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Product of Fuzzy Metric Spaces and Fixed Point Theorems

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Abstract

In this paper the notion of product of probabilistic metric spaces is extended to some family of fuzzy metric spaces. We also study the topological expect of product of fuzzy metric spaces and a fixed point theorem on it.

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1 Introduction

The theory of fuzzy sets was introduced by Zadeh in 1965 [18]. Many authors have introduced the concepts of fuzzy metric in different ways [3, 5]. In particular, Kramosil and Michalek [8] generalized the concept of probabilistic metric space given by Menger [14] to the fuzzy framework. In [7, 8] George and Veeramani modified the concept of fuzzy metric space introduced by Kramosil and Michalek and obtained a Hausdorff and first countable topology on this modified fuzzy metric space. On the other hand, the study on product spaces in the probabilistic framework was initiated by Istratescu and Vaduva [11], and subsequently by Egbert [4], Alsina [1] and Alsina and Schweizer [2]. Recently, Lafuerza-Guillen [13] has studied finite products of probabilistic normed spaces and proved some interesting results. The main purpose of this paper is to introduce the product spaces in the fuzzy framework. In section 3, we generalize the concepts of product of probabilistic metric (normed) spaces studied by Egbert (Lafuerza -Guillen). In section 4, we study on the fixed point theorem on this newly developed fuzzy product spaces by generalizing the fixed point theorem introduced in product spaces by Nedler [15].

2 Preliminaries

Throughout this paper we shall use all symbols and basic definitions of George and Veeramani [7, 8].

Definition 2.1. *A binary operation* $*\colon [0,1]\times[0,1]\to[0,1]$ *is a continuous t-norm if it satisfies the following conditions:*
(i) $*$ *is associative and commutative;*
(ii) $*$ *is continuous;*
(iii) $a*1=a$ *for all* $a\in[0,1]$*;*
(iv) $a*b\le c*d$ *whenever* $a\le c$ *and* $b\le d$*, for each* $a,b,c,d\in[0,1]$.

Two typical examples of continuous t-norm are

$$a*b=ab \quad \text{and} \quad a*b=Min(a,b).$$

The following definition is due to George and Veeramani [7].

Definition 2.2. *[7] A Fuzzy Metric Space is a triple* $(X,M,*)$ *where* X *is a nonempty set,* $*$ *is a continuous t-norm and* $M\colon X\times X\times[0,\infty)\to[0,1]$ *is a mapping (called fuzzy metric) which satisfies the following properties: for every* $x,y,z\in X$ *and* $t,s>0$
(FM1) $M(x,y,t)>0$*;*
(FM2) $M(x,y,t)=1$ *if and only if* $x=y$*;*
(FM3) $M(x,y,t)=M(y,x,t)$*;*
(FM4) $M(x,z,t+s)\ge M(x,y,t)*M(y,z,s)$*;*
(FM5) $M(x,y,\cdot)\colon[0,\infty)\to[0,1]$ *is continuous.*

Lemma 2.3. *[9]* $M(x,y,\cdot)$ *is nondecreasing for all* $x,y\in X$

Remark 2.4. *[7] In a fuzzy metric space* $(X,M,*)$*, for any* $r\in(0,1)$ *there exists an* $s\in(0,1)$ *such that* $s*s\ge r$.

In [7] it has been proved that every fuzzy metric M on X generates a topology τ_M on X which has a base the family of sets of the form

$$\{B_x(r,t)\colon x\in X, r\in(0,1), t>0\},$$

where $B_x(r,t) = \{y \in X \colon M(x,y,t) > 1-r\}$ is a neighborhood of $x \in X$ for all $r \in (0,1)$ and $t > 0$. In addition, (X, τ_M) is a Hausdorff first countable topological space. Moreover, if (X,d) is a metric space, then the topology generated by d coincides with the topology τ_M generated by the induced metric M_d.

