An Inverse Eigenvalue Problem for General Tridiagonal Matrices

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Abstract

The following inverse eigenvalue problem is considered: Given the minimal and maximal eigenvalues of all the leading submatrices of a tridiagonal matrix, we construct the matrix and give the conditions that are necessary and sufficient for the existence of such a solution.

Mathematics Subject Classification: 65D32; 65Q05; 39A05; 65F15

Keywords: Inverse Eigenvalue problem; Tridiagonal Matrices; Symmetrization

1 Introduction

In [5], an algorithm that solves a special version of the inverse eigenvalue problem is introduced. This problem is summarized as follows: Let the real symmetric tridiagonal $k \times k$ matrix $A$ be of the form:

$$A = \begin{bmatrix} \alpha_1 & \beta_1 & 0 & . & . & 0 \\ \beta_1 & \alpha_2 & \beta_2 & 0 & . & 0 \\ 0 & \beta_2 & \alpha_3 & \beta_3 & . & 0 \\ 0 & 0 & . & . & . & . \\ . & . & . & \beta_{k-2} & \alpha_{k-1} & \beta_{k-1} \\ 0 & . & 0 & \beta_{k-1} & \alpha_k \end{bmatrix}$$

..........................(1)
Where $\beta_i \neq 0, i = 1, 2, ..., k - 1$. This matrix can be constructed when the minimal and maximal eigenvalues of each leading principal submatrix of $A$ are known.

This problem of constructing a symmetric tridiagonal matrix from some spectral information—called the inverse eigenvalue problem—has proved to be important in many applications, such as vibration theory, structural design, and control theory. Many articles have appeared that discuss this problem recently, such as [2,3,4], to name a few.

The present work extends the results of [5] in the sense that we start with a general nonsymmetric tridiagonal matrix $T$ which satisfies certain structural conditions, and it is assumed that the minimal and maximal eigenvalues of each principal submatrix of $T$ are known, this matrix is symmetrized so that a symmetric tridiagonal matrix $J$ is generated, see [1].

Since the matrices $T$ and $J$ are similar, they both have the same eigenvalues. A solution of the inverse eigenvalue problem that constructs the matrix $J$ using the algorithm in [5] is implemented, and finally the matrix $T$ is constructed, which in itself is a solution of the inverse eigenvalue problem.

Section 2 contains the similarity transformation that is used to symmetrize the general tridiagonal matrix $T$ in order to get the matrix $J$, and it also includes the theoretical justification used in [5] which produces the computation of the entries of matrix $J$ as a solution of the inverse eigenvalue problem, and how to obtain the matrix $T$ from the matrix $J$. Finally, section 3 contains the extended algorithm, a numerical example, and some concluding remarks.

## 2 The Symmetrization Process

Let $T$ be a general tridiagonal matrix of the form:

$$T = \begin{bmatrix}
  a_1 & b_1 & 0 & . & . & . & 0 \\
  c_2 & a_2 & b_2 & 0 & . & . & 0 \\
  0 & c_3 & a_3 & b_3 & . & . & 0 \\
  0 & 0 & . & . & . & . & . \\
  . & . & . & c_{k-1} & a_{k-1} & b_{k-1} & . \\
  0 & . & . & 0 & c_k & a_k & . \\
\end{bmatrix}$$

(2)

Where $b_i \neq 0, i = 1, ..., k - 1$, and $c_i \neq 0, i = 2, ..., k$. The following theorem constructs a symmetric tridiagonal matrix $J$ which is similar to $T$ via a diagonal similarity transformation.
\textbf{Theorem 1} Let \( D \) be a diagonal matrix of the form: \( D = \text{diag}(\gamma_1, \gamma_2, \ldots, \gamma_k) \), where \( \gamma_i = \sqrt{\frac{\gamma_{i+1}c_{i+1} \cdots c_n}{b_ib_{i+1} \cdots b_{n-1}}} \), \( i = 1, \ldots, k-1 \), and \( \gamma_k = 1 \). Then the matrix \( J = D^{-1}TD \) is a symmetric tridiagonal matrix of the form:

\[
J = \begin{bmatrix}
a_1 & \sqrt{b_1c_2} & 0 & \cdots & 0 \\
\sqrt{b_1c_2} & a_2 & \sqrt{b_2c_3} & 0 & \cdots \\
0 & \sqrt{b_2c_3} & a_3 & \sqrt{b_3c_4} & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots & \sqrt{b_{k-1}c_k} & a_{k-1} \\
0 & \cdots & \cdots & 0 & \sqrt{b_{k-1}c_k} & a_k
\end{bmatrix}
\]

\textbf{Proof.} The entries of \( DT D^{-1} \) are classified as follows:

(i) The diagonal entries are just \( a_j \) for \( j = 1, \ldots, n \).

