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Improving the Power of the Hybrid Test of Symmetry

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Abstract

In this note we show how to improve the power of the Hybrid test of symmetry introduced by (Modarres, R., and Gastwirth, J. L. (1998). Hyprid test for the hypothesis of symmetry. Journal of Applied Statistics, 25(6), p. 777 – 783). This can be accomplished by ignoring stage 1 of the test completely. An explanation of the reason of this is given and the simulation results support our claim.

Mathematics Subject Classification: 62G10

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1. Introduction

Let $x_1, \ldots, x_N$ be N independent observations from a continuous distribution with known median, assumed zero without any loss of generality. The problem under consideration is testing whether the distribution is symmetric about zero. This problem has received a considerable attention in the literature, see for example McWilliams (1990), Tajuddin (1994), Modarres and Gastwirth (1996) and Baklizi (2003).

Let $I_k = I(x_k > 0), k = 1, \ldots, N$, be the usual indicator function. Define the sign statistic as $S = \sum_{k=1}^{N} I_k$. Consider the ordered data set in magnitude (signs are retained) and let $x_{|k|}$ be the k-th ordered observation in magnitude, $V_k = I\left(x_{|k|} > 0\right)$. Modarress and Gastwirth (1998) proposed a two stage test for symmetry as follows; in stage 1 reject the null hypothesis of symmetry if $|Z_s| > z_{\alpha_1/2}$, where $Z_s = \dfrac{S - N/2}{\sqrt{N/4}}$ and z_α is the upper $\alpha -$quantile of the standard normal distribution. If the null hypothesis is not rejected, go to stage 2 which rejects the null hypothesis if $|Z_w| > z_{\alpha_2/2}$, where α_1 and α_2 satisfy $\alpha_1 + (1-\alpha_1)\alpha_2 = \alpha$, the nominal size of the test and $Z_w = \dfrac{W_p - E(W_p)}{(\mathrm{var}(W_P))^{1/2}}$, where $W_p = \sum_{k=Np+1}^{N}(k - Np)V_k$, the percentile modified rank statistic, p is a trimming proportion, and

$$E(W_p) = \frac{1}{2}n(1-p)(N(1-p)+1)$$

$$\mathrm{var}(W_p) = \frac{mn}{12(N-1)}(1-p)(N(1-p)+1)(N(1-p)(3p+1)+3p-1)$$

n and m are the numbers of positive and negative sample values, respectively.

Modarres and Gastwirth (1998), using simulation techniques, showed that the proposed test is very powerful and performs very well compared to other known tests of symmetry.

2. Modification of the test

It can be see that the first stage of the hybrid test is in fact not effective in detecting departures from symmetry. The reason is that both the null and non null distributions of S are binomial with parameter $\theta = 1/2$. This is because the median is known and unchanged whether or not the distribution is symmetric. Therefore, it is expected that, by removing stage 1 completely from the test, one can get a more powerful test. A simulation study is carried out using the same simulation design as in Modarres and Gastwirth (1998) to compare the two stage test (MG) with the test based on the second stage only (Modified MG). The results support our claim and show the superiority of the one stage test. The reason for the inferior power of the (MG) test is that the size of the sign test corresponds to stage 1 does not contribute to the power beyond the value of the size itself. This is because the sign test is not sensitive to departures from symmetry. In the modified MG test however, the whole critical region is based on the percentile modified rank statistic which is sensitive for departures from symmetry.

Table 1: Powers of the MG and Modified MG tests

Case	$N = 20$		$N = 30$		$N = 50$		$N = 100$	
	MG	Modified	MG	Modified	MG	Modified	MG	Modified
1	0.038	0.045	0.041	0.045	0.048	0.050	0.052	0.051
2	0.299	0.420	0.605	0.653	0.879	0.894	0.994	0.994
3	0.449	0.593	0.816	0.854	0.975	0.980	1.000	1.000
4	0.127	0.187	0.251	0.287	0.463	0.499	0.793	0.818
5	0.174	0.254	0.362	0.404	0.633	0.668	0.929	0.939
6	0.042	0.058	0.053	0.062	0.068	0.075	0.099	0.106
7	0.083	0.114	0.133	0.156	0.229	0.253	0.456	0.486
8	0.520	0.666	0.887	0.913	0.992	0.993	1.000	1.000
9	0.536	0.692	0.900	0.924	0.994	0.995	1.000	1.000

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