Theorem 2.5. *[7] Let $(X, M, *)$ be a fuzzy metric space and τ_M be the topology induced by the fuzzy metric M. Then for a sequence (x_n) in X, $x_n \to x$ if and only if $M(x_n, x, t) \to 1$ as $n \to \infty$.*

Definition 2.6. *A sequence (x_n) in a fuzzy metric space $(X, M, *)$ is a Cauchy sequence if and only if for each $\epsilon > 0$ and $t > 0$ there exists $n_0 \in \mathbb{N}$ such that $M(x_n, x_m, t) > 1 - \epsilon$ for all $n, m \geq n_0$, i.e., $\lim_{n,m\to\infty} M(x_n, x_m, t) = 1$ for every $t > 0$.*

A fuzzy metric space in which every Cauchy sequence is convergent is called a complete fuzzy metric space.

3 Product of Fuzzy Metric Space

The study on product spaces in the probabilistic framework was initiated by Istratescu and Vaduva [11] followed by Egbert [4], Alsina [1] and Alsina and Schweizer [2]. In this section, we define the product of two fuzzy metric spaces in the sense of Egbert [4] in the following way:

Definition 3.1. *Let $(X, M_X, *)$ and $(Y, M_Y, *)$ are two fuzzy metric spaces defined with same continuous t-norms $*$. Let Δ be a continuous t-norm. The Δ-product of $(X, M_X, *)$ and $(Y, M_Y, *)$ is the product space $(X \times Y, M_\Delta, *)$ where $X \times Y$ is the Cartesian product of the sets X and Y, and M_Δ is the mapping from $(X \times Y \times (0,1)) \times (X \times Y \times (0,1))$ into $[0,1]$ given by*

$$M_\Delta(p, q, t+s) = M_1(x_1, x_2, t) \Delta M_2(y_1, y_2, s) \tag{1}$$

for every $p = (x_1, y_1)$ and $q = (x_2, y_2)$ in $X \times Y$ and $t + s \in (0,1)$.

As an immediate consequence of Definition 3.1, we have

Theorem 3.2. *If $(X, M_X, *)$ and $(Y, M_Y, *)$ are fuzzy metric spaces under the same continuous t-norm $*$, then their $*$-product $(X \times Y, M_*, *)$ is a fuzzy metric space under $*$.*

We noted that for a metric space (X, d_X), if $a*b = ab$ (or $a*b = Min(a,b)$) for all $a, b \in [0,1]$ and $M_{d_X}(x_1, x_2, t) = \frac{kt^n}{kt^n + m d_X(x_1, x_2)}$ for each $x_1, x_2 \in X$ and $k, m, n > 0$, then $(X, M_{d_X}, *)$ is a fuzzy metric space induced by the metric d_X. Hence, we have

Example 3.3. *Let (X, d_X) and (Y, d_Y) are metric spaces and $(X \times Y, d)$ be their product with $d(p, q) = Max\{d_X(x_1, x_2), d_Y(y_1, y_2)\}$ for each $p = (x_1, y_1)$ and $q = (x_2, y_2)$ in $X \times Y$. Denote $a\Delta b = Min(a, b)$ for all $a, b \in [0, 1]$ and let $M_d(p, q, t) = \frac{t}{t+d(p,q)}$. Then $(x \times Y, M_d, *)$ is a Δ-product of (X, d_X) and (Y, d_Y).*

Proof: It is suffices to prove the condition (1). To this end,

$$\begin{aligned} M_d(p,q,t) &= \frac{t}{t+d(p,q)} = \frac{t}{t+Max\{d_X(x_1,x_2), d_Y(y_1,y_2)\}} \\ &= \frac{t}{Max\{t+d_X(x_1,x_2), d_Y(y_1,y_2)\}} \\ &= Min\left(\frac{t}{t+d_X(x_1,x_2)}, \frac{t}{t+d_Y(y_1,y_2)}\right) \\ &= \left(\frac{t}{t+d_X(x_1,x_2)}\right)\Delta\left(\frac{t}{t+d_Y(y_1,y_2)}\right). \end{aligned}$$

Whence, $M_d(p, q, t) = M_{d_X}\Delta M_{d_Y}$.

