A Note on Characterization of Prime Ideals of $\Gamma$-Semigroups in terms of Fuzzy Subsets

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Abstract

In this paper the notion of fuzzy prime ideal in $\Gamma$-semigroups has been introduced and studied. Relationship between prime ideals of a $\Gamma$-semigroup and that of its operator semigroups have been obtained which are used to revisit analogous results on ideals of $\Gamma$-semigroups and its operator semigroups.

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1 Introduction

Γ-semigroup was introduced by Sen and Saha[8] as a generalization of semigroup and ternary semigroup. Many results of semigroups have been extended to Γ-semigroups directly and via operator semigroups[1, 2, 3] of a Γ-semigroup. Fuzzy semigroups have been introduced by Kuroki[4] as a generalization of classical semigroups, using the concept of fuzzy set introduced by Zadeh[9]. Since then many authors have studied semigroups in terms of fuzzy sets. Motivated by Kuroki[4], Mustafa et al[5] we have initiated the study of Γ-semigroups in terms of fuzzy sets[7]. This paper is a continuation of our study of Γ-semigroups in terms of fuzzy sets. We introduce here the notion of fuzzy prime ideals in Γ-semigroups. They are found to satisfy characteristic function criterion and level subset criterion. As we did for fuzzy ideals of a Γ-semigroup in [7], in order to make operator semigroups to work, we establish here various relationships between fuzzy prime ideals of a Γ-semigroup and that of its operator semigroups. Among other results we obtain an inclusion preserving bijection between the set of all prime ideals of a Γ-semigroup (not necessarily with unities) and that of its operator semigroups. As an immediate application of this we obtain a new proof of an important result of Γ-semigroup.

2 Preliminaries

We recall the following definitions and results which will be used in the sequel.

**Definition 2.1** [9] A fuzzy subset of a non-empty set $X$ is a function $\mu : X \rightarrow [0, 1]$.

**Definition 2.2** [3] Let $S$ and $\Gamma$ be two non-empty sets. $S$ is called a Γ-semigroup if there exist mappings from $S \times \Gamma \times S$ to $S$, written as $(a, \alpha, b) \mapsto a\alpha b$, and from $\Gamma \times S \times \Gamma$ to $\Gamma$, written as $(\alpha, a, \beta) \mapsto \alpha a\beta$ satisfying the following associative laws $(a\alpha b)\beta c = a(\alpha b)c = a\alpha (b\beta c)$ and $\alpha (a\beta b)\gamma = (\alpha a\beta)b\gamma = \alpha a(b\beta\gamma)$ for all $a, b, c \in S$ and for all $\alpha, \beta, \gamma \in \Gamma$.

**Definition 2.3** [7] A non-empty fuzzy subset $\mu$ of a Γ-semigroup $S$ is called a fuzzy left ideal (right ideal) of $S$ if $\mu(x\gamma y) \geq \mu(y)$ (resp. $\mu(x\gamma y) \geq \mu(x)$) $\forall x, y \in S, \forall \gamma \in \Gamma$.

**Definition 2.4** [7] A non-empty fuzzy subset $\mu$ of a Γ-semigroup $S$ is called a fuzzy ideal of $S$ if it is both fuzzy left ideal and fuzzy right ideal of $S$.

**Definition 2.5** [7] Let $\mu$ be a fuzzy subset of a set $S$. Then for $t \in [0, 1]$ the set $\mu_t = \{x \in S : \mu(x) \geq t\}$ is called $t$-level subset or simply level subset of $\mu$. 
Proposition 2.6 [7] Let $I$ be a non-empty subset of a $\Gamma$-semigroup $S$ and $\mu_I$ be the characteristic function of $I$, then $I$ is a left ideal (resp. right ideal, ideal) of $S$ if and only if $\mu_I$ is a fuzzy left ideal (resp. fuzzy right ideal, fuzzy ideal) of $S$.

Proposition 2.7 [7] A non-empty fuzzy subset $\mu$ in a $\Gamma$-semigroup $S$ is a fuzzy ideal iff for any $t \in [0,1]$, the $t$-level subset of $\mu$ (if non-empty), is an ideal of $S$.

