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A Ranked Voting System For Consolidation of Different Results of Ranking Methods in DEA

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Abstract

Within data envelopment analysis is a subgroup of papers in which many researchers have sought to improve the differential capabilities of DEA and to fully rank the decision making units (DMU). However, whilst each technique is useful in a specialist area, no one methodology can be prescribed as the complete solution to the question of ranking. Different results which are reached in applying the alternative ranking methods, is a common drawback of proposed ranking methods. This paper addresses ranked voting system to determine an ordering of top t DMUs in terms of the aggregate vote by using the results of ranking methods.

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1 Introduction

Data envelopment analysis (DEA) is basically an LP based technique, Charnes et al. (1978), used for estimating production frontiers and evaluating the relative efficiency of organizational units, referred to as decision making unit (DMU). DEA by focusing on certain simple ratios, provides for each DMU just a score and based on these scores a set of DMUs can be partitioned into two groups: frontier DMUs (efficient) and non-frontier DMUs (inefficient).

Often decision makers (DM) are interested in a complete ranking, beyond the dichotomized classification. Therefore, subgroup of papers have been developed in this field in which many researchers have sought to improve the differential capabilities of DEA and to fully rank both efficient, as well as inefficient DMUs. Since these ranking methods have been developed based on some antithetical preferences, different calculations are reached in applying the alternative ranking methods; see Torgersen et al. (1996). Hence, whilst each ranking technique is useful in a specialist area, no one methodology can be prescribed as the complete solution to the question of ranking. Clearly, the logic behind the reason for ranking the DMUs will decide the ultimate ranking procedure chosen and consequently the results.

In what follows, we will introduce a procedure to aggregate the results of proposed ranking methods. This procedure applies ranked voting system in which each ranking method, as a voter, selects and ranks the top t DMUs. The paper unfolds as follows. Section 2 includes a brief literature review. In section 3, Aggregated Vote-Ranking Method accompanied by a computational experiment is reported. Finally, section 4 gives our conclusive remarks.

2 Literature Review

Adler et al. (2002), reviewed the ranking methods developed in the literature. They grouped the ranking methods into six basic areas. The first area involves the evaluation of a cross-efficiency matrix, in which the units are self and peer evaluated. The second idea, generally known as the super efficiency method, ranks through the exclusion of the unit being scored from the dual linear program (this proved popular and many papers sprouted from this idea). The third group is based on benchmarking, in which a DMU is highly ranked if it is chosen as a useful target for many other DMUs (this is of substantial use when looking to benchmark industries). The fourth group of papers developed a connection between multivariate statistical techniques and DEA. The fifth group discussed the ranking of inefficient units through proportional measures of inefficiency and in the last set of papers, which crosses multi-criteria decision making models with DEA, some concepts used additional, preferential information in order to aid the ranking process.

As for ranking of alternatives, one of the most familiar methods is to compare the weighted sum of their votes, after determining suitable weights of each alternatives. The problem of aggregating individual ordinal preferences has been approached by several authors (see [4, 8, 9, 11]). Most of the solution methods proposed are oriented to social choice theory and voting systems. Borda (1781) initially proposed the 'Method of Marks' so as to obtain an agreement among different opinions. His method is surely a useful method in evaluating consumers' preferences of commodities in marketing, or in ranking social policies in political sciences, for instance. This method ranks the candidates according to the sum of the rankings assigned by the electors.

However, in ranked voting system, the problem is to determine an ordering of all n candidates by obtaining a total score $s_j = \sum_{r=1}^{t} u_r y_{rj} \quad j = 1, \cdots, n$, for each candidate j, where y_{rj} is the number of r-th place votes candidate j receives, and u_r, r=$1, \cdots, t$, is the sequence of weights given to the r-th place vote. Because of no established ways to determine the weights, many arbitrary choices of the sequence of weights can exist. The well known Borda method, $u_r = \ t - r + 1, \ \ r = 1, \cdots, t,$ is an example. However, in order to obtain a total ordering of candidates, we need to specify the sequence of weights satisfying two conditions below.

Decreasing Sequence of Weights: $u_r > u_{r+1} > 0 \qquad\qquad r = 1, \cdots, t-1$

Convex Sequence of Weights : $u_r - u_{r+1} \geq u_{r+1} - u_{r+2} \quad r = 1, \cdots, t-2$

It is, however, difficult to determine suitable weight of each alternative a priori. In this context, Cook and Kress (1990) considered an alternative method which does not require specifying the sequence of weights. This type of procedure which optimizes each alternative individually is based on DEA methodology and has also been applied in Cook and Kress (1994, 1996) and in

Puerto et al. (2000) in order to construct multiple criteria indices. Here, the candidates in ranked voting system are as DMUs in DEA and each DMU is considered to have t outputs (ranked votes) and only one input with amount unity.