Definition 3.4. *[15] Let Δ and $*$ are continuous t-norms. We say that Δ is stronger than $*$, if for each $a_1, a_2, b_1, b_2 \in [0, 1]$,*

$$(a_1 * b_1)\Delta(a_2, b_2) \geq (a_1\Delta a_2) * (b_1\Delta b_2).$$

Lemma 3.5. *If Δ is stronger than $*$ then $\Delta \geq *$.*

Proof: From Definition 3.4, by setting $a_2 = b_1 = 1$, yields $a_1\Delta b_2 \geq a_1 * b_2$, i.e., $\Delta \geq *$.

Theorem 3.6. *Let $(X, M_X, *)$ and $(Y, M_Y, *)$ are fuzzy metric spaces under the same continuous t-norm $*$. If there exists a continuous t-norm Δ stronger than $*$, then the Δ-product $(X \times Y, M_\Delta, *)$ is a fuzzy metric space under $*$.*

Proof: It is suffices to prove the axiom (FM4) and (FM6). Let $p = (x_1, y_1), q = (x_2, y_2), r = (x_3, y_3)$ are in $X \times Y$. Then

$$\begin{aligned} & M_\Delta(p, r, 2\alpha) = (M_X(x_1, x_3, \alpha)\Delta M_Y(y_1, y_3, \alpha)) \\ &\geq (M_X(x_1, x_2, \alpha/2) * M_X(x_2, x_3, \alpha/2))\Delta(M_Y(y_1, y_2, \alpha/2) * M_Y(y_2, y_3, \alpha/2)) \\ &\geq (M_X(x_1, x_2, \alpha/2)\Delta M_X(x_2, x_3, \alpha/2)) * (M_Y(y_1, y_2, \alpha/2)\Delta M_Y(y_2, y_3, \alpha/2)) \\ &= M_\Delta(p, q, \alpha) * M_\Delta(q, r, \alpha). \end{aligned}$$

The continuity of the t-norms implies the function $M_\Delta(p, q, \cdot) \colon (0, \infty) \to [0, 1]$ is continuous. Hence completes the proof.

Corollary 3.7. *If $(X, M_X, *^1)$ and $(Y, M_Y, *^2)$ are fuzzy metric spaces and if there exists a continuous t-norm Δ stronger than $*^1$ and $*^2$ then their Δ-product is a fuzzy metric space under Δ.*

We now turn to the question of topologies in the -product spaces and give the following result:

Theorem 3.8. *Let $(X_1, M_1, *)$ and $(X_2, M_2, *)$ be fuzzy metric spaces under the same continuous t-norm $*$. Let U denote the neighborhood system in $(X_1 \times X_2, M_*, *)$ and let V denote the neighborhood system in $(X_1 \times X_2, M_*, *)$ consisting of the Cartesian products $B_{x_1}(r,t) \times B_{x_2}(r,t)$ where $x_1 \in X_1$, $x_2 \in X_2$, $r \in (0,1)$ and $t > 0$. Then U and V induce the same fuzzy topology on $(X_1 \times X_2, M_*, *)$.*

Proof: Clearly, since $*$ is continuous, U and V are bases for their respective topology. So, it is suffices to prove that for each $V' \in V$ there exists a $U' \in U$ such that $U' \subseteq V'$, and conversely. Let $A_1 \times A_2 \in V$. Then there exist neighborhoods $B_{x_1}(r,t)$ and $B_{x_2}(r,t)$ contained in A_1 and A_2 respectively. Let $r = Min(r_1, r_2)$, $t = Min(t_1, t_2)$, and let $x = (x_1, x_2)$. Here, we shall show that $B_x(r,t) \in A_1 \times A_2$. Let $y = (y_1, y_2) \in B_x(r,t)$, then we have

$$\begin{aligned} M_1(x_1, y_1, t_1) &= M_1(x_1, y_1, t_1) * 1 \geq M_1(x_1, y_1, t_1) * M_1(x_2, y_2, t_2) \\ &\geq M_1(x_1, y_1, t) * M_1(x_2, y_2, t) \\ &= M(x, y, t) > 1 - r \geq 1 - r_1. \end{aligned}$$