Definition 2.8 [2] Let $S$ be a $\Gamma$-semigroup. An ideal $P$ of $S$ is said to be prime if, for any two ideals $A$ and $B$ of $S$, $A \Gamma B \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$.

Theorem 2.9 Let $I$ be an ideal of a $\Gamma$-semigroup $S$. Then the following are equivalent.

(i) $I$ is prime.
(ii) For $x, y \in S$, $x \Gamma S \Gamma y \subseteq I \Rightarrow x \in I$ or $y \in I$.
(iii) For $x, y \in S$, $x \Gamma y \subseteq I \Rightarrow x \in I$ or $y \in I$.

Proof. By Theorem 3.4[2] (i) $\Rightarrow$ (ii).

(ii) $\Rightarrow$ (iii)
Let us suppose that (ii) holds and $x \Gamma y \subseteq I$. Then $x \Gamma s \Gamma y \subseteq I$ as $x \Gamma s \Gamma y \subseteq x \Gamma y$
Hence by (ii), $x \in I$ or $y \in I$.

(iii) $\Rightarrow$ (i)
Let us suppose that (iii) holds. Let for two ideals $A, B$ of $S$, $A \Gamma B \subseteq I$. If possible, suppose $A \not\subseteq I$ or $B \not\subseteq I$. Then $x \in A$ and $y \in B$ such that $x \not\in I$ and $y \not\in I$. This implies that $x \Gamma y \subseteq I$ with $x \not\in I$, $y \not\in I$. This is a contradiction to (iii). Hence either $A \subseteq I$ or $B \subseteq I$. Consequently $I$ is a prime ideal of $S$. ■

3 Fuzzy Prime Ideal

Definition 3.1 A fuzzy ideal $\mu$ of a $\Gamma$-semigroup $S$ is called fuzzy prime ideal if $\inf_{\gamma \in \Gamma} \mu(x \gamma y) = \max\{\mu(x), \mu(y)\}$ $\forall x, y \in S$.

Example: Let $S$ be the set of all $1 \times 2$ matrices over $GF_2$ (the finite field with two elements) and $\Gamma$ be the set of all $2 \times 1$ matrices over $GF_2$. Then $S$ is a $\Gamma$-semigroup where $a \alpha b$ and $a \alpha \beta (a, b \in S$ and $\alpha, \beta \in \Gamma)$ denote the usual matrix product. Let $\mu : S \rightarrow [0,1]$ be defined by

$\mu(x) = \begin{cases} 0.3 & \text{if } a=(0,0) \\ 0.2 & \text{otherwise} \end{cases}$

Then $\mu$ is a fuzzy prime ideal of $S$. 

Theorem 3.2 Let $S$ be a $\Gamma$-semigroup and $\emptyset \neq I \subseteq S$. Then the following are equivalent.

(i) $I$ is a prime ideal of $S$.

(ii) The characteristic function $\mu_I$ of $I$ is a fuzzy prime ideal of $S$.

Proof. (i) $\Rightarrow$ (ii)

Let $I$ be a prime ideal of $S$ and $\mu_I$ be the characteristic function of $I$. Since $I \neq \emptyset$, $\mu_I$ is non-empty. Let $x, y \in S$. Suppose $x\Gamma y \subseteq I$. Then $\mu_I(x\gamma y) = 1$ for $\gamma \in \Gamma$. Hence $\inf_{\gamma \in \Gamma} \mu_I(x\gamma y) = 1$. Now $I$ being prime, $x \in I$ or $y \in I$ (cf. Theorem 2.9). Hence $\mu_I(x) = 1$ or $\mu_I(y) = 1$ which gives $\max\{\mu_I(x), \mu_I(y)\} = 1$. Thus we see that $\inf_{\gamma \in \Gamma} \mu_I(x\gamma y) = \max\{\mu_I(x), \mu_I(y)\}$. Now suppose that $x\Gamma y \not\subseteq I$.

