\textit{s}^g - Locally Closed Sets in Bitopological Spaces

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Abstract

The aim of this paper is to introduce the concepts of semi star generalized locally closed sets, \textit{s}^g submaximal spaces and study their basic properties in bitopological spaces.

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1 Introduction

The study of generalization of closed sets has been found to ensure some new separation axioms which have been very useful in the study of certain objects of digital topology. In recent years many generalizations of closed sets have been developed by various authors. K. Chandrasekhar Rao and K. Joseph [3] introduced the concepts of semi star generalized open sets and semi star generalized closed sets in unital topological spaces. On the other hand K. Chandrasekhar Rao and K. Kannan [4] introduced the concepts of semi star generalized open sets and semi star generalized closed sets in bitopological spaces.


of semi star generalized locally closed sets and $s^*g$ - submaximal spaces in unital topological spaces.

In this paper, we introduce the concept of semi star generalized locally closed sets, $s^*g$ - submaximal spaces and study their basic properties in bitopological spaces.

2 Preliminaries

Let $(X, \tau_1, \tau_2)$ or simply $X$ denote a bitopological space. By $\tau_1 - S^*GO(X, \tau_1, \tau_2)$ {resp. $\tau_1 - S^*GC(X, \tau_1, \tau_2)$}, we shall mean the collection of all $\tau_1 - s^*g$ open sets (resp. $\tau_1 - s^*g$ closed sets) in $(X, \tau_1, \tau_2)$. For any subset $A \subseteq X$, $\tau_1$ - int $(A)$ and $\tau_1$ - cl $(A)$ denote the interior and closure of a set $A$ with respect to the topology $\tau_1$ respectively. $A^C$ denotes the complement of $A$ in $X$ unless explicitly stated. We shall require the following known definitions.

Definition 2.1 A subset of a bitopological space $(X, \tau_1, \tau_2)$ is called

(a) $\tau_1 \tau_2$ - semi open if there exists a $\tau_1$ - open set $U$ such that $U \subseteq A \subseteq \tau_2$ - cl $(U)$.

(b) $\tau_1 \tau_2$ - semi closed if $X - A$ is $\tau_1 \tau_2$ - semi open.

Equivalently, a subset $A$ of a bitopological space $(X, \tau_1, \tau_2)$ is called $\tau_1 \tau_2$ - semi closed if there exists a $\tau_1$ - closed set $F$ such that $\tau_2$ - int $(F) \subseteq A \subseteq F$.

(c) $\tau_1 \tau_2$ - generalized closed ($\tau_1 \tau_2 - g$ closed) if $\tau_2$ - cl $(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\tau_1$ - open in $X$.

(d) $\tau_1 \tau_2$ - generalized open ($\tau_1 \tau_2 - g$ open) if $X - A$ is $\tau_1 \tau_2 - g$ closed.

(e) $\tau_1 \tau_2$ - semi star generalized closed ($\tau_1 \tau_2 - s^*g$ closed) if $\tau_2$ - cl $(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\tau_1$ - semi open in $X$.

(f) $\tau_1 \tau_2$ - semi star generalized open ($\tau_1 \tau_2 - s^*g$ open) if $X - A$ is $\tau_1 \tau_2 - s^*g$ closed in $X$.

3 Semi Star Generalized Locally Closed Sets

Definition 3.1 A subset $A$ of a bitopological space $(X, \tau_1, \tau_2)$ is said to be

(a) $\tau_1 \tau_2 - s^*g$ locally closed set if $A = G \cap F$ where $G$ is $\tau_1 - s^*g$ open set and $F$ is $\tau_2 - s^*g$ closed set in $X$. 
(b) $\tau_1 \tau_2 - s^{*}g$ locally closed* if $A = G \cap F$ where $G$ is $\tau_1 - s^{*}g$ open set and $F$ is $\tau_2$ - closed in $X$.

(c) $\tau_1 \tau_2 - s^{*}g$ locally closed** if $A = G \cap F$ where $G$ is $\tau_1$ - open and $F$ is $\tau_2 - s^{*}g$ closed in $X$.

Remark 3.2  
(a) The class of all $\tau_1 \tau_2 - s^{*}g$ locally closed sets in $(X, \tau_1, \tau_2)$ is denoted by $\tau_1 \tau_2 - S^{*}GLC(X, \tau_1, \tau_2)$.