Similarly, we can show that $M_2(x_2, y_2, t_2) > 1 - r_2$. Thus $y_1 \in B_{x_1}(r_1, t_1)$ and $y_2 \in B_{x_2}(r_2, t_2)$ which implies that $B_x(r,t) \in A_1 \times A_2$. Conversely, suppose that $B_x(r,t) \in U$. Since $*$ is continuous, there exists an $\eta \in (0,1)$ such that $(1-\eta) * (1-\eta) > 1 - r$. Let $y \in (y_1, y_2) \in B_{x_1}(\eta, t) \times B_{x_2}(\eta, t)$. Then

$$M_*(x, y, t) = M_1(x_1, y_1, t) * M_2(x_2, y_2, t) \geq (1-\eta) * (1-\eta) > 1 - r$$

so that $y \in B_x(r,t)$ and $B_{x_1}(r,t) \times B_{x_2}(r,t) \subseteq B_x(r,t)$. This completes the proof.

4 Fixed Point Theorems

Grabiec [9] proved a fuzzy Banach contraction theorem whenever fuzzy metric space was considered in the sense of Kramosil and Michalek and was complete in Grabiec's sense. Meanwhile, Gregori and Sapena [10] gave fixed point theorems for complete fuzzy metric space in the sense of George and Veeramani and also for Kramosil and Michalek's fuzzy metric space which are complete in Grabiec's sense. Recently, Zikic [19] proved that the fixed point theorem of Gregori and Sapena holds under general conditions (theory of countable extension of a t-norm).

We begin with the definition of contraction mappings in fuzzy metric spaces.

Definition 4.1. *Let $(X, M, *)$ be a fuzzy metric space. A mapping $f\colon X \to X$ is said to be fuzzy contraction if there exists a $k \in (0,1)$ such that*

$$M(fx, fy, t) \geq M(x, y, t/k) \quad \text{for all} \quad x, y \in X.$$

Theorem 4.2. *[9] Let $(X, M, *)$ be a complete fuzzy metric space such that $\lim_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$. Let $f\colon X \to X$ be a contractive mapping. Then f has a unique fixed point.*

We prove the following theorem on continuity of fixed points as a fuzzy version of Theorem 1 in [15].

Theorem 4.3. *Let $(X, M*)$ be a fuzzy metric space with $a * b = Min(a, b)$. Let $f_i\colon X \to X$ be a function with at least one fixed point x_i for each $i = 1, 2, \cdots$, and $f_0\colon X \to X$ be a fuzzy contraction mapping with fixed point x_0. If the sequence (f_i) converges uniformly to f_0, then the sequence (x_i) converges to x_0.*

Proof: Let $k \in (0,1)$ and choose a positive number $N \in \mathbb{N}$ such that $i \geq N$ implies

$$M(f_i x, f_0 x, (1-k)t) > 1 - r$$

where $r \in (0,1)$ and $x \in X$. Then, if $i \geq N$, we have

$$\begin{aligned} M(x_i, x_0, t) &= M(f_i x_i, f_0 x_0, t) \\ &\geq M(f_i x_i, f_0 x_i, (1-k)t) * M(f_0 x_i, f_0 x_0, kt) \\ &> Min(1 - r, M(x_i, x_0, t)). \end{aligned}$$

Hence, $M(x_i, x_0, t) \to 1$ as $i \to \infty$. This proves that (x_i) converges to x_0.

In what follows $\pi_1\colon X \times Y \to X$ will denote the first projection mapping defined by $\pi_1(x, y) = x$, while $\pi_2\colon X \times Y \to Y$ will denote the second projection mapping defined by $\pi_2(x, y) = y$.