(ii) The upper diagonal entries for \( j = 1, \ldots, n-2 \) are of the form:

\[
b_j \frac{\gamma_j}{\gamma_{j+1}} = b_j \sqrt{\frac{c_{j+1}c_{j+2} \cdots c_n}{b_jb_{j+1} \cdots b_{n-1}}} \sqrt{\frac{b_{j+1} \cdots b_{n-1}}{c_{j+2} \cdots c_n}} = b_j \sqrt{\frac{c_{j+1}}{b_j}} = \sqrt{b_jc_{j+1}}.
\]

(iii) The lower diagonal entries for \( j = 2, \ldots, n-1 \) are of the form:

\[
c_j \frac{\gamma_j}{\gamma_{j-1}} = c_j \sqrt{\frac{c_{j+1}c_{j+2} \cdots c_n}{b_jb_{j+1} \cdots b_{n-1}}} \sqrt{\frac{b_{j-1}b_j \cdots b_{n-1}}{c_{j+1} \cdots c_n}} = c_j \sqrt{\frac{b_{j-1}}{c_j}} = \sqrt{b_{j-1}c_j}.
\]

(iv) The upper diagonal entry in the position \((n-1, n)\) has the form:

\[
b_{n-1} \gamma_{n-1} = b_{n-1} \sqrt{\frac{c_n}{b_{n-1}}} = \sqrt{b_{n-1}c_n}.
\]

(v) Finally, the lower diagonal entry in the position \((n, n-1)\) has the form:

\[
\frac{c_n}{\gamma_{n-1}} = c_n \sqrt{\frac{b_{n-1}}{c_n}} = \sqrt{b_{n-1}c_n}.
\]

\textbf{Remark 2} Notice that \( b_jc_{i+1} \neq 0, i = 1, \ldots, k-1 \).

Now, let \( A \) be the symmetric tridiagonal matrix given in (1), and suppose that the minimal and maximal eigenvalues \( \lambda_1^{(j)} \) and \( \lambda_k^{(j)} \) of its leading principal submatrices \( A_j, j = 1, \ldots, k \), are given.

Pickman \textit{et.al.} in \cite{5} introduce an algorithm that poses the inverse eigenvalue problem as follows: Given the real numbers
\[ \lambda_1^{(k)} < \lambda_1^{(k-1)} < \ldots < \lambda_1^{(2)} < \lambda_1^{(1)} < \lambda_2^{(1)} < \ldots < \lambda_{k-1}^{(k-1)} < \lambda_k^{(k)} \]

Then what are the necessary and sufficient conditions for the existence of a \( k \times k \) symmetric tridiagonal matrix of the form in (1) such that \( \lambda_1^{(j)} \) and \( \lambda_j^{(j)} \) are respectively the minimal and maximal eigenvalues of the leading principal submatrix \( A_j, j = 1, \ldots, k \). The following theorem constitutes the theoretical cornerstone of the work in [5], and the constructive proof of this theorem generates the algorithm.

**Theorem 3** The necessary and sufficient condition for the existence of a \( k \times k \) symmetric tridiagonal matrix \( A \) of the form in (1) such that \( \lambda_1^{(j)} \) and \( \lambda_j^{(j)} \) are respectively the minimal and the maximal eigenvalues of the leading principal submatrices \( A_j, j = 1, \ldots, k \), is

\[ \lambda_1^{(k)} < \lambda_1^{(k-1)} < \ldots < \lambda_1^{(2)} < \lambda_1^{(1)} < \lambda_2^{(1)} < \ldots < \lambda_{k-1}^{(k-1)} < \lambda_k^{(k)} \]

