1, \lambda_2, \lambda_3$ must satisfy

$$\frac{d\lambda_1}{dt} = -\frac{\partial H}{\partial x} = -\left[e^{-\delta t} pqx + \lambda_1 \left\{ r - \frac{2rx}{K} - \sigma_1 - \frac{2maxz}{\left(a + x^2\right)^2} - qE \right\} + \lambda_2 \sigma_1 + \lambda_3 \frac{2ma\alpha xz}{\left(a + x^2\right)^2} \right] \tag{5.7}$$

$$\frac{d\lambda_2}{dt} = -\frac{\partial H}{\partial y} = -\left[\sigma_2 \lambda_1 + \lambda_2 \left\{ s - \frac{2sy}{L} - \sigma_2 \right\} \right] \tag{5.8}$$

$$\frac{d\lambda_3}{dt} = -\frac{\partial H}{\partial z} = -\left[-\lambda_1 \frac{mx^2}{\left(a + x^2\right)} + \lambda_3 \left\{ -d + \frac{m\alpha x^2}{\left(a + x^2\right)} \right\} \right] \tag{5.9}$$

Considering the interior equilibrium $P_2\left(\hat{x},\hat{y},\hat{z}\right)$ and the equation $(5.6),(5.8)$ can be written as

$$\frac{d\lambda_2}{dt} - A_1 \lambda_2 = -A_2 e^{-\delta t}$$

Where,

$$A_1 = \frac{sy}{L} + \sigma_1 \frac{x}{y}, A_2 = \sigma_2 \left(p - \frac{c}{qx} \right)$$

whose solution is given by

$$\lambda_2 = A_2 \frac{e^{-\delta t}}{A_1 + \delta} \tag{5.10}$$

from (5.9), we get

$$\lambda_3 = -\frac{mx^2}{\left(a + x^2\right)\delta} \left(p - \frac{c}{qx} \right) e^{-\delta t} \tag{5.11}$$

from (5.7), we get

$$\frac{d\lambda_1}{dt} - B_1\lambda_1 = -B_2 e^{-\delta t}$$

Where

$$B_1 = \frac{r}{K}x + \sigma_2\frac{y}{x} - \frac{mxz\left(a - x^2\right)}{\left(a + x^2\right)^2}, B_2 = pqE + \frac{\sigma_1 A_2}{A_1 + \delta} - \frac{2m^2\alpha x^3 z}{\delta\left(a + x^2\right)^3}\left(p - \frac{c}{qx}\right)$$

whose solution is given by

$$\lambda_1 = B_2\frac{e^{-\delta t}}{B_1 + \delta} \qquad (5.12)$$

from (5.6) and (5.12), we get the singular path

$$\left(p - \frac{c}{qx}\right) = \frac{B_2}{B_1 + \delta} \qquad (5.13)$$

Equation (3.4) together with equation (5.13) gives the optimal equilibrium population $\hat{x} = x_\delta, \hat{y} = y_\delta, \hat{z} = z_\delta$. Then the optimal harvesting effort is given by

$$\hat{E} = E_\delta = \frac{1}{q}\left[r\left(1 - \frac{x_\delta}{K}\right) - \sigma_1 + \sigma_2\frac{y_\delta}{x_\delta} - \frac{mx_\delta z_\delta}{a + x_\delta^2}\right].$$

6. NUMERICAL SIMULATION AND CONCLUSION

Using the parameters r = 3.0, K = 110, $\sigma_1 = 0.5$, $\sigma_2 = 0.5$, m = 2.5, a = 12.0, q = 0.01, s = 0.4, L = 200, d = 0.01, $\alpha = 0.006$, p = 15, c = 1.4, $\delta = 0.005$.for the above values of parameters we find the optimal equilibrium as (4.9, 18.01, 6.78).Using the above parameters, the sensitivity analysis is performed to study the effect of predation on optimal solution. The following table shows the variation of $\left(x_\delta, y_\delta, z_\delta\right)$

m	x_δ	y_δ	z_δ
2	7.75	25.6	9.51
2.5	4.9	18.01	6.78
3	3.87	14.9	5.63

Thus we observe that the equilibrium level goes down by using new transmission function.

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