

A Maple Program for Solving Systems of Linear and Nonlinear Integral Equations by Adomian Decomposition Method

Jafar Biazar and Masumeh Pourabd

Department of Mathematics
Guilan University
Rasht, P.O.Box 1914, Iran
biazar@guilan.ac.ir

Abstract. Mathematical modeling for many problems in different disciplines, such as engineering, chemistry, physics and biology leads to integral equation, or system of integral equations. It's the reason of great interest for solving these equations.

There are some analytical and numerical methods for solving Volterra integral equations, but extension of these methods to systems of such integral equations is not easy to employ.

Adomian decomposition method, well address in [1,2] has been used to solved some of these systems such as systems of differential equations, systems of integral equations and even systems of integro-differential equation [3,4,5].

Applying this method needs some computations which is sometimes boring, having a program to do all computations would be interesting and helpful.

In this article a maple program is prepared to solve a system of Volterra integral equations of the second kind, linear or non-linear.

Keywords: Adomian decomposition, Systems of Volterra integral equations of second kind

1. INTRODUCTION

Consider the following system of second kind Volterra integral equations:

$$F(t) = g(t) + \int_0^t G(s, t, F(s))ds, \quad (1.1)$$

Where

$$F(t) = (f_1(t), f_2(t), \dots, f_n(t))^t,$$

$$G(t) = (g_1(t), g_2(t), \dots, g_n(t))^t,$$

$$G(s, t, F(s)) = (G_1(s, t, F(s)), G_2(s, t, F(s)), \dots, G_n(s, t, F(s))).$$

Especial case of linear systems comes from linearity of the function $V(s, t, F(s))$. Consider the i th equation in (1),

$$F_i(t) = g_i(t) + \int_0^t G_i(s, t, f_1(t), f_2(t), \dots, f_n(t)) ds, \quad (1.2)$$

Adomian decomposition method considers the Solution as the summation of a series say:

$$F_{i\lambda} = \sum_{j=0}^{\infty} f_{ij}(t) \lambda^j \quad (1.3)$$

and $G_i(s, t, f_1(t), f_2(t), \dots, f_n(t))$, as a series, say:

$$\begin{aligned} & G_i(s, t, f_1(t), f_2(t), \dots, f_n(t)) \\ &= \sum_{j=0}^{\infty} A_{ij}(f_{10}, \dots, f_{1j}, f_{20}, \dots, f_{2j}, \dots, f_{1j}, f_{n0}, \dots, f_{nj}) \lambda^j \end{aligned} \quad (1.4)$$

where $A_{ij}(f_{10}, \dots, f_{1j}, f_{20}, \dots, f_{2j}, \dots, f_{1j}, f_{n0}, \dots, f_{nj})$, which depends on f_{k0}, \dots, f_{kj} , $k = 1, 2, \dots$ called Adomian polynomial and are define as:

$$A_{ij} = \frac{1}{j!} \left[\frac{d^j}{d\lambda^j} G_{i\lambda}(s, t, \sum_{i=0}^{\infty} f_{1i} \lambda^i, \dots, \sum_{i=0}^{\infty} f_{ni} \lambda^i) \right]_{\lambda=0} \quad (1.5)$$

2. MAPLE PROGRAM

Inputs of the algorithm are the following:

n , the number of equations in the system of integral equations,

m , the number of terms of the approximations to the solutions,

$g_i(t)$, the first term in the right hand side of the canonical form, equation (2).

Here is the main program:

>restart;

>with(student):

INPUT

1 >G[1]:=G₁(s, t, f₁, f₂, ..., f_N);

 G[2]:=G₂(s, t, f₁, f₂, ..., f_N);

 ⋮

 G[k]:=G_N(s, t, f₁, f₂, ..., f_N);

2 >g[1]:=f[1];

```

g[2]:=f[2]
⋮
g[N]:=f[N]);
3 >m:=M:
4 >n:=N:

5 > for i from 1 by 1 while i <= n
    do f[i,lambda]:=sum(f[i,b]*(lambda)^b,b=0..m):
    end do:
6 >for i from 1 by 1 while i <= n
    do for j from 1 by 1 while j <= n
        do G[i,lambda]:=subs(f[j](t)=f[j,lambda],G[i]):
        G[i]:=G[i,lambda]
        end do:
    end do;
7 >for i from 1 by 1 while i <= n
    do s[i]:=expand(G[i,lambda],lambda):
    ft[i]:= unapply(s[i],lambda):
    end do:
8 >for j from 1 by 1 while j <= n
    do for i from 0 by 1 while i <= m
        do A[j][i]:=((D@@i)(ft[j])(0)/i!):
        print( A[j,i], "=", A[j][i])
        end do;
    end do;
9 >for j from 1 by 1 while j <= n
    do F[j][0]:=g[j]
    end do:
10 > for i from 0 by 1 while i < m
    do for j from 1 by 1 while j <= n
        do f[j,i]:=subs(x=t,F[j][i])
        end do:
        for j from 1 by 1 while j <= n
            do F[j][i+1]:=int(A[j][i],t=0..x):
            print( F[j,i+1], "=", F[j][i+1])
            end do
        end do:
11 > for j from 1 by 1 while j <= n
    do F[j][0]:=subs(t=x,F[j][0]):
    end do:
12 >for j from 1 by 1 while j <= n
    do f[j]:=sum('F[j][i]',i'=0..b-1)
    end do:

```

In statement(1) the integrands $G_i(s, t, f_1, \dots, f_n)$ should be entered.