Then for $\gamma \in \Gamma$, $x\gamma y \not\subseteq I$ which means that $\mu_I(x\gamma y) = 0$. Consequently, $\inf_{\gamma \in \Gamma} \mu_I(x\gamma y) = 0$. Now since $I$ is a prime ideal of $S$, $x \not\in I$ and $y \not\in I$. Hence $\mu_I(x) = 0$ and $\mu_I(y) = 0$. Consequently, $\max\{\mu_I(x), \mu_I(y)\} = 0$. Thus we see that in this case also $\inf_{\gamma \in \Gamma} \mu_I(x\gamma y) = \max\{\mu_I(x), \mu_I(y)\}$.

(ii) $\Rightarrow$ (i)

Let $\mu_I$ be a fuzzy prime ideal of $S$. Then $\mu_I$ is a fuzzy ideal of $S$. So by Proposition 2.6, $I$ is an ideal of $S$. Let $x, y \in S$ such that $x\Gamma y \subseteq I$. Then $\mu_I(x\gamma y) = 1$. Hence $\inf_{\gamma \in \Gamma} \mu_I(x\gamma y) = 1$. Let $x \not\in I$ and $y \not\in I$. Then $\mu_I(x) = 0 = \mu_I(y)$, which means $\max\{\mu_I(x), \mu_I(y)\} = 0$. This implies that $\inf_{\gamma \in \Gamma} \mu_I(x\gamma y) = 0$.

Thus we get a contradiction. Hence $x \in I$ or $y \in I$. Thus we see that $I$ is a prime ideal of $S$(cf. Theorem 2.9). □

Theorem 3.3 Let $S$ be a $\Gamma$-semigroup and $\mu$ be a non-empty fuzzy subset of $S$. Then the following are equivalent.

(i) $\mu$ is fuzzy prime ideal of $S$

(ii) For any $t \in [0, 1]$ the $t$-level subset of $\mu$(if it is non-empty) is a prime ideal of $S$.

Proof. (i) $\Rightarrow$ (ii)

Let $\mu$ be a fuzzy prime ideal of $S$. Let $t \in [0, 1]$ such that $\mu_t$ is non-empty. Let for $x, y \in S$, $x\Gamma y \subseteq \mu_t$. Then $\mu(x\gamma y) \geq t \ \forall \gamma \in \Gamma$. So $\inf_{\gamma \in \Gamma} \mu(x\gamma y) \geq t$. Since $\mu$ is a fuzzy prime ideal, it follows that $\max\{\mu(x), \mu(y)\} \geq t$. So $\mu(x) \geq t$ or $\mu(y) \geq t$. Hence $x \in \mu_t$ or $y \in \mu_t$. So $\mu_t$ is a prime ideal of $S$(cf. Theorem 2.9).

(ii) $\Rightarrow$ (i)

Let every non-empty level subset $\mu_t$ of $\mu$ be a prime ideal of $S$. Let $x, y \in S$. Let $\inf_{\gamma \in \Gamma} \mu(x\gamma y) = t$ (we note here that since $\mu(x\gamma y) \in [0, 1] \ \forall \gamma \in \Gamma$, $\inf_{\gamma \in \Gamma} \mu(x\gamma y)$ exists). Then $\mu(x\gamma y) \geq t \ \forall \gamma \in \Gamma$. So $x\gamma y \in \mu_t \ \forall \gamma \in \Gamma$. So $\mu_t$ is non-empty and $x\Gamma y \subseteq \mu_t$. Since $\mu_t$ is a prime ideal of $S$, $x \in \mu_t$ or $y \in \mu_t$(cf. Theorem 2.9). So $\mu(x) \geq t$ or $\mu(y) \geq t$. So $\max\{\mu(x), \mu(y)\} \geq t$, i.e., $\max\{\mu(x), \mu(y)\} \geq t$. □
inf \( \mu(x\gamma y) \) ......(1). Again by Proposition 2.7, \( \mu \) is a fuzzy ideal of \( S \). So \( \forall \gamma \in \Gamma \), \( \mu(x\gamma y) \geq \mu(x) \) and \( \mu(x\gamma y) \geq \mu(y) \). So \( \mu(x\gamma y) \geq \max\{\mu(x), \mu(y)\} \) \( \forall \gamma \in \Gamma \). Hence \( \inf_{\gamma \in \Gamma} \mu(x\gamma y) \geq \max\{\mu(x), \mu(y)\} \) ......(2). Combining (1) and (2), thus \( \inf_{\gamma \in \Gamma} \mu(x\gamma y) = \max\{\mu(x), \mu(y)\} \). Hence \( \mu \) is a fuzzy prime ideal of \( S \). \( \blacksquare \)

4 Corresponding Fuzzy Prime Ideals

Unless otherwise stated, throughout this section \( S \) denotes a \( \Gamma \)-semigroup and \( L, R \) its left and right operator semigroups respectively.