**Proof.** See [5].

If \( P_j(x) \) is the characteristic polynomial of the \( j \times j \) leading principal submatrix of \( A \), then it is known that these characteristic polynomials satisfy the following recursion relation:

\[
P_j(\lambda) = (\lambda - \alpha_j)P_{j-1}(\lambda) - \beta_{j-1}^2 P_{j-2}(\lambda), \quad j = 1, \ldots, k, \quad \text{where} \quad P_0(\lambda) = 1, \text{and} \quad \beta_0 = 0.
\]

The entries of \( A \) are given by, see [5]:

\[
\alpha_1 = \lambda_1^{(1)}
\]

\[
\alpha_j = \frac{\lambda_j^{(j)}P_{j-1}(\lambda_1^{(j)})P_{j-2}(\lambda_1^{(j)}) - \lambda_j^{(j)}P_{j-1}(\lambda_j^{(j)})P_{j-2}(\lambda_j^{(j)})}{P_{j-1}(\lambda_1^{(j)})P_{j-2}(\lambda_1^{(j)}) - \lambda_j^{(j)}P_{j-1}(\lambda_j^{(j)})P_{j-2}(\lambda_j^{(j)})}, \quad j = 2, \ldots, k
\]

\[
\beta_{j-1}^2 = \frac{(\lambda_1^{(j)} - \lambda_j^{(j)})P_{j-1}(\lambda_1^{(j)})P_{j-2}(\lambda_1^{(j)}) - (\lambda_1^{(j)} - \lambda_j^{(j)})P_{j-1}(\lambda_1^{(j)})P_{j-2}(\lambda_1^{(j)})}{P_{j-1}(\lambda_1^{(j)})P_{j-2}(\lambda_1^{(j)}) - \lambda_j^{(j)}P_{j-1}(\lambda_j^{(j)})P_{j-2}(\lambda_j^{(j)})}, \quad j = 2, \ldots, k
\]

Thus, the matrix \( A \) is now filled up using these formulas.

**Remark 4** It is worth to mention that this algorithm gives two choices for each \( \beta_i, i = 1, \ldots, k - 1 \). Thus, the algorithm generates a class of \( 2^{k-1} \) different solutions for the inverse eigenvalue problem.

Now, comparing the matrix \( A \) in (1) with the matrix \( J \) in (3), we can see that the above algorithm can be used to fill up the entries of \( J \), with the observation that

\[
\left\{ \begin{array}{l}
\alpha_j = \alpha_j, \quad j = 1, \ldots, k \\
\text{and} \quad b_j c_{j+1} = \beta_j^2, \quad j = 1, \ldots, k - 1
\end{array} \right. \tag{4}
\]
So, equation (4) can be used to fill up the entries of the matrix $J$ as well as the entries of the original nonsymmetric matrix $T$. But we have to observe that the second equation of (4) generates a class of infinitely many solutions to the inverse eigenvalue problem. As a matter of fact, any choice of $b_j$ and $c_{j+1}$ such that $b_j c_{j+1} = \beta_j^2$ will give a solution, and there are infinitely many such choices.

3 The Algorithm

Given the real numbers

$$
\lambda_1^{(k)} < \lambda_1^{(k-1)} < \ldots < \lambda_1^{(2)} < \lambda_1^{(1)} < \lambda_2^{(1)} < \ldots < \lambda_{k-1}^{(1)} < \lambda_k^{(k)}
$$

that represent the minimal and maximal eigenvalues of all principal leading submatrices $T_j, j = 1, \ldots, k$ of the matrix $T$ in (2). This algorithm is used to compute the entries of $T$ as follows:

1. Set $a_1 = \lambda_1^{(1)}, P_0(\lambda) = \lambda - a_1, \beta_0 = 0.$

2. For $j = 2, \ldots, k$, set

$$
a_j = \frac{\lambda_j^{(j)} P_{j-1}(\lambda_j^{(j)}) P_{j-2}(\lambda_j^{(j)}) - \lambda_j^{(j)} P_{j-1}(\lambda_j^{(j)}) P_{j-2}(\lambda_j^{(j)})}{P_{j-1}(\lambda_j^{(j)}) P_{j-2}(\lambda_j^{(j)}) - P_{j-1}(\lambda_j^{(j)}) P_{j-2}(\lambda_j^{(j)})}
$$