In statement (2) $g_i(t)$ the first term in the right hand side of canonical forms in equation (2) should be entered.

In statement (5), regarding equation (3) and the chosen m , an approximation to $f_{i\lambda}$ here in after referred as f_{im} , will be considered symbolically.

In statement (6), $f'_{i\lambda}$ are substitute in G_i to compute $G_{i\lambda}$,

In statement (9) according to formula (4) Adomian polynomials will be computed,

In statement (10), f_{im} are computed,

In statement (12) f_{im} are added together to compute an approximation to f_i .

3. NUMERICAL EXAMPLES

in this part three examples are provided to illustrate the program.

Example1:

In the first example we solve the following nonlinear system of two integral equations. with the exact solutions are $f_1(x) = \sin(x)$ and $f_2(x) = \cos(x)$

$$\left\{ \begin{array}{l} f_1(t) = \sin(t) - t + \int_0^t (f_1^2(s) + f_2^2(s)) ds \\ f_2(t) = \cos(t) - \frac{1}{2} \sin^2(t) + \int_0^t f_1(s) f_2(s) ds \end{array} \right\}.$$

Inputs are $n = 2$ and $m = 6$.

The results and the exact solutions are plotted in Fig1 – 1 and Fig 1 – 2.

Example2:

In the second example we solve the following nonlinear system of three integral equations. the exact solutions are $f_1(x) = \ln(x)$, $f_2(x) = x$ and $f_3(x) = x^2$.

$$\left\{ \begin{array}{l} f_1(t) = \ln(t) + \ln^2(t)t + t^2 + \int_0^t (4f_1(s)f_2(s)) ds \\ f_2(t) = t - \frac{1}{6} \ln(t)t^6 + \frac{1}{36} t^6 + \int_0^t (\frac{1}{3} s f_3^2(s) f_1(s)) ds \\ f_3(t) = t^2 + \frac{1}{15} t^5 + \int_0^t (s f_3(s) f_2(s)) ds \end{array} \right.$$

Inputs are $m = 3$ and $n = 6$. The results and the exact solutions are plotted in Fig2-1, Fig 2-2 and Fig 2-3.

Example 3:

In the third example we solve the following nonlinear system of four integral equations. the exact solutions are $f_1(x) = x$, $f_2(x) = x^2$, $f_3(x) = x^3$ and $f_4(x) = x^4$.

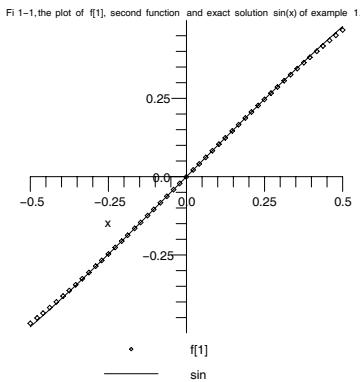


FIGURE 1. the plots of the exact solution, $\sin(x)$ and the approximated solution, $f[1]$ for example 1.

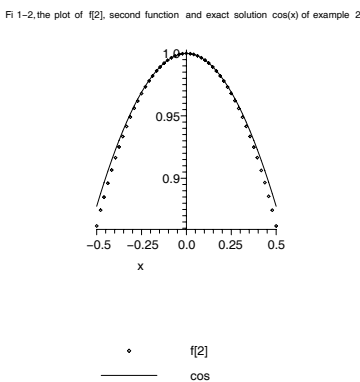


FIGURE 2. the plots of the exact solution, $\cos(x)$ and the approximated solution, $f[2]$ for example 1.

$$\begin{cases} f_1(t) = t + t^4 + \int_0^t (4f_1(s)f_2(s))ds \\ f_2(t) = t^2 - t^8 + \int_0^t 8f_4(s)f_3(s)ds \\ f_3(t) = \frac{2}{3}t^2 - \frac{1}{2}t^2 - \frac{1}{2}t^2 + \int_0^t (f_1(s) + f_2(s) + f_4(s))ds \\ f_4(t) = t^4 - t^6 + \int_0^t (6f_2(s)f_3(s))ds \end{cases}$$

Inputs are $n = 4$ and $m = 6$. The results and the exact solutions are plotted in Fig2-1, Fig 2-2 and Fig 2-3.