(b) The class of all $\tau_1 \tau_2 - s^{*}g$ locally closed* sets in $(X, \tau_1, \tau_2)$ is denoted by $\tau_1 \tau_2 - S^{*}GLC^{*}(X, \tau_1, \tau_2)$.

(c) The class of all $\tau_1 \tau_2 - s^{*}g$ locally closed** sets in $(X, \tau_1, \tau_2)$ is denoted by $\tau_1 \tau_2 - S^{*}GLC^{**}(X, \tau_1, \tau_2)$.

Example 3.3 Let $X = \{a, b, c\}, \tau_1 = \{\phi, X, \{b, c\}\}, \tau_2 = \{\phi, X, \{a\}\}$. Then $\tau_1 - s^{*}g$ open sets in $(X, \tau_1, \tau_2)$ are $\phi, X, \{b, c\}$ and $\tau_2 - s^{*}g$ closed sets in $(X, \tau_1, \tau_2)$ are $X, \phi, \{b, c\}$. Then

(a) $\tau_1 \tau_2 - s^{*}g$ locally closed sets in $(X, \tau_1, \tau_2)$ are $\phi, X, \{b, c\}$.

(b) $\tau_1 \tau_2 - s^{*}g$ locally closed* sets in $(X, \tau_1, \tau_2)$ are $\phi, X, \{b, c\}$.

(c) $\tau_1 \tau_2 - s^{*}g$ locally closed** sets in $(X, \tau_1, \tau_2)$ are $\phi, X, \{b, c\}$.

Remark 3.4 Every $\tau_1 \tau_2 - s^{*}g$ locally closed set in $(X, \tau_1, \tau_2)$ are not $\tau_2$ - closed in general as can be seen from the following example.

Example 3.5 In Example 3.3, $\{b\}$ is $\tau_1 \tau_2 - s^{*}g$ locally closed set in $(X, \tau_1, \tau_2)$, but $\{b\}$ is not $\tau_2$ - closed in $(X, \tau_1, \tau_2)$.

Remark 3.6 Every $\tau_1 \tau_2 - s^{*}g$ locally closed set in $(X, \tau_1, \tau_2)$ are not $\tau_1$ - open in general as can be seen from the following example.

Example 3.7 In Example 3.3, $\{c\}$ is $\tau_1 \tau_2 - s^{*}g$ locally closed set in $(X, \tau_1, \tau_2)$, but $\{c\}$ is not $\tau_1$ - open in $(X, \tau_1, \tau_2)$.

Theorem 3.8 In any bitopological space $(X, \tau_1, \tau_2)$,

(i) $A \in \tau_1 \tau_2 - S^{*}GLC^{*}(X, \tau_1, \tau_2) \Rightarrow A \in \tau_1 \tau_2 - S^{*}GLC(X, \tau_1, \tau_2)$.

(ii) $A \in \tau_1 \tau_2 - S^{*}GLC^{**}(X, \tau_1, \tau_2) \Rightarrow A \in \tau_1 \tau_2 - S^{*}GLC(X, \tau_1, \tau_2)$.

(iii) $A \in \tau_2 - S^{*}GC(X, \tau_1, \tau_2) \Rightarrow A \in \tau_1 \tau_2 - S^{*}GLC(X, \tau_1, \tau_2)$.

(iv) $A \in \tau_1 - S^{*}GO(X, \tau_1, \tau_2) \Rightarrow A \in \tau_1 \tau_2 - S^{*}GLC(X, \tau_1, \tau_2)$. 
Proof. (i) Since $A$ is $\tau_1, \tau_2 - s^* g$ locally closed subset in $(X, \tau_1, \tau_2)$, we have $A = G \cap F$ where $G$ is $\tau_1 - s^* g$ open set and $F$ is $\tau_2$ - closed in $X$. Since every $\tau_2$ - closed sets are $\tau_2 - s^* g$ closed in $(X, \tau_1, \tau_2)$, $A = G \cap F$ where $G$ is $\tau_1 - s^* g$ open and $F$ is $\tau_2 - s^* g$ closed in $(X, \tau_1, \tau_2)$. Therefore $A \in \tau_1, \tau_2 - S^* GLC(X, \tau_1, \tau_2)$.