Choose $\beta_{j-1}$ so that $\beta_{j-1}^2 = \frac{(\lambda_j^{(j)} - \lambda_1^{(j)}) P_{j-1}(\lambda_j^{(j)}) P_{j-2}(\lambda_j^{(j)})}{P_{j-1}(\lambda_j^{(j)}) P_{j-2}(\lambda_j^{(j)}) - P_{j-1}(\lambda_j^{(j)}) P_{j-2}(\lambda_j^{(j)})}$

$$
P_j(\lambda) = (\lambda - a_j) P_{j-1}(\lambda) - \beta_{j-1}^2 P_{j-2}(\lambda)
$$

Choose $b_{j-1}$ and $c_j$ such that $b_{j-1} c_j = \beta_{j-1}^2$.

3.1 Numerical Example

This example is adopted from [5].

Let

$$
\lambda_1^{(6)} < \lambda_1^{(5)} < \lambda_1^{(4)} < \lambda_1^{(3)} < \lambda_1^{(2)} < \lambda_1^{(1)} < \lambda_2^{(1)} < \lambda_3^{(1)} < \lambda_4^{(4)} < \lambda_5^{(5)} < \lambda_6^{(6)}
$$

Where

$$
\lambda_1^{(6)} = -8, \lambda_1^{(5)} = -5, \lambda_1^{(4)} = -3, \lambda_1^{(3)} = -2, \lambda_1^{(2)} = -1, \lambda_1^{(1)} = 2, \lambda_2^{(2)} = 6, \lambda_3^{(3)} = 7, \lambda_4^{(4)} = 9, \lambda_5^{(5)} = 12, \lambda_6^{(6)} = 15.
$$
Applying the above algorithm to generate $A$, yields the following results:

$P_0(\lambda) = 1, \beta_0 = 0, \alpha_1 = 2,$
$P_1(\lambda) = \lambda - 2, \beta_1^2 = 12, \alpha_2 = 3,$
$P_2(\lambda) = \lambda^2 - 5\lambda - 6, \beta_2^2 = 8, \alpha_3 = 3,$
$P_3(\lambda) = \lambda^3 - 7\lambda^2 - 4\lambda + 28, \beta_3^2 = 21.6292, \alpha_4 = 4.7865,$
$P_4(\lambda) = \lambda^4 - 11.7865\lambda^3 + 7.8763\lambda^2 + 155.292\lambda - 4.2468, \beta_4^2 = 45.3846, \alpha_5 = 2.5515,$
$P_5(\lambda) = \lambda^5 - 14.338\lambda^4 - 7.435\lambda^3 + 452.8878\lambda^2 - 218.9359\lambda - 1260.4117,$
$\beta_5^2 = 76.0231, \alpha_6 = 4.2603$

(I) Possible choices for the matrix $A$ are:

1. $\begin{bmatrix} 2 & 3.4641 & 0 & 0 & 0 & 0 \\ 3.4641 & 3 & 2.8284 & 0 & 0 & 0 \\ 0 & 2.8284 & 2 & 4.6507 & 0 & 0 \\ 0 & 0 & 4.6507 & 4.7865 & 6.7368 & 0 \\ 0 & 0 & 0 & 6.7368 & 2.5515 & 8.7191 \\ 0 & 0 & 0 & 0 & 8.7191 & 4.2603 \end{bmatrix}$

2. $\begin{bmatrix} 2 & -3.4641 & 0 & 0 & 0 & 0 \\ -3.4641 & 3 & 2.8284 & 0 & 0 & 0 \\ 0 & 2.8284 & 2 & -4.6507 & 0 & 0 \\ 0 & 0 & -4.6507 & 4.7865 & 6.7368 & 0 \\ 0 & 0 & 0 & 6.7368 & 2.5515 & -8.7191 \\ 0 & 0 & 0 & 0 & -8.7191 & 4.2603 \end{bmatrix}$

3. $\begin{bmatrix} 2 & 3.4641 & 0 & 0 & 0 & 0 \\ 3.4641 & 3 & -2.8284 & 0 & 0 & 0 \\ 0 & -2.8284 & 2 & 4.6507 & 0 & 0 \\ 0 & 0 & 4.6507 & 4.7865 & -6.7368 & 0 \\ 0 & 0 & 0 & -6.7368 & 2.5515 & 8.7191 \\ 0 & 0 & 0 & 0 & 8.7191 & 4.2603 \end{bmatrix}$