4. DISCUSSION

The goal of this work was to write a Maple program for solving systems of Volterra integral equations. We have achieved the goal successfully. The program accepts the number of the equations in the system of integral equations and the number of terms in the series form solution.

Those are familiar with Adomian decomposition method are well aware of the amount of computational work of this method.

Fi 2-1,the plot of f[1] , second function and exact solution ln(x) of example 2.

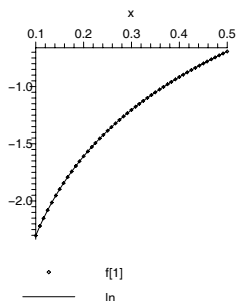


FIGURE 3. the plots of Ex.2, $f[1]$ approximated solution, $\ln(x)$ the exact solution.

Fi 2-2,the plot of f[2], second function and exact solution x of example 2.

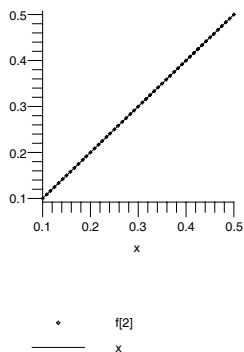


FIGURE 4. the plots of Ex.2, $f[2]$ approximated solution, x the exact solution.

Fi 2-3,the plot of f[3], second function and exact solution x^2 of example 2.

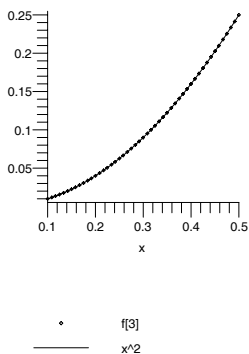


FIGURE 5. the plots of the exact solution, x^2 and the approximated solution, $f[3]$ for example 2.

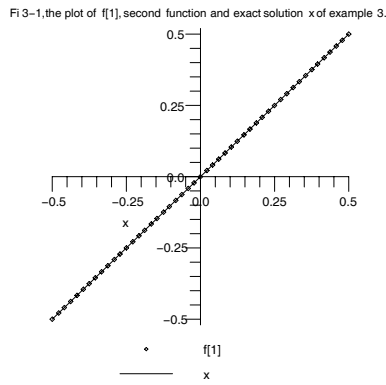


FIGURE 6. the plots of the exact solution, x and the approximated solution, $f[1]$ for example 3.

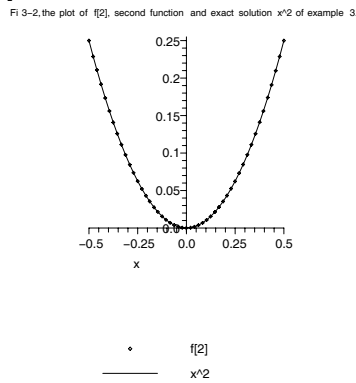


FIGURE 7. the plots of Ex.3, $f[2]$ approximated solution, x^2 the exact solution.

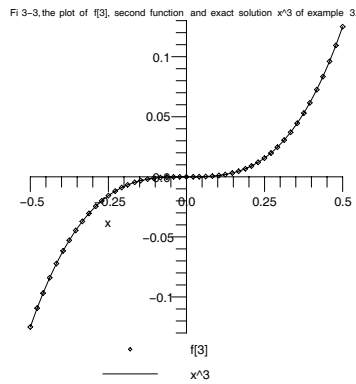


FIGURE 8. the plots of Ex.3, $f[3]$ approximated solution, x^3 the exact solution.

Having such a program enable us to have a solutions easily and there is no need to do that much computations.

A computational program, with maple, has been written by the second author. Our research group are working of maple program for solving system of

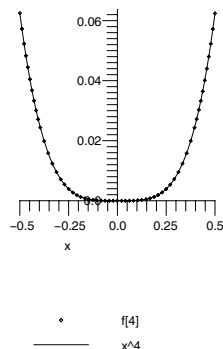
Fi.3-4, the plot of $f[4]$, second function and exact solution x^4 of example 3.

FIGURE 9. the plots of the exact solution, x^4 and the approximated solution, $f[4]$ for example 3.

differential and integro differential equations.

REFERENCE

- [1] G. Adomian, Nonlinear stochastic systems and Applications to Physics, Kluwer,(1989).
- [2] G. Adomian, G.E. Adomian, A global method for solution of complex systems, Math. Model, 5 (1984) 521-568.
- [3] E. Babolian and J. Biazar, Solution of a system of nonlinear Volterra integral equations of the second kind, Far East J. Math. Sci. 2(6) (2000) pp.935-945.
- [4] J. Biazar, E. Babolian, R. Islam, Solution of a system of ordinary differential equations with Adomian decomposition method, Appl. Math. Comput., 147(3) (2004) 731-719.
- [5] J. Biazar, Solution of a system integral- differential equations by Adomian decomposition method, Appl. Math. Comput. 168 (2005) 1232-1238.