(ii) Since $A$ is $\tau_1, \tau_2 - s^* g$ locally closed** subset in $(X, \tau_1, \tau_2)$, we have $A = G \cap F$ where $G$ is $\tau_1 - open$ and $F$ is $\tau_2 - s^* g$ closed in $(X, \tau_1, \tau_2)$. Since every $\tau_1$ - open sets are $\tau_1 - s^* g$ open in $(X, \tau_1, \tau_2)$, $A = G \cap F$ where $G$ is $\tau_1 - s^* g$ open and $F$ is $\tau_2 - s^* g$ closed in $(X, \tau_1, \tau_2)$. Therefore $A \in \tau_1, \tau_2 - S^* GLC(X, \tau_1, \tau_2)$.

(iii) Since $A = A \cap X$ and $A$ is $\tau_2 - s^* g$ closed and $X$ is $\tau_1 - s^* g$ open in $(X, \tau_1, \tau_2)$, we have $A \in \tau_1, \tau_2 - S^* GLC(X, \tau_1, \tau_2)$.

(iv) Since $A = A \cap X$ and $A$ is $\tau_1 - s^* g$ open and $X$ is $\tau_2 - s^* g$ closed in $(X, \tau_1, \tau_2)$, we have $A \in \tau_1, \tau_2 - S^* GLC(X, \tau_1, \tau_2)$. \hfill \Box

Remark 3.9 The converses of (i), (ii), (iii) and (iv) of the above theorem are not true in general as can be seen from the following examples.

Example 3.10 Let $X = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a\}, \{c, d\}\}$. Then $\{a, c\}$ is $\tau_1, \tau_2 - s^* g$ locally closed in $(X, \tau_1, \tau_2)$, but not $\tau_1, \tau_2 - s^* g$ locally closed* in $(X, \tau_1, \tau_2)$.

Example 3.11 In Example 3.3, $\{b\}$ is $\tau_1, \tau_2 - s^* g$ locally closed in $(X, \tau_1, \tau_2)$, but not $\tau_1, \tau_2 - s^* g$ locally closed** in $(X, \tau_1, \tau_2)$.

Example 3.12 In Example 3.10, $\{a, c\}$ is $\tau_1, \tau_2 - s^* g$ locally closed in $(X, \tau_1, \tau_2)$, but not $\tau_1 - s^* g$ open in $(X, \tau_1, \tau_2)$ and $\{a\}$ is $\tau_1, \tau_2 - s^* g$ locally closed in $(X, \tau_1, \tau_2)$, but not $\tau_2 - s^* g$ closed in $(X, \tau_1, \tau_2)$.

Theorem 3.13 If $(X, \tau_1, \tau_2)$ is pairwise door space, then every subset of $X$ is both $\tau_1, \tau_2 - s^* g$ locally closed and $\tau_2, \tau_1 - s^* g$ locally closed.

Proof. Since $(X, \tau_1, \tau_2)$ is pairwise door space, every subset of $(X, \tau_1, \tau_2)$ is either $\tau_1$ - open or $\tau_2$ - closed and $\tau_2$ - open or $\tau_1$ - closed. Since every $\tau_1$ - open (resp. $\tau_2$ - closed) subset of $(X, \tau_1, \tau_2)$ is $\tau_1 - s^* g$ open (resp. $\tau_2 - s^* g$ closed), we have every subset of $(X, \tau_1, \tau_2)$ is either $\tau_1 - s^* g$ open or $\tau_2 - s^* g$ closed. Since every $\tau_1 - s^* g$ open and $\tau_2 - s^* g$ closed subset of $(X, \tau_1, \tau_2)$ is $\tau_1, \tau_2 - s^* g$ locally closed, we have every subset of $X$ is $\tau_1, \tau_2 - s^* g$ locally closed. Similarly we can prove that every subset of $X$ is $\tau_2, \tau_1 - s^* g$ locally closed. \hfill \Box

Theorem 3.14 For a subset $A$ of a bitopological space $(X, \tau_1, \tau_2)$, the following are equivalent.

(a) $A \in \tau_1, \tau_2 - S^* GLC^*(X, \tau_1, \tau_2)$.

(b) $A = G \cap [\tau_2 - cl (A)]$ for some $\tau_1 - s^* g$ open set $G$. 
(c) $A \cup \{X - [\tau_2 - \cl(A)]\}$ is $\tau_1 - s^*g$ open.

(d) $[\tau_2 - \cl(A)] - A$ is $\tau_1 - s^*g$ closed.

