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Stochastic Analysis of Business with Two Levels and Manpower with Three Levels

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Abstract

In this paper we consider a business organization under varying conditions which are restricted to depend on business and manpower under fluctuating conditions of full availability, modeate availability and nil availability in the case of manpower and full availability and nil availability in the case business. The different states have been discussed under the assumption that the transitions from one state to another in both manpower and business occur in exponential times with different parameters. An expression for "Rate of Crisis under steady state (C_∞)" is arrived and steady state cost have also been worked by assuming different costs for the parameters under different conditions.

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1 Introduction

Nowadays we find that labor has become a buyers market as well as seller's market. Any company normally runs on commercial basis wishes to keep only the optimum level of any resources needed to meet company's requirement at any time during the course of the business and manpower is not an exception. This is spelt in the sense that a company does not want to keep manpower more than what is required. Hence, retrenchment and recruitment are common and frequent in most of the companies now. Recruitment is done when the business is busy and shed manpower when the business is lean. Equally true with the labor, has the option to switch over to other jobs because of better working condition, better emolument, proximity to their living place or other reasons. Under such situations the company may face crisis because business may be there but manpower may not be available. If skilled laborers and technically qualified persons leave the business the seriousness is worst felt and the company has to hire paying heavy price or pay overtime to employees.

Approach to manpower problems have been dealt in very many different ways as early as 1947 by Vajda [10] and others. Models in manpower planning has been dealt in depth in Barthlomew [1], Grinold & Marshal [3] and Vajda [10]. The methods to compute wastages (Resignation, dismissal and death) and promotion intensities which produce the proportions corresponding to some desired planning proposals has been dealt by Lesson [4]. Markov models are designed for wastages and promotion in manpower system by Vassilou [11]. V. Subramaniam [9] in his thesis has made an attempt to provide optimal policy for recruitment training, promotion, and wastages in man power planning models with special provisions such as time bound promotions, cost of training and voluntary retirement scheme. For application of Markov chains in a manpower system with efficiency and seniority and Stochastic structures of graded size in manpower planning systems one may refer to Setlhare [8]. A two unit stand by system has been investigated by Chandrasekar and Natrajan [2] with confidence limits under steady state. For n unit standby system one may refer to Ramanarayanan and Usha [7]. Yadhavalli and Botha [12] have examined the same for two unit system with introduction of preparation time for the service facility and the confidence limits for stationary rate of disappointment of an intermittently used system. For three characteristics system involving manpower, money and machine one may refer to C. Mohan and R. Ramanarayanan [6] For the study of Semi Markov Models for Manpower planning one may refer to the paper by Sally Meclean [5].

In this paper we consider two characteristics namely manpower and business. We derive a formula for the steady state rate of crisis and the steady state probabilities. The situations may be that the manpower may be fully available, insufficiently available or hardly available, but business may fluctuate between full availability to nil availability. The steady state probabilities of the continuous Markov chain describing the transitions in various states are derived and critical states are identified for presenting the cost analysis. Numerical illustrations are provided.

2 Assumptions

1. There are three levels of Manpoer namely Manpower is full, is moderate and Manpower is nil.

2. There are two levels of business namely (I) business is fully available (2) business is lean or nil.

3. The time T during which the manpower remains continuously moderate and becomes nil has exponential distribution with parameter λ_{10}. The time R required to complete recruitment for filling up of vacancies from level nil to moderate level is exponentially distributed with parameter μ_{01}.

4. The time T' during which the Manpower remains continuously full and becomes nil has exponential distribution with parameter λ_{20} and the time R' required to complete full recruitment from nil level is exponentially distributed with parameter μ_{02}.

5. The period of time T'' during which the Manpower is continuously full becomes moderate has exponential distribution with parameter λ_{21} and the period of time R'' required for recruitment from insufficient to full is exponentially distributed with parameter μ_{12}. Random variables T and R; T' and R'; T'' and R'' are all independent.

6. The busy and lean periods of the business are exponentially distributed with parameters 'a' and 'b' respectively.

3 System Analysis

The Stochastic Process $X(t)$ describing the state of the system is a continuous time Markov chain with 6 points state space as given below in the order of Manpower and Business

$$S = \{(0\ 0), (0\ 1), (1\ 0), (1\ 1), (2\ 0), (2\ 1)\} \tag{1}$$

where
2 - Refers to full availability in the case of manpower
1 - Refers to semi availability or insufficiently available manpower and it refers

to busy period in the case of business.
0 - Refers to shortage/lean/non availability manpower or business.
The infinitesimal generator Q of the continuous time Markov chain of the state space is given below which is a matrix of order 6.

