An Interactive Approach for Solving Nash Cooperative Continuous Static Games (NCCSG)

M. M. K. ElShafei

Department of Mathematics
Faculty of Science, Helwan University
Ain- Helwan, Cairo, Egypt
mervat_elshafei@yahoo.com

Abstract
This paper, presents a solution method for Nash cooperative continuous static games (which is another type of continuous static games are constructs in this paper) by using interactive approach. This is achieved by using the method of compromise programming and the method of compromise weights from the pay-off table of membership function for each cost function. Also we obtain the stability set of the first kind for the solution. The method, called interactive stability compromise programming (ISCP).

Keywords: Game theory, Continuous static games, Interactive decision making, compromise weights, the stability set of the first kind

1. Introduction
Many decision making problems that arise in the real world need to be modeled as vector optimization, continuous static games are another formulations
of vector optimization problem [3] by considering the more general case of multiple decision makers, each with their own cost criterion. This generalization introduces the possibility of competition among the system controllers, called "players" and the optimization problem under consideration is therefore termed a "game". Each player in the game controls a specified subset of the system parameters (called his control vector) and seeks to minimize his own scalar cost criterion, subject to specified constraints. Several solution concepts are possible as Nash equilibrium concept, Pareto-minimal concept, min-max concept, min-max counterpoint concept, and Stackelberg leader-follower concept [3].

i) Nash equilibrium solution
The player act independently, without collaboration with any of the other players, and that each player seek to minimize his cost function. The information available to each player consists of the cost functions and consists for each player.

ii) Min-Max solutions
Each player chooses his control under the assumption that all of the other players have formed a coalition to maximize his cost. The information available to each player consists of his cost function and constraints for each player.

iii) Min-Max counter point solutions
One of the players has complete knowledge of the cost functions and constraints for the other players and seeks to minimize his own cost, assuming that the other players select min-max controls.

iv) Pareto-minimal solution
Cooperation among all the players is possible. It is assumed that each player helps the others up to the point of disadvantage to himself.

v) Stackelberg leader-follower solution
One player (the leader) announces his control first. Then the remaining players (the followers) announce their composite control simultaneously.

In practice most decision problems have multiple objectives conflicting among themselves. The solution for such problems can only be obtained by trying to get
Interactive approach

compromises based on the information provided by the decision maker (DM). Several methods have been developed to solve multiobjective decision making (MODM) problems [8]. In [6] some of these methods are based on prior information required from the DM. This information may be in the form of the desired achievement levels of the objective functions and the ranking of the levels indicating their importance, such as in goal programming. It may also be in the form of weights showing the importance of the objectives. The disadvantage with these methods is that the DM cannot easily provide this prior information since he has no idea about the solution process of the problem. Other methods, called interactive methods have been developed in order to overcome this disadvantage [9, 5, 6, 7]. There are two categories of interactive methods. Interactive methods of the first type require the DM to provide some trad-offs among the attained values of the objective functions in order to determine the new solution [1]. The interactive methods of the second type require the DM to provide some preference information by comparing the various efficient solutions in the space of the objective functions or the decision variables [10]. The quantity and complexity of the information required from the DM in such methods are important factors affecting the chances of reaching the best compromise solution.

This paper, presents formulation of another type of continuous static games called Nash cooperative continuous static games (NCCSG) in which the players are divided into two groups, each one is cooperative and both are playing according to Nash equilibrium solution concept. Also, an interactive stability compromise programming (ISCP) method for solving this type of games is introduced. Finally an algorithm to clarify this interactive approach is introduced.

2. Problem formulation

Let us consider the following Nash cooperative continuous static games (NCCSG) problem, each player \( i = 1, \ldots, r \), selects his control vector \( u_i \in R^n \) seeking to minimize a scalar-valued criterion.
Subject to \( n \) equality constraints
\[
g(x,u) = 0 \quad (1.2)
\]
where \( x \in \mathbb{R}^n \) is the state vector and \( u = (u_1, \ldots, u_r) \in \mathbb{R}^r \),
\[s = s_1 + s_2 + \ldots + s_r,\]
is the composite control. The composite control is required
to be an element of a regular control constraint set \( \Omega \subseteq \mathbb{R}^s \) of the form
\[
\Omega = \{ u \in \mathbb{R}^s \mid h(x,u) \geq 0 \} \quad (1.3)
\]
where \( x = \xi(u) \) is the solution to (1.2) given \( u \). The functions \( G_i(x,u) : \mathbb{R}^n \times \mathbb{R}^r \to \mathbb{R}^1 \), \( g(x,u) : \mathbb{R}^n \times \mathbb{R}^r \to \mathbb{R}^n \), and \( h(x,u) : \mathbb{R}^n \times \mathbb{R}^r \to \mathbb{R}^q \) are assumed to be \( C^1 \), with
\[
\left| \frac{\partial g(x,u)}{\partial x} \right| \neq 0, \quad (1.4)
\]
in a ball about a solution point \((x,u)\).