Definition 4.4. *Let $(X, M, *)$ be a fuzzy metric space and Y be any space. A mapping $f\colon X \times Y \to X \times Y$ is said to be locally fuzzy contraction in the first variable if and only if for each $y \in Y$ there exists an open ball $B_y(\epsilon)$, $\epsilon \in (0,1)$ containing y and a real number $k(y) \in (0,1)$ such that*

$$M(\pi_1 \circ f(x_1, y), \pi_1 \circ f(x_2, y), t) \geq M(x_1, x_2, t/k(y)) \quad \text{for all} \quad x_1, x_2 \in X.$$

A mapping $f\colon X\times Y\to X\times Y$ is called fuzzy contraction in the first variable if and only if there exists a real number $k\in(0,1)$ such that for any $y\in Y$

$$M(\pi_1\circ f(x_1,y),\pi_1\circ f(x_2,y),t)\geq M(x_1,x_2,t/k)\quad\text{for all}\quad x_1,x_2\in X.$$

It is obvious that every fuzzy contraction mapping is locally fuzzy contraction in the first variable.

We define a fuzzy contraction mapping in the second variable in an analogous fashion.

Definition 4.5. *The fuzzy metric space $(X,M,*)$ has fixed point property (f.p.p) if every continuous mapping $f\colon X\to X$ has fixed point.*

The following theorem is a fuzzy version of a theorem in [15].

Theorem 4.6. *Let $(X,M_X,*)$ be a complete fuzzy metric space, $(Y,M_Y,*)$ be a fuzzy metric space with the f.p.p., and let $f\colon X\times Y\to X\times Y$ be uniformly continuous and a fuzzy contraction mapping in the first variable. Then, f has a fixed point.*

Proof: For $y\in Y$, let $f_y\colon X\to X$ be defined by $f_y(x)=\pi_1\circ f(x,y)$ for all $x\in X$. Since, for every $y\in Y$, f_y is a fuzzy contraction mapping, therefore f_y has a unique fixed point (see, Theorem 4.2). Let $G\colon Y\to X$ be given by $G(y)=f_y(G(y))$ is the unique fixed point of f_y. Now, let $y_0\in Y$ and let (y_n) be a sequence of points of Y which converges to y_0. Since f is uniformly continuous, the sequence (f_{y_n}) converges uniformly to f_{y_0}. Hence, by Theorem 4.3, the sequence $(G(y_n))$ converges to $G(y_0)$. This shows that the function G is continuous on Y. Now, let $g\colon Y\to Y$ be a continuous function defined via $g(y)=\pi_2\circ f(G(y),y)$ for each $y\in Y$. Since, $(Y,M_Y,*)$ has f.p.p., there is a point $z\in Y$ such that $g(z)=z, i.e., z=g(z)=\pi_2\circ f(G(z),z)$. It follows that $(G(z),z)$ is a fixed point of f. This completes the proof.

To prove the following theorem, we require:

Lemma 4.7. *Let $(X,M,*)$ be a fuzzy metric space with $a*a\geq a$ for every $a\in[0,1]$ and Y be a fuzzy topological space with f.p.p. Let $f\colon X\times Y\to X\times Y$ be locally fuzzy contraction in the first variable. Let $x_0\in X$ and $y\in Y$. Define the sequence $(p_n(y))$ in X as follows:*

$$p_0(y)=x_0\quad \textit{and}\quad p_n=p_n(y)=\pi_1\circ f(p_{n-1}(y),y).$$

Then,
(i) $(P_n(y))$ is a Cauchy sequence in X.
(ii) If $p_n\to p_y$, then $\pi_1\circ f(p_y,y)=p_y$.
(iii) Define $g\colon Y\to Y$ as $g(y)=\pi_2\circ f(p_y,y)$. Then, g is a continuous function.