Proof. $(a) \Rightarrow (b):$
Since $A$ is $\tau_1 \tau_2 - s^*g$ locally closed* set in $(X, \tau_1, \tau_2)$, we have $A = G \cap F$ where
$G$ is $\tau_1 - s^*g$ open set and $F$ is $\tau_2$-closed in $X$. Since $A \subseteq \tau_2 - \cl(A)$ and
$A \subseteq G$, we have $A \subseteq G \cap [\tau_2 - \cl(A)]$ ..........(1)
Since $A \subseteq F$ and $F$ is $\tau_2$-closed in $X$, we have $\tau_2 - \cl(A) \subseteq F$. Therefore
$G \cap [\tau_2 - \cl(A)] \subseteq G \cap F = A$. Hence $G \cap [\tau_2 - \cl(A)] \subseteq A$ ..........(2)
From (1) and (2), we have $A = G \cap [\tau_2 - \cl(A)]$ for some $\tau_1 - s^*g$ open set $G$ in
$(X, \tau_1, \tau_2)$.

(b) $\Rightarrow (a) :$
Suppose that $A = G \cap [\tau_2 - \cl(A)]$ for some $\tau_1 - s^*g$ open set $G$ in $(X, \tau_1, \tau_2)$.
Since $\tau_2 - \cl(A)$ is $\tau_2$-closed in $(X, \tau_1, \tau_2)$ and $G$ is $\tau_1 - s^*g$ closed in $(X, \tau_1, \tau_2)$,
we have $A \in \tau_1 \tau_2 - S^*GLC^*(X, \tau_1, \tau_2)$

(c) $\Rightarrow (b):$
Suppose that $A \cup \{X - [\tau_2 - \cl(A)]\}$ is $\tau_1 - s^*g$ open in $(X, \tau_1, \tau_2)$. Let $G =
A \cup \{X - [\tau_2 - \cl(A)]\}$. Then $G$ is $\tau_1 - s^*g$ open in $(X, \tau_1, \tau_2)$.

Now,

$$
G \cap [\tau_2 - \cl(A)] = [A \cup \{X - [\tau_2 - \cl(A)]\}] \cap [\tau_2 - \cl(A)]
= \{[A \cup [\tau_2 - \cl(A)]^C] \cap [\tau_2 - \cl(A)]
= \{A \cap [\tau_2 - \cl(A)]\} \cup \{[\tau_2 - \cl(A)]^C \cap [\tau_2 - \cl(A)]\}
= A \cup \phi
= A.
$$

Therefore $A = G \cap [\tau_2 - \cl(A)]$ for some $\tau_1 - s^*g$ open set $G$ in $(X, \tau_1, \tau_2)$.

(c) $\Rightarrow (d):$
Suppose that $A \cup \{X - [\tau_2 - \cl(A)]\}$ is $\tau_1 - s^*g$ open in $(X, \tau_1, \tau_2)$. Let $G =
A \cup \{X - [\tau_2 - \cl(A)]\}$. Since $G$ is $\tau_1 - s^*g$ open in $(X, \tau_1, \tau_2)$, we have $X - G$
is $\tau_1 - s^*g$ closed in $(X, \tau_1, \tau_2)$.

Now,

$$
X - G = X - [A \cup \{X - [\tau_2 - \cl(A)]\}]
= (X - A) \cap \{X - [\tau_2 - \cl(A)]\}
= (X - A) \cap [\tau_2 - \cl(A)]
= \tau_2 - \cl(A) - A.
$$
Therefore, $\tau_2 - \text{cl} (A) - A$ is $\tau_1 - s^*g$ closed in $(X, \tau_1, \tau_2)$.

(d) $\Rightarrow$ (c):

Suppose that $\tau_2 - \text{cl} (A) - A$ is $\tau_1 - s^*g$ closed in $(X, \tau_1, \tau_2)$. Let $F = \tau_2 - \text{cl} (A) - A$. Then $F$ is $\tau_1 - s^*g$ closed in $(X, \tau_1, \tau_2)$ implies that $X - F$ is $\tau_1 - s^*g$ open in $(X, \tau_1, \tau_2)$.