$$Q = \begin{array}{|c|c|c|c|c|c|c|} \hline MP/B & (0\ 0) & (0\ 1) & (1\ 0) & (1\ 1) & (2\ 0) & (2\ 1) \\ \hline (0\ 0) & \epsilon_1 & b & \mu_{01} & 0 & \mu_{02} & 0 \\ \hline (0\ 1) & a & \epsilon_2 & 0 & \mu_{01} & 0 & \mu_{02} \\ \hline (1\ 0) & \lambda_{10} & 0 & \epsilon_3 & b & \mu_{12} & 0 \\ \hline (1\ 1) & 0 & \lambda_{10} & a & \epsilon_4 & 0 & \mu_{12} \\ \hline (2\ 0) & \lambda_{20} & 0 & \lambda_{21} & 0 & \epsilon_5 & b \\ \hline (2\ 1) & 0 & \lambda_{20} & 0 & \lambda_{21} & a & \epsilon_6 \\ \hline \end{array} \quad (2)$$

$$\epsilon_1 = -(\mu_{01} + \mu_{02} + b), \quad \epsilon_2 = -(\mu_{01} + \mu_{02} + a), \quad \epsilon_3 = -(\lambda_{10} + \mu_{12} + b),$$
$$\epsilon_4 = -(\lambda_{10} + \mu_{12} + a), \quad \epsilon_5 = -(\lambda_{20} + \lambda_{21} + b), \quad \epsilon_6 = -(\lambda_{21} + \lambda_{21} + a). \quad (3)$$

The occurrences of transitions in both manpower and business are independent, the individual infinitesimal generaor of them are given by:

1. The infinitesimal generator of business is given below by a matrix of order 2.

$$B = \begin{array}{|c|c|c|} \hline B & 0 & 1 \\ \hline 0 & -b & b \\ \hline 1 & a & -a \\ \hline \end{array}$$

   and the steady state probabilities are $\pi_{B0} = \dfrac{a}{a+b}$ and $\pi_{B1} = \dfrac{b}{a+b}$.

2. The infinitesimal generator of manpower is given below by the matrix of order 3,

$$M = \begin{array}{|c|c|c|c|} \hline M & 0 & 1 & 2 \\ \hline 0 & -(\mu_{01} + \mu_{02}) & \mu_{01} & \mu_{02} \\ \hline 1 & \lambda_{10} & -(\lambda_{10} + \mu_{12}) & \mu_{12} \\ \hline 2 & \lambda_{20} & \lambda_{21} & -(\lambda_{21} + \lambda_{20}) \\ \hline \end{array}$$

   The steady state probabilities of manpower are:

$$\pi_{M0} = \frac{d_0}{d_0 + d_1 + d_2}, \quad \pi_{M1} = \frac{d_1}{d_0 + d_1 + d_2}, \quad \pi_{M2} = \frac{d_2}{d_0 + d_1 + d_2}$$

where
$d_0 = \lambda_{20}\mu_{12} + \lambda_{20}\lambda_{10} + \lambda_{21}\lambda_{10}$
$d_1 = \lambda_{20}\mu_{01} + \lambda_{21}\mu_{01} + \lambda_{21}\mu_{02}$
$d_2 = \lambda_{10}\mu_{02} + \mu_{12}\mu_{02} + \mu_{12}\mu_{01}$

The steady state probabilitiy vector of thr matrix Q can be derived easily by using $\underline{\pi}\ Q = 0$ and $\underline{\pi}\ e = 1$

$$\begin{aligned} \pi_{0\ 0} &= \frac{ad_0}{X\sum_0^2 d_i} \quad \pi_{0\ 1} = \frac{bd_0}{X\sum_0^2 d_i} \quad \pi_{1\ 0} = \frac{ad_1}{X\sum_0^2 d_i} \\ \pi_{1\ 1} &= \frac{bd_1}{X\sum_0^2 d_i} \quad \pi_{2\ 0} = \frac{ad_0}{X\sum_0^2 d_i} \quad \pi_{2\ 1} = \frac{bd_1}{X\sum_0^2 d_i} \end{aligned} \tag{4}$$

where $\sum_0^2 d_i = [d_0 + d_1 + d_2]$ and $X = (a + b)$
When the business is available, either full manpower or moderate manpower must be available. When it is not so this will create heavy loss, We shall call this situation as crisis.
The crisis state is $\{(0\ 1)\}$ and the crises occur when there is full business but manpower is NIL.
Now the rate of crisis in steady state (C_∞) is obtained as follows.

$$\begin{aligned} P(\text{crisis in } [t\ t + \Delta t] &= p[X(t + \Delta t) = (0\ 1)/X(t) = (0\ 0)]xp[(X(t) = (0\ 0)] \\ &+ p[X(t + \Delta t) = (0\ 1)/X(t) = (2\ 1)]xp[(Xt) = (2\ 1)] \\ &+ p[X(t + \Delta t) = (0\ 1)/xXt) = (1\ 1)]xp[(X(t) = (1\ 1)] \\ &+ O\Delta(t). \end{aligned}$$