A coalition \( T_1 = \{1, 2, \ldots, m\} \subset \{1, 2, \ldots, m, m+1, \ldots, r\} \) (the set of all players) is
formed and another coalition \( T_2 = \{m+1, \ldots, r\} \) is formed by the other players.
Therefore pareto-optimal control of each coalition must be appearing, where
coopera/tion among all of the players is possible in each coalition. It is assumed
that each player in coalition \( T_1 \) helps the others up to the point of disadvantage to
himself also each player in coalition \( T_2 \) helps the others up to the point of
disadvantage to himself. Both two coalitions are playing according to Nash
equilibrium solution concept, without collaboration with any of the other
coalition.

Let \( u = (u_1, u_2, \ldots, u_m) \in \mathbb{R}^m \) be a composite control for coalition \( T_1 \) and
\[\nu = (u_{m+1}, u_{m+2}, \ldots, u_r) \in \mathbb{R}^{r-m} \]
denote the composite control for the other
coalition \( T_2 \). The composite control \((u, \nu) \in \mathbb{R}^s \).

Pareto- minimal solutions for coalition \( T_i \) may be attained by seeking to:
\[
\text{Min } G_i(\eta, x, u, \nu) = \sum_{i=1}^{m} \eta_i G_i(x, u, \nu)
\]
S.T \( (1.5) \)
Interactive approach

\[ g(x,u) = 0 , \]
\[ \Omega = \{ u \in R^s \mid h(x,u) \geq 0 \} , \]
\[ \eta_i \geq 0, \sum_{i=1}^{r} \eta_i = I , \quad i = 1, \ldots, m . \]

Also pareto-minimal solutions for coalition \( T_2 \) may be attained by seeking to:

\[
\text{Min } G_i (\eta, \xi (u, \nu), u, \nu) = \sum_{i=m+1}^{r} \eta G_i (\xi (u, \nu), u, \nu)
\]
S.T
\[
g(x,u) = 0
\]
\[
\Omega = \{ u \in R^s \mid h(x,u) \geq 0 \}
\]
\[ \eta_i \geq 0, \sum_{i=m+1}^{r} \eta_i = I , \quad 0 \leq i = m + 1, \ldots, r . \]

Definition 1

A point \((u^*, \nu^*) \in \Omega\) is a pareto-minimal solution for any coalition \((T_1\) or \(T_2\)) if and only if there does not exist a \((u, \nu) \in \Omega\) such that

\[ G_i (\xi (u, \nu), u, \nu) \leq G_i (\xi (u^*, \nu^*), u^*, \nu^*) , \]

for all \( i \in \{1, \ldots, m\} \) for coalition \(T_1\) or for all \( i \in \{m+1, \ldots, r\} \) for coalition \(T_2\) and

\[ G_j (\xi (u, \nu), u, \nu) < G_j (\xi (u^*, \nu^*), u^*, \nu^*) , \]

for some \( j \in \{1, \ldots, m\} \) for coalition \(T_1\) or for some \( j \in \{m+1, \ldots, m\} \) for coalition \(T_2\).

Theorem 1

If \((u^*, \nu^*) \in \Omega\) is a regular local pareto-minimal solution for any coalition \(T_1, \ell = 1\) or 2 and if \(x^* = \xi (u^*, \nu^*)\) is the corresponding solution to \(g (x, u^*, \nu^*) = 0\), then there exist vectors \(\eta \in R^r, \lambda \in R^n, \phi \in R^d\) and \(\phi \geq 0\), such that

\[
\frac{\partial L_i (x^*, u^*, \nu^*, \eta, \lambda, \phi)}{\partial x} = 0
\]
\[
\frac{\partial L_i (x^*, u^*, \nu^*, \eta, \lambda, \phi)}{\partial u} = 0
\]
\[ \frac{\partial L_i(x^*, u^*, \nu^*, \eta, \lambda, \phi)}{\partial \nu} = 0 \]

\[ g(x^*, u^*, \nu^*) = 0 \]

\[ \phi h(x^*, u^*, \nu^*) = 0 \]

\[ h(x^*, u^*, \nu^*) \geq 0 \]

\[ \eta_i \geq 0, \sum_{i \in T_l} \eta_i = 1, \quad \ell = 1 \, \text{or} \, 2 \]

Where,

\[ L_i(x, u, \nu, \eta, \lambda, \phi) = \sum_{i \in T_l} \eta_i^T G_i(x, u, \nu) - \sum_{j=1}^{n} \lambda_j^T g_j(x, u, \nu) - \sum_{k=1}^{q} \phi_k^T h_k(x, u, \nu). \]