Now,

\[
X - F = X - \{[\tau_2 - \text{cl}(A)] - A\} \\
= X \cap \{[\tau_2 - \text{cl}(A)] - A\}^C \\
= X \cap \{[\tau_2 - \text{cl}(A)] \cap A^C\}^C \\
= X \cap \{[\tau_2 - \text{cl}(A)]^C \cup (A^C)^C\} \\
= X \cap \{[\tau_2 - \text{cl}(A)]^C \cup A\} \\
= \{X \cap [\tau_2 - \text{cl}(A)]\}^C \cup \{X \cap A\} \\
= [\tau_2 - \text{cl}(A)]^C \cup A \\
= \{X - [\tau_2 - \text{cl}(A)]\} \cup A.
\]

Hence $A \cup \{X - [\tau_2 - \text{cl}(A)]\}$ is $\tau_1 - s^*g$ open in $(X, \tau_1, \tau_2)$.

\[\square\]

**Theorem 3.15** In a bitopological space $(X, \tau_1, \tau_2)$, the following are equivalent.

(a) $A - [\tau_1 - \text{int} (A)]$ is $\tau_2 - s^*g$ open in $(X, \tau_1, \tau_2)$.

(b) $[\tau_1 - \text{int} (A)] \cup [X - A]$ is $\tau_2 - s^*g$ closed in $(X, \tau_1, \tau_2)$.

(c) $G \cup [\tau_1 - \text{int} (A)] = A$ for some $\tau_2 - s^*g$ open set $G$ in $(X, \tau_1, \tau_2)$.

**Proof.** (a) $\Rightarrow$ (b):

Now,

\[
X - \{A - [\tau_1 - \text{int}(A)]\} = X \cap \{A - [\tau_1 - \text{int}(A)]\}^C \\
= X \cap \{A \cap \{\tau_1 - \text{int}(A)\}\}^C \\
= X \cap \{A^C \cup ([\tau_1 - \text{int}(A)]\}^C\} \\
= X \cap \{A^C \cup [\tau_1 - \text{int}(A)]\} \\
= \{A^C \cup [\tau_1 - \text{int}(A)]\} \\
= [\tau_1 - \text{int}(A)] \cup [X - A].
\]

Since $A - [\tau_1 - \text{int} (A)]$ is $\tau_2 - s^*g$ open, we have $X - \{A - [\tau_1 - \text{int} (A)]\} = [\tau_1 - \text{int}(A)] \cup [X - A]$ is $\tau_2 - s^*g$ closed in $(X, \tau_1, \tau_2)$.

(b) $\Rightarrow$ (a):

Suppose that $[\tau_1 - \text{int} (A)] \cup [X - A]$ is $\tau_2 - s^*g$ closed in $(X, \tau_1, \tau_2)$. Since $[\tau_1$
- $\text{int} (A) \cup [X - A]$ is $\tau_2 - s^*g$ closed, we have $X - \{\tau_1 - \text{int} (A) \cup [X - A]\}$ is $\tau_2 - s^*g$ open. Now,

\[
X - \{\tau_1 - \text{int}(A) \cup [X - A]\} = X \cap \{\tau_1 - \text{int}(A) \cup [X - A]\}^C
\]

\[
= X \cap \{\tau_1 - \text{int}(A) \cup A^C\}^C
\]

\[
= X \cap \{\tau_1 - \text{int}(A)^C \cap (A^C)^C\}
\]

\[
= X \cap \{\tau_1 - \text{int}(A)^C \cap A\}
\]

\[
= A \cap [\tau_1 - \text{int}(A)]^C
\]

\[
= A - [\tau_1 - \text{int}(A)].
\]

Therefore $A - [\tau_1 - \text{int}(A)]$ is $\tau_2 - s^*g$ open in $(X, \tau_1, \tau_2)$. (b) $\Rightarrow$ (c):

Suppose that $[\tau_1 - \text{int}(A)] \cup [X - A]$ is $\tau_2 - s^*g$ closed. Let $U = [\tau_1 - \text{int}(A)] \cup [X - A]$. Then $U$ is $\tau_2 - s^*g$ closed. Then $U^C$ is $\tau_2 - s^*g$ open.