Proof: (i) Since f is a locally fuzzy contraction mapping in the first variable, there exists a real number $k \in (0,1)$ such that

$$M(p_n, p_{n+1}, t) = M(\pi_1 \circ f(p_{n-1}, y), \pi_1 \circ f(p_n, y), t) \geq M(p_{n_1}, p_n, t/k) \quad n \geq 1.$$

By a simple induction we get

$$M(p_n, p_{n+1}, t) \geq M(p_0, p_1, t/k^n)$$

for all n and $t > 0$. We note that, for every positive integer m, n with $m > n$ and $k \in (0,1)$, we have

$$(1-k)(1+k+k^2+\cdots+k^{m-n-1}) = 1-k^{m-n} < 1.$$

Therefore, $t > (1-k)(1+k+k^2+\cdots+k^{m-n-1})t$. Since M is nondecreasing, we have

$$M(p_n, p_m, t) \geq M(p_n, p_m, (1-k)(1+k+k^2+\cdots+k^{m-n-1})t).$$

Thus, by (FM4), we notice that, for $m > n$,

$$\begin{aligned}
&M(p_n, p_m, (1-k)(1+k+k^2+\cdots+k^{m-n-1})t) \\
&\geq \; M(p_n, p_{n+1}, (1-k)t) * \cdots * M(p_{m-1}, p_m, (1-k)k^{m-n-1}t) \\
&\geq \; M(p_0, p_1, (1-k)t/k^n) * \cdots * M(p_0, p_1, (1-k)k^{m-n-1}t/k^{m-1}) \\
&= \; M(p_0, p_1, (1-k)t/k^n) * \cdots * M(p_0, p_1, (1-k)t/k^n)
\end{aligned}$$

Since $a * a \geq a$, we conclude that

$$M(p_n, p_m, t) \geq M(p_0, p_1, (1-k)t/k^n).$$

By letting $n \to \infty$ and $m > n$, we get

$$\lim_{n,m\to\infty} M(p_n, p_m, t) = \lim_{n\to\infty} M(p_o, p_1, (1-k)t/k^n) = 1.$$

This implies that $(p_n(y))$ is a Cauchy sequence in X.

(ii) Let $u = \pi_1 \circ f(p_y, y)$. By contradiction, suppose that $u \neq p_y$. Then $M(u, p_y, t) = \epsilon < 1$ for every $t > 0$. Since $f\colon X \times Y \to X \times Y$ is continuous, there exists an open set $U \times V$ in $X \times Y$ and a real number $\lambda \in (0, \epsilon)$ such that $(p_y, y) \in U \times V$, $U \subseteq B_{p_y}(\lambda, t)$ and $f(U \times V) \subseteq B_u(\lambda, t) \times Y$. Since $p_n \to p_y$, there is a positive number $N \in \mathbb{N}$ such that $p_n \in U$ for all $n \geq N$. But $\pi_1 \circ f(p_k, y) = p_{k+1} \in U$. Therefore $f(p_k, y) \notin B_u(\lambda, t) \times Y$ which contradicts the fact that $f(U \times V) \subseteq B_u(\lambda, t) \times Y$. Therefore our assumption is incorrect.

(iii) Follows Lemma 3 in [6].

Theorem 4.8. *Let $(X, M, *)$ be a complete fuzzy metric space with $a*a \geq a$ for every $a \in [0,1]$ and let Y be a fuzzy topological space with f.p.p. If the mapping $f\colon X \times Y \to X \times Y$ is a locally fuzzy contraction in the first variable, then f has a fixed point.*

Proof: Let $x_0 \in X$ and $y \in Y$. Define the sequence $(p_n(y))$ as follows:

$$p_0(y) = x_0 \quad \text{and} \quad p_n(y) = \pi_1 \circ f(p_{n-1}(y), y).$$

By Lemma 4.7(i), the sequence $(p_n(y))$ is a Cauchy sequence in X. Since $(X, M, *)$ is complete, there exists a point $p_y \in X$ such that $\lim_{n\to\infty} p_y(y) = p_y$. Now define a continuous mapping $g\colon Y \to Y$ by $g(y) = \pi_2 \circ f(p_y, y)$. Since Y has the f.p.p., there exists a point $y_0 \in Y$ such that $g(y_0) = y_0$. By Lemma 4.7(ii) we have $\pi_1 \circ f(p_{y_0}, y_0) = p_{y_0}$. But $y_0 = g(y_0) = \pi_2 \circ f(p_{y_0}, y_0)$. Hence, $f(p_{y_0}, y_0) = (p_{y_0}, y_0)$ which completes the proof.

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