Zeleny [8] has suggested that the set of efficient solutions can be reduced by introducing "the compromise set" concept. To obtain the compromise solution for each coalition in problems (1.5) and (1.6), find the solution which has a minimum distance with respect to the solution \( G_i^l(\xi (u, \nu), u, \nu), i \in T_l, \ell = 1, 2 \). This idea requires normalization of the objective functions and appropriate choice for the distance measure. The solutions found in this way are a reduced set of all efficient solutions. The set of compromise solutions may be large, and also the choice of weights by players in each coalition may be difficult. These difficulties could be reduced by combining the basic ideas for the methods of compromise programming and compromise weights.

### 3. Compromise weights for coalition \( T_1 \) and \( T_2 \)

The interactive compromise Programming (ICP) method is based on two main ideas:

First, the players in coalition \( T_1, \ell = 1, 2 \) could state his preference among some alternative solutions more easily if the values of cost functions were
Interactive approach

measured on the same scale varying between zero and one. This could be done by employing "the membership functions" for the cost functions concept in the compromise programming for each coalition. In this method, the following definition of the membership functions is used for scaling for problem (1.5) and (1.6) of coalition $T_1$ and $T_2$:

$$
\mu_{G_i}(\xi(u, v), u, v) = \frac{G_i(\xi(u, v), u, v) - G_i^L}{G_i^U - G_i^L}, \quad i \in T_\ell, \quad \ell = 1 \text{ for coalition } T_1 \text{ and } \ell = 2 \text{ for coalition } T_2,
$$

(1.7)

where $G_i(\xi(u, v), u, v)$ are the cost functions, $G_i^U$ are the maximum possible values of $G_i(\xi(u, v), u, v)$, and $G_i^L$ are the minimum possible values of $G_i(\xi(u, v), u, v)$ satisfying the constraints $\Omega$, $i \in T_\ell, \ell = 1, 2$.

The $\mu_{G_i}(\xi(u, v), u, v)$ are defined as the membership functions of $G_i(\xi(u, v), u, v)$ to the minimum possible value $G_i(\xi(u, v), u, v), i \in T_\ell, \ell = 1, 2$.

The scalarization problem for coalition $T_1$ is proposed as the following problem:

Min $\mu_{G_i}(\xi(u, v), u, v) = \sum_{i=1}^{n} \eta_i \mu_{G_i}(\xi(u, v), u, v)$

S.T

\[ g(x, u, v) = 0 \]

$$
\Omega = \{ u \in R^l \mid h(x, u, v) \geq 0 \} \tag{1.8}
$$

$$
\eta_i \geq 0, \quad \sum_{i=m}^{r} \eta_i = 1, \quad i = 1, ..., m.
$$

Also the scalarization problem for coalition $T_2$ is:

Min $\mu_{G_i}(\xi(u, v), u, v) = \sum_{i=m+1}^{r} \eta_i \mu_{G_i}(\xi(u, v), u, v)$

S.T

\[ g(x, u, v) = 0 \]

$$
\Omega = \{ u \in R^l \mid h(x, u, v) \geq 0 \} \tag{1.9}
$$
\[ \eta_i \geq 0, \quad \sum_{i=m+1}^r \eta_i = 1, \quad i = m+1, \ldots, r. \]

The second main idea, one of the main drawbacks of the interactive methods is the difficulty of getting the weights of the cost function from the players even if the values of the cost functions are presented to him on the same scale. In this method, the compromise weights of the cost functions can be obtained by means of the pay-off matrix \( p \) of order \( mxm \) of coalition \( T_1 \) and \( mxr \) of coalition \( T_2 \) of which \( m, r \) successive columns show the effects of the \( i \) instrument vector \( (x_i^*, u_i^*, v_i^*) \) on the membership cost functions for coalition \( T_1 \) and \( T_2 \) respectively:

\[
\begin{align*}
P_{t_1} &= (\overline{\mu}_G(x_1^*, u_1^*, v_1^*), \ldots, \overline{\mu}_G(x_m^*, u_m^*, v_m^*)), \\
P_{t_2} &= (\overline{\mu}_G(x_{m+1}^*, u_{m+1}^*, v_{m+1}^*), \ldots, \overline{\mu}_G(x_r^*, u_r^*, v_r^*)).
\end{align*}
\]

The compromise weights \( \eta_i, \quad i \in T_\ell, \quad \ell = 1, 2 \) (for each coalition) can be obtained from the normalized version of the pay-off matrix \( P \) as in the form:

\[
\eta_i = \frac{(P_{t_\ell}^T)^{-1} L_i}{L (P_{t_\ell}^T)^{-1} L_i}, \quad i \in T_\ell, \quad \ell = 1 \text{ for coalition } T_1, \text{ and } \ell = 2 \text{ for coalition } T_2 ,(1.10)
\]

where \( L_i \) is the unit vector and \( L_i^T \) is the transpose unit vector. Also \( P_{t_\ell}^T \) is the transpose pay-off matrix \( p \).