Now,

\[
U^C \cup [\tau_1 - \text{int}(A)] = \{\tau_1 - \text{int}(A) \cup [X - A]\}^C \cup [\tau_1 - \text{int}(A)]
\]

\[
= \{\tau_1 - \text{int}(A)^C \cap (A^C)^C\} \cup [\tau_1 - \text{int}(A)]
\]

\[
= \{\tau_1 - \text{int}(A)^C \cap A\} \cup [\tau_1 - \text{int}(A)]
\]

\[
= \{\tau_1 - \text{int}(A)^C \cup [\tau_1 - \text{int}(A)]\} \cap \{A \cup [\tau_1 - \text{int}(A)]\}
\]

\[
= X \cap A
\]

\[
= A.
\]

Take $G = U^C$. Then $A = G \cup [\tau_1 - \text{int}(A)] = A$ for some $\tau_2 - s^*g$ open set in $(X, \tau_1, \tau_2)$. (c) $\Rightarrow$ (b):

Suppose that $A = G \cup [\tau_1 - \text{int}(A)] = A$ for some $\tau_2 - s^*g$ open set $G$ in $(X, \tau_1, \tau_2)$. Now,

\[
[\tau_1 - \text{int}(A)] \cup [X - A] = \tau_1 - \text{int}(A) \cup A^C
\]

\[
= [\tau_1 - \text{int}(A)] \cup \{G \cup [\tau_1 - \text{int}(A)]\}^C
\]

\[
= [\tau_1 - \text{int}(A)] \cup \{G^C \cap [\tau_1 - \text{int}(A)]^C\}
\]

\[
= \{[\tau_1 - \text{int}(A)] \cup G^C\} \cap \{[\tau_1 - \text{int}(A)] \cup [\tau_1 - \text{int}(A)]^C\}
\]

\[
= \{[\tau_1 - \text{int}(A)] \cup G^C\} \cap X
\]

\[
= \{[\tau_1 - \text{int}(A)] \cup G^C\}
\]

\[
= X - G.
\]

Since $G$ is $\tau_2 - s^*g$ open in $(X, \tau_1, \tau_2)$, we have $X - G$ is $\tau_2 - s^*g$ closed in $(X, \tau_1, \tau_2)$. Therefore $[\tau_1 - \text{int}(A)] \cup [X - A]$ is $\tau_2 - s^*g$ closed in $(X, \tau_1, \tau_2)$. □
**Remark 3.16** The union of two \( \tau_1 \tau_2 - s^*g \) locally closed sets in \((X, \tau_1, \tau_2)\) is not \( \tau_1 \tau_2 - s^*g \) locally closed in general as can be seen from the following example.

**Example 3.17** Let \( X = \{a, b, c, d\}, \tau_1 = \{\emptyset, X, \{a\}, \{a, b\}\}, \tau_2 = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}. \) Then \( A = \{a, d\}, B = \{b, d\} \) are \( \tau_1 \tau_2 - s^*g \) locally closed sets in \((X, \tau_1, \tau_2)\), but \( A \cup B = \{a, b, d\} \) is not \( \tau_1 \tau_2 - s^*g \) locally closed set in \((X, \tau_1, \tau_2)\).

**Remark 3.18** Even \( A \) and \( B \) are not \( \tau_1 \tau_2 - s^*g \) locally closed sets in \((X, \tau_1, \tau_2)\), \( A \cup B \) is \( \tau_1 \tau_2 - s^*g \) locally closed in general as can be seen from the following example.

**Example 3.19** Let \( X = \{a, b, c, d\}, \tau_1 = \{\emptyset, X, \{a\}, \{a, b\}\}, \tau_2 = \{\emptyset, X, \{a\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}. \) Then \( A = \{b\}, B = \{a, d\} \) are not \( \tau_1 \tau_2 - s^*g \) locally closed sets in \((X, \tau_1, \tau_2)\), but \( A \cup B = \{a, b, d\} \) is \( \tau_1 \tau_2 - s^*g \) locally closed set in \((X, \tau_1, \tau_2)\).

### 4 \( s^*g \) - Submaximal Spaces

**Definition 4.1** A bitopological space \((X, \tau_1, \tau_2)\) is

(i) \( \tau_1 \tau_2 \) - submaximal space if every \( \tau_1 \) - dense subset of \( X \) is \( \tau_2 \) - open in \( X \).

(ii) \( \tau_2 \tau_1 \) - submaximal space if every \( \tau_2 \) - dense subset of \( X \) is \( \tau_1 \) - open in \( X \).

(iii) \( \tau_1 \tau_2 - s^*g \) submaximal space if every \( \tau_1 \) - dense subset of \( X \) is \( \tau_2 - s^*g \) open in \( X \).

(iv) \( \tau_2 \tau_1 - s^*g \) submaximal space if every \( \tau_2 \) - dense subset of \( X \) is \( \tau_1 - s^*g \) open in \( X \).

**Example 4.2** In Example 3.17,

(i) \( \tau_1 \) - dense subsets of \((X, \tau_1, \tau_2)\) are
\( X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\).