Definition 4.1 [2] Let \( S \) be a \( \Gamma \)-semigroup. Let us define a relation \( \rho \) on \( S \times \Gamma \) as : \( (x, \alpha)\rho(y, \beta) \) if and only if \( x\alpha s = y\beta s \) for all \( s \in S \) and \( \gamma x\alpha = \gamma y\beta \) for all \( \gamma \in \Gamma \). Then \( \rho \) is an equivalence relation. Let \( [x, \alpha] \) denote the equivalence class containing \( (x, \alpha) \). Let \( L = \{[x, \alpha] : x \in S, \alpha \in \Gamma\} \). Then \( L \) is a semigroup with respect to the multiplication defined by \( [x, \alpha][y, \beta] = [x\alpha y, \beta] \). This semigroup \( L \) is called the left operator semigroup of the \( \Gamma \)-semigroup \( S \).

Dually the right operator semigroup \( R \) of \( \Gamma \)-semigroup \( S \) is defined where the multiplication is defined by \( [\alpha, a][\beta, b] = [aa\beta, b] \).

Definition 4.2 For a fuzzy subset \( \mu \) of \( R \) we define a fuzzy subset \( \mu^{*} \) of \( S \) by \( \mu^{*}(a) = \inf_{\gamma \in \Gamma} \mu(\gamma, a) \), where \( a \in S \). For any subset \( \sigma \) of \( S \) we define a fuzzy subset \( \sigma^{*} \) of \( R \) by \( \sigma^{*}(\alpha, a) = \inf_{s \in S} \sigma(saa) \), where \( \alpha, a \in R \). For a fuzzy subset \( \delta \) of \( L \), we define a fuzzy subset \( \delta^{+} \) of \( S \) by \( \delta^{+}(a) = \inf_{\gamma \in \Gamma} \delta([a, \gamma]) \) where \( a \in S \).

For any fuzzy subset \( \eta \) of \( S \) we define a fuzzy subset \( \eta^{+} \) of \( L \) by \( \eta^{+}(\alpha, a) = \inf_{s \in S} \eta(saa) \), where \( \alpha, a \in L \).

Lemma 4.3 [7] If \( \mu \) is a fuzzy subset of \( R \), then \( (\mu_{t})^{*} = (\mu^{*})_{t} \) where \( t \in Im(\mu) \), provided the sets are non-empty.

Lemma 4.4 [7] If \( \sigma \) is a fuzzy subset of \( S \), then \( (\sigma_{t})^{*} = (\sigma^{*})_{t} \) where \( t \in Im(\sigma) \), provided the sets are non-empty.

Proposition 4.5 Suppose \( \mu \) is a fuzzy prime ideal of \( R \). Then \( \mu^{*} \) is a fuzzy prime ideal of \( S \).

Proof. Since \( \mu \) is a fuzzy prime ideal of \( R \), \( \mu_{t} \) is a prime ideal of \( R[6] \) \( \forall t \in Im(\mu) \). Hence \( (\mu_{t})^{*} \) is a prime ideal of \( S[2] \). Now \( (\mu_{t})^{*} \) and \( (\mu^{*})_{t} \) are non-empty. Hence \( (\mu_{t})^{*} = (\mu^{*})_{t} \)(cf. Lemma 4.3). This gives \( (\mu^{*})_{t} \) is a prime ideal of \( S \) for all \( t \in Im(\mu) \). Hence \( \mu^{*} \) is a fuzzy prime ideal of \( S \)(cf. Theorem 3.3). \( \blacksquare \)
Proposition 4.6 Suppose \( \sigma \) is a fuzzy prime ideal of \( S \). Then \( \sigma^* \) is a fuzzy prime ideal of \( R \).