4. $\begin{bmatrix} 2 & 3.4641 & 0 & 0 & 0 & 0 \\ 3.4641 & 3 & 2.8284 & 0 & 0 & 0 \\ 0 & 2.8284 & 2 & -4.6507 & 0 & 0 \\ 0 & 0 & -4.6507 & 4.7865 & -6.7368 & 0 \\ 0 & 0 & 0 & -6.7368 & 2.5515 & 8.7191 \\ 0 & 0 & 0 & 0 & 8.7191 & 4.2603 \end{bmatrix}$
5.
\[
\begin{bmatrix}
2 & -3.4641 & 0 & 0 & 0 & 0 \\
-3.4641 & 3 & -2.8284 & 0 & 0 & 0 \\
0 & -2.8284 & 2 & -4.6507 & 0 & 0 \\
0 & 0 & -4.6507 & 4.7865 & -6.7368 & 0 \\
0 & 0 & 0 & -6.7368 & 2.5515 & -8.7191 \\
0 & 0 & 0 & 0 & -8.7191 & 4.2603
\end{bmatrix}
\]

Out of a total number of 32 choices.

(II) Possible choices of the matrix $T$ are:

1.
\[
\begin{bmatrix}
2 & 3 & 0 & 0 & 0 & 0 \\
4 & 3 & 4 & 0 & 0 & 0 \\
0 & 2 & 2 & 7.2097 & 0 & 0 \\
0 & 0 & 3 & 4.7865 & 9 & 0 \\
0 & 0 & 0 & 5.0427 & 2.5515 & 11 \\
0 & 0 & 0 & 0 & 6.912 & 4.2603
\end{bmatrix}
\]

2.
\[
\begin{bmatrix}
2 & -3 & 0 & 0 & 0 & 0 \\
-4 & 3 & 4 & 0 & 0 & 0 \\
0 & 2 & 2 & -7.2097 & 0 & 0 \\
0 & 0 & -3 & 4.7865 & 9 & 0 \\
0 & 0 & 0 & 5.0427 & 2.5515 & -11 \\
0 & 0 & 0 & 0 & -6.912 & 4.2603
\end{bmatrix}
\]

3.
\[
\begin{bmatrix}
2 & 4 & 0 & 0 & 0 & 0 \\
3 & 3 & 2 & 0 & 0 & 0 \\
0 & 4 & 2 & 3 & 0 & 0 \\
0 & 0 & 7.2097 & 4.7865 & 5.0427 & 0 \\
0 & 0 & 0 & 9 & 2.5515 & 6.912 \\
0 & 0 & 0 & 0 & 11 & 4.2603
\end{bmatrix}
\]

4.
\[
\begin{bmatrix}
2 & 2 & 0 & 0 & 0 & 0 \\
6 & 3 & 1 & 0 & 0 & 0 \\
0 & 8 & 2 & 21.6292 & 0 & 0 \\
0 & 0 & 1 & 4.7865 & 5.0427 & 0 \\
0 & 0 & 0 & 9 & 2.5515 & 11 \\
0 & 0 & 0 & 0 & 6.912 & 4.2603
\end{bmatrix}
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Out of infinitely many choices.

3.2 Conclusion

In this work, we extended the results of [5] in the sense of constructing a general tridiagonal matrix as a solution of a special inverse eigenvalue problem instead of having a symmetric tridiagonal matrix as such a solution. To be more precise, [5] introduces a solution of the inverse eigenvalue problem in which the minimal and maximal eigenvalues of every leading principal submatrix of a $k \times k$ symmetric tridiagonal matrix are known, and consequently such a matrix is constructed using the spectral known information. The extension we introduce here is to use the symmetric tridiagonal matrix, which the algorithm in [5] computes, in order to construct a general tridiagonal matrix which is also a solution of the inverse eigenvalue problem. This means that this general tridiagonal matrix has the same minimal and maximal eigenvalues for every leading principal submatrix as the symmetric matrix which [5] presents. Moreover, it should be noticed that when a symmetric matrix is sought as a solution, the algorithm in [5] gives a choice from a class that contains $2^{k-1}$ such matrices. The extension we presented here generates a class of general tridiagonal matrices which contains infinitely many solutions.

References


Received: December, 2008