(ii) \( \tau_2 - s^*g \) open sets of \((X, \tau_1, \tau_2)\) are
\( \emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\).

(iii) \( \tau_2 \) - open sets of \((X, \tau_1, \tau_2)\) are
\( \emptyset, X, \{a\}, \{a, b\}\{a, c\}\{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}. \) Therefore \((X, \tau_1, \tau_2)\) is both \( \tau_1 \tau_2 - s^*g \) submaximal space and \( \tau_1 \tau_2 \) - submaximal space.

**Theorem 4.3** If \((X, \tau_1, \tau_2)\) is \( \tau_1 \tau_2 \) - submaximal space then \( X \) is \( \tau_1 \tau_2 - s^*g \) submaximal space.
**Proof.** Since \((X, \tau_1, \tau_2)\) is \(\tau_1\tau_2\)-submaximal space, we have every \(\tau_1\)-dense subset of \(X\) is \(\tau_2\)-open in \(X\). Since every \(\tau_2\)-open set in \(X\) is \(\tau_2 - s^*g\) open in \(X\), we have every \(\tau_1\)-dense subset of \(X\) is \(\tau_2 - s^*g\)-open in \(X\). Therefore \((X, \tau_1, \tau_2)\) is \(\tau_1\tau_2 - s^*g\) submaximal space. \(\square\)

**Remark 4.4** The converse of the above theorem is not true in general as can be seen from the following example.

**Example 4.5** Let \(X = \{a, b, c, d\}, \tau_1 = \{\phi, X, \{a\}, \{a, b\}\}, \tau_2 = \{\phi, X, \{a\}, \{b, c\}\}\). Then \(\tau_1\)-dense subsets of \((X, \tau_1, \tau_2)\) are \(X, \{a\}\), \(\{a, b\}\), \(\{a, c\}\), \(\{a, b, c\}\), \(\{a, b, d\}\), \(\{a, c, d\}\). Therefore \((X, \tau_1, \tau_2)\) is \(\tau_1\tau_2 - s^*g\) submaximal space but not \(\tau_1\tau_2\)-submaximal space.

**Theorem 4.6** A bitopological space \((X, \tau_1, \tau_2)\) is \(\tau_1\tau_2 - s^*g\) submaximal space if and only if \(\tau_2\tau_1 - S^*GLC^*(X, \tau_1, \tau_2) = P(X)\).

**Proof.** Suppose that \((X, \tau_1, \tau_2)\) is \(\tau_1\tau_2 - s^*g\) submaximal space. Obviously \(\tau_2\tau_1 - S^*GLC^*(X, \tau_1, \tau_2) \subseteq P(X)\). Let \(A \in P(X)\) and \(U = A \cup \{X - [\tau_1 - \text{cl}(A)]\}\). Since \(\tau_1 - \text{cl}(U) = X\), we have \(U\) is \(\tau_1\)-dense subset of \(X\). Since \((X, \tau_1, \tau_2)\) is \(\tau_1\tau_2 - s^*g\) submaximal space, we have \(U\) is \(\tau_2 - s^*g\) open in \(X\). Since every \(\tau_2 - s^*g\) open set in \(X\) is \(\tau_2\tau_1 - s^*g\) locally closed* set in \((X, \tau_1, \tau_2)\), we have \(U \in \tau_2\tau_1 - S^*GLC^*(X, \tau_1, \tau_2)\). Therefore \(P(X) \subseteq \tau_2\tau_1 - S^*GLC^*(X, \tau_1, \tau_2)\). Hence \(\tau_2\tau_1 - S^*GLC^*(X, \tau_1, \tau_2) = P(X)\).

Conversely, suppose that \(\tau_2\tau_1 - S^*GLC^*(X, \tau_1, \tau_2) = P(X)\). Let \(A\) be the \(\tau_1\)-dense subset of \((X, \tau_1, \tau_2)\). Then \(A \cup \{X - [\tau_1 - \text{cl}(A)]\} = A \cup [\tau_1 - \text{cl}(A)]^C = A\). Therefore \(A \in \tau_2\tau_1 - S^*GLC^*(X, \tau_1, \tau_2)\) implies that \(A\) is \(\tau_2 - s^*g\) open in \(X\). By Theorem 3.14. Hence \(X\) is \(\tau_1\tau_2 - s^*g\) submaximal space. \(\square\)

**References**


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