**Proof.** Since \( \sigma \) is a fuzzy prime ideal of \( S \), \( \sigma_t \) is a prime ideal of \( S \) \( \forall t \in \text{Im}(\sigma) \) (cf. Theorem 3.3). Hence \( (\sigma_t)^* \) is a prime ideal of \( R[2] \). Also \( (\sigma_t)^* \) and \( (\sigma^*)_t \) are non-empty. So \( (\sigma_t)^* = (\sigma^*)_t \) (cf. Lemma 4.4), \( (\sigma^*)_t \) is a prime ideal of \( R \) for all \( t \in \text{Im}(\mu) \). Consequently \( \sigma^* \) is a fuzzy prime ideal of \( R[6] \). 

**Remark:** The left operator analogous of the above two propositions are also true.

**Theorem 4.7** Let \( S \) be a \( \Gamma \)-semigroup and \( R \) be its right operator semigroup. Then there exist an inclusion preserving bijection \( \mu \mapsto \mu^* \) between the set of all fuzzy prime ideals of \( R \) and set of all fuzzy prime ideals of \( S \), where \( \mu \) is a fuzzy prime ideal of \( R \).

**Proof.** Let \( x \in S \). Then \( (\mu^*)^*[\alpha, x] = \inf_{s \in S} \mu^*[\alpha, x] = \inf_{s \in S} \mu(s \alpha x) \geq \mu(x) \) (since \( \mu \) is a fuzzy ideal). Again for \( x \in S \), \( (\mu^*)^*[\alpha, x] = \inf_{s \in S} \mu(s \alpha x) = \inf_{s \in S} \mu(\alpha x) \) (since \( \mu \) is a fuzzy prime ideal) \( \leq \max(\mu(x), \mu(x)) = \mu(x) \).

Thus we see that \( (\mu^*)^* = \mu \). Hence the mapping is one-one. Now for \( [\alpha, x] \in R \), \( (\mu^*)^*[\alpha, x] = \inf_{s \in S \beta \in \Gamma} \mu^*[\alpha, x] = \inf_{s \in S \beta \in \Gamma} \mu(\beta, s)[\alpha, x] \geq \mu([\alpha, x]) \).

Hence \( \mu \subseteq (\mu^*)^* \). Since \( \mu \) is a fuzzy prime ideal, \( \mu([\alpha, x], [\beta, s]) = \max(\mu([\alpha, x], [\beta, s])) \forall s \in S \) and \( \forall \beta \in \Gamma \). Hence for \( s = x \) and \( \beta = \alpha \), \( \mu([\alpha, x], [\beta, s]) = \mu([\alpha, x]) \). This together with the relation \( (\mu^*)^*[\alpha, x] = \inf_{s \in S \beta \in \Gamma} \mu([\alpha, x], [\beta, s]) \) gives \( (\mu^*)^*[\alpha, x] \leq \mu([\alpha, x]) \) for all \( [\alpha, x] \in R \). This means \( (\mu^*)^* \subseteq \mu \). Thus we see that \( \mu = (\mu^*)^* \). This proves that the mapping is onto. Now let \( \mu_1, \mu_2 \in FI(S) \) be such that \( \mu_1 \subseteq \mu_2 \). Then for all \( [\alpha, x] \in R \), \( \mu_1^*([\alpha, x]) = \inf_{s \in S} \mu_1(s \alpha x) \leq \inf_{s \in S} \mu_2(s \alpha x) = \mu_2^*([\alpha, x]) \). Thus \( \mu_1^* \subseteq \mu_2^* \). Similarly we can show that if \( \mu_1 \subseteq \mu_2 \) where \( \mu_1, \mu_2 \in FI(R)(FLI(R)) \) then \( \mu_1^* \subseteq \mu_2^* \). Hence \( \mu \mapsto \mu^* \) is an inclusion preserving bijection. 

**Remark:** Similar result holds for the \( \Gamma \)-semigroup \( S \) and the left operator semigroup \( L \) of \( S \).