Taking limit as $\Delta t \to 0$, we get,
$C_t = bP_{00}(t) + \lambda_{20}P_{21}(t) + \lambda_{10}P_{11}(t)$
$C_\infty = lt_{t\to\infty}[bP_{00}(t) + \lambda_{20}P_{21}(t) + \lambda_{10}P_{11}(t)]$
that is
$C_\infty = [b\pi_{00} + \lambda_{20}\pi_{21} + \lambda_{10}\pi_{11}]$
Using the steady state probabilities, we get

$$C_\infty = \frac{b}{XY}\{ad_0 + \lambda_{20}d_0 + \lambda_{10}d_1\} \tag{5}$$

where $X = (a + b)$ and $Y = (d_0 + d_1 + d_2)$.

4 Numerical Illustration

Now taking the values of the parameters in the model as below, we can find the steady probabilities and the the rate of crises using the formulas (4) and

(5) respectively.

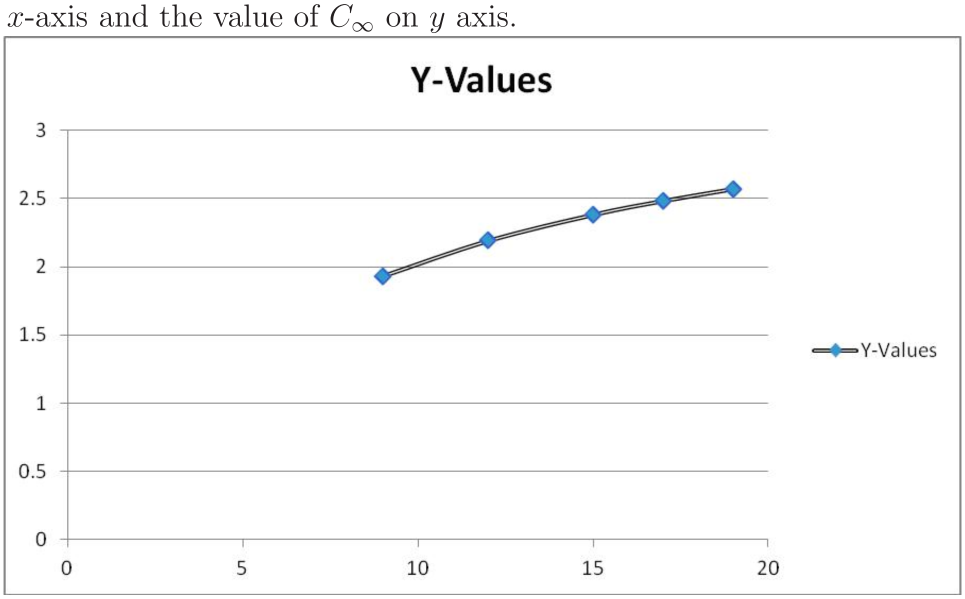
$$\lambda_{10} = 4,\ \lambda_{21} = 5,\ \lambda_{20} = 2,\ \mu_{12} = 8,\ \mu_{01} = 6,\ \mu_{02} = 7,\ a = 8 \text{ and } b = 9.$$

Steady state probability	Value
π_{00}	0.0818
π_{01}	0.0922
π_{10}	0.1432
π_{11}	0.1611
π_{20}	0.2455
π_{21}	0.2762
Total	1.0000

Now assigning the values $b = 9, 12, 15, 17$ and 19 we calculate the corresponding rate of crisis and is given below in the table:

b	C_∞
9	1,9355
12	2.1913
15	2.3819
17	2.4835
19	2.5701

The graph of the steady state crisis is given below taking the values of b on the x-axis and the value of C_∞ on y axis.



The steady state costs in different situations are determined by taking the values:
$C^0_{MP} = 25 \quad C^1_{MP} = 15 \quad C^2_{MP} = 10 : C^0_B = 15 \quad C^1_B = 8$

S. No.	Steady state probability	Cost of state
1	π_{00}	3.2720
2	π_{01}	3.0393
3	π_{10}	4.2960
4	π_{11}	3.7053
5	π_{20}	6.1375
6	π_{21}	4.9716
	Total	24.4217

We find that as the value of parameter b increases the crisis rate also increases. Also we observe that the cost of doing business is very heavy if the manpower is full but there is no business. Under circumstances we should fetch business at premium rate or offer heavy discount to get business. The cost of business is comparatively low when the business is full and the manpower is also full. The same holds in the case of manpower is moderate whereas the business may be dull or busy.

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