Nevertheless, depending on the decisional context, it may be convenient to impose certain conditions on the weights associated to the ordinal categories in order to better represent the range of weights that the voters are likely to consider. Hashimoto (1997) incorporate the condition of DSW and CSW into DEA as the assurance region (DEA/AR model). He used the DEA/AR exclusion model. In this model, The DMU being evaluated is excluded from the comparison set. Hence, the exclusion model allows DEA efficiency scores to exceed unity. The Hashimoto (1997) formulation is as follows:

$$\begin{aligned} E_o = \ Max \quad & \sum_{r=1}^{t} u_r\, y_{ro} & \\ s.t. \quad & \sum_{r=1}^{t} u_r\, y_{rj} \ \leq 1 & j = 1, 2, \ldots, n, \ j \neq o \\ & u_r - u_{r+1} \ \geq \epsilon & r = 1, 2, \ldots, t-1 \\ & u_t \ \geq \epsilon & \\ & u_r - 2u_{r+1} + u_{r+2} \geq 0 & r = 1, 2, \ldots, t-2 \end{aligned} \tag{1}$$

Where ϵ is a positive non-Archimedean infinitesimal.

3 Aggregated Ranking Method

We are in a position to propose our aggregated ranking method in the following manner. Consider n DMUs to be evaluated indexed by j=1,$\cdots$,n. Each DMU$_j$ is assumed to use m different inputs x$_{ij}$ (i=1,$\cdots$,m) to produce s different outputs y$_{rj}$ (r=1,$\cdots$,s).

Step 1: Consider p methods of ranking are available. Rank the DMUs by using these methods (it is better that the ranking methods take all the aspect of ranking with different preferences). In other words, each ranking method, as a voter, selects and ranks the top t DMUs (candidates) in view of its preferences.

Step 2: Consider n DMUs to be evaluated indexed by j=1,$\cdots$,n. Each DMU$_j$ is assumed to have t outputs, y$_{rj}$ (r=1,$\cdots$,t), and only one input with amount unity, where y$_{rj}$ is the number of r-th place votes DMU$_j$ received in step 1.

Step 3: Apply the DEA/AR exclusion model, like Hashimoto (1997), on this production possibility set (PPS) and obtain a total ordering of DMUs in accordance with E$_o^*$ scores.

Table 1. The raw data and the results of the DEA ranking methods

	In1	In2	Out1	Out2	1*	2*	3*	4*	5*	6*	7*	8*	9*	10*
A	150	0.2	14000	3500	1	1	1	1	1	3	1	4	1	1
B	400	0.7	14000	21000	2	3	2	2	2	1	5	5	3	4
C	320	1.2	42000	10500	4	4	3	4	3	2	3	2	4	5
D	520	2.0	28000	42000	2	1	4	4	4	4	4	1	1	1
E	350	1.2	19000	25000	3	2	5	3	5	5	2	3	2	2
F	320	0.7	14000	15000	5	5	6	5	6	6	6	6	5	3

* Ranking Methods: Cross-efficiency (aggressive), Cross-efficiency (benevolent), Super-efficiency, Benchmarking, CCA, DDEA, DR/DEA, Maximin efficiency ratio, MOLP-minimax, MOLP-minisum, respectively.

Example: In order to illustrate the results of using the proposed ranking method, we apply a simple example which has been analyzed in Adler et al. (2002) using 10 ranking methods; p=10. Table 1 shows the raw data and the results of the DEA ranking methods presented in Adler et al. (2002), in which six DMUs are compared over four variables. Table 2 shows the raw data and the results of the Aggregated Vote-Ranking Method, $\epsilon = 0.0001$, to determining an ordering of top t(=2) DMUs.

Table 2. The result of Aggregated Vote-Ranking Method

	Out1	Out2	In	E_o^*	Aggregated Vote Ranking
A	8	-	1	1.9999	1
B	1	4	1	0.6246	3
C	-	2	1	0.2498	5
D	4	1	1	0.6249	2
E	-	4	1	0.4996	4
F	-	-	1	0.0000	6

4 Conclusion

A common drawback that permeates to the most of the studies that produce the ranking of DMUs, is different calculations which are reached in applying the alternative ranking methods. The procedure proposed in this paper attempts to rectify this deficiency in the literature by applying a ranked voting system, using a DEA/AR exclusion model, to aggregate the results of ranking methods. In this method, each ranking method, as a voter, selects and ranks the top t DMUs. Then, the ranked voting system determine an ordering of DMUs.

This procedure could be especially suitable in many real world contexts, mainly when the information that the ranking methods (voters) have about the decision process is not unified, in the sense that different methods may assume different preferences for ranking and so, different calculations are reached in applying the alternative ranking methods.

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