Now we establish the following two lemmas to deduce the inclusion preserving bijections between the set of all prime ideals of a \( \Gamma \)-semigroup and that of its operator semigroups with the fuzzy notions of \( \Gamma \)-semigroups.

**Lemma 4.8** Let \( I \) be an ideal, \( \mu \) a fuzzy ideal of a \( \Gamma \)-semigroup \( S \) and \( R \) the right operator semigroup of \( S \). Then \( (\mu_I)^* = \mu_{I^*} \), where \( \mu_I \) is the characteristic function of \( I \).
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Proof. Let $[\beta, y] \in R$. Then $(\mu_I)^s([\beta, y]) = \inf_{s \in S} \mu(s\beta y)$. Suppose $[\beta, y] \in I^s$. Then $s\beta y \in I$ for all $s \in S$. Hence $\mu_I(s\beta y) = 1$ for all $s \in S$. This gives $\inf_{s \in S} \mu(s\beta y) = 1$ whence $(\mu_I)^s([\beta, y]) = 1$. Also $\mu_{I^s}([\beta, y]) = 1$. Hence $(\mu_I)^s([\beta, y]) = \mu_{I^s}([\beta, y])$. Suppose $[\beta, y] \notin I^s$. Then for some $t \in S$, $t\beta y \notin I$. So $\mu_I(t\beta y) = 0$. This gives $\inf_{s \in S} \mu_I(s\beta y) = 0$ i.e., $(\mu_I)^s([\beta, y]) = 0$. Again $\mu_{I^s}([\beta, y]) = 0$. Thus in this case also $(\mu_I)^s([\beta, y]) = \mu_{I^s}([\beta, y])$. Hence $(\mu_I)^s = \mu_{I^s}$. ■

Similar is the proof of the following lemma.

Lemma 4.9 Let $I$ be a (left) ideal of the right operator semigroup $R$ of a $\Gamma$-semigroup $S$. Then $(\lambda_I)^+ = \lambda_{I^+}$, where $\lambda_I$ denotes the characteristic function of $I$.

Remark 4.10 Dually we can deduce for left operator semigroup $L$ of the $\Gamma$-semigroup $S$, (i) $(\lambda_I)^+ = \lambda_{I^+}$, (ii) $(\lambda_I)^+ = \lambda_{I^+}$, where $\lambda_I$ denotes the characteristic function of $I$.

Theorem 4.11 [2] Let $S$ be a $\Gamma$-semigroup (not necessarily with unities). Then there exists an inclusion preserving bijection between the set of all prime ideals of $S$ and that of its right operator semigroup $R$ via the mapping $I \rightarrow I^s$.

Proof. Let us denote the mapping $I \rightarrow I^s$ by $\phi$. This is actually a mapping follows from dual of Lemma 3.12[2]. Now let $\phi(I_1) = \phi(I_2)$. Then $I_1^s = I_2^s$. This implies that $\lambda_{I_1^s} = \lambda_{I_2^s}$. (where $\lambda_I$ is the characteristic function $I$.) Hence by Lemma 4.8, $(\lambda_{I_1})^s = (\lambda_{I_2})^s$. This together with Theorem 4.7 gives $\lambda_{I_1} = \lambda_{I_2}$ whence $I_1 = I_2$. Consequently $\phi$ is one-one. Let $I$ be a prime ideal of $R$. Then its characteristic function $\lambda_I$ is a fuzzy prime ideal of $R$. Hence by Theorem 4.7, $((\lambda_I)^s)^s = \lambda_I$. This implies that $\lambda_{(I^s)^s} = \lambda_I$. Let $I_1, I_2$ be two prime ideals of $S$ with $I_1 \subseteq I_2$. Then $\lambda_{I_1} \subseteq \lambda_{I_2}$. Hence by Theorem 4.7, we see that $(\lambda_{I_1})^s \subseteq (\lambda_{I_2})^s$ i.e., $\lambda_{I_1^s} \subseteq \lambda_{I_2^s}$. (cf. Lemma 4.8) which gives $I_1^s \subseteq I_2^s$. ■

Remark: The result similar to the above for the left operator semigroup $L$ of the $\Gamma$-semigroup $S$ can be deduced by the Remark 4.10 and Theorem 4.7.

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