The process is terminated when one of the following occures:

1) The players in coalition \( T_\ell, \quad \ell = 1, 2 \) satisfied with the current solution.

2) The inverse of matrix \( p \) does not exist in this case the original set of normalized weights \( \eta_i, \quad i = 1, \ldots, m \) (for coalition \( T_1 \)) is computed from another formula \([4]\) :

\[
\eta_i = \frac{e^{a_i}}{\sum_{j=1}^m e^{a_j}}, \quad i = 1, \ldots, m,
\]

where
\[ \alpha_1 = \frac{1}{a_m - a_{m-1}} \ln \left| \sum_{i=1}^{m} \frac{a_i}{a_m} \right|, \quad (1.11) \]

\[ a_i = \hat{G}_i - \hat{G}_i^*, \quad i \in T_1, \quad i = \{1, \ldots, m\}, \]

\[ \hat{G}_i = \max G_i (\xi (u^i, v^i), u^i, v^i), \quad \hat{G}_i = \min G_i (\xi (u^i, v^i), u^i, v^i), \]

also the original set of normalized weights \( \eta_i, \quad i = m+1, \ldots, r \) (for coalition \( T_2 \)) computed from :

\[ \eta_i = \frac{e^{\alpha_2 u_i}}{\sum_{i \in T_2} e^{\alpha_2 u_i}}, \quad i \in T_2, \]

where

\[ \alpha_2 = \frac{1}{a_r - a_{r-1}} \ln \left| \sum_{i=1}^{r} \frac{a_i}{a_r} \right|, \quad (1.12) \]

\[ a_i = \hat{G}_i - \hat{G}_i^*, \quad i \in T_2 = \{m+1, \ldots, r\}, \]

\[ \hat{G}_i = \max G_i (\xi (u^i, v^i), u^i, v^i), \quad \hat{G}_i = \min G_i (\xi (u^i, v^i), u^i, v^i). \]

4. Stability set of the first kind

**Definition 2**

The solvability set of coalition \( T_1 \) for problem (1.5) is defined by:

\[ B_{i_1} = \{ \eta \in \mathbb{R}^m | \min G_{i_1} (\eta, \xi (u, v)) u, \psi \} \text{ exists} \}, \]

also the solvability set of coalition \( T_2 \) for problem (1.6) is defined by :

\[ B_{i_2} = \{ \eta \in \mathbb{R}^{s-m} | \min G_{i_2} (\eta, \xi (u, v)) u, \psi \} \text{ exists} \}. \]
Definition 3

Suppose that $B_\ell \neq \emptyset$ for coalition $T_\ell$, ($\ell = 1$ or 2) with a corresponding pareto-minimal solution $(\bar{x}, \bar{u}, \bar{v})$, then the stability set of the first kind of coalition $T_\ell$ corresponding to $(\bar{x}, \bar{u}, \bar{v})$ is defined by

$$S_\ell (\bar{x}, \bar{u}, \bar{v}) = \{ \eta \in B_\ell | \sum_{i \in T_\ell} \eta_i G_i(\bar{x}, \bar{u}, \bar{v}) = \min \sum_{i \in T_\ell} \eta_i G_i(\bar{x}, \bar{u}, \bar{v}) \}, \quad \ell = 1 \text{ for coalition } T_1 \text{ and } \ell = 2 \text{ for coalition } T_2.$$

It is clear that the stability set of the first kind is the set of all parameters corresponding to pareto-minimal solution of the scalarization problem (1.5) for coalition $T_1$ or problem (1.6) for coalition $T_2$.

To determine the stability set of the first kind $S_\ell (x^*, u^*, \nu^*)$ for coalition $T_1$ or the stability set of the first kind $S_\ell (x^*, u^*, \nu^*)$ for coalition $T_2$ substituting in the system of equations given by theorem 1.1, we obtain the set

$$D_\ell = \{ (\eta, \lambda, \phi) | \sum_{i \in T_\ell} \eta_i \frac{\partial G_i(x^*, u^*, \nu^*)}{\partial x} - \sum_{j=1}^{n} \lambda_j \frac{\partial g_j(x^*, u^*, \nu^*)}{\partial x} - \sum_{k=1}^{q} \phi_k \frac{\partial h_k(x^*, u^*, \nu^*)}{\partial x} = 0, \quad \ell = 1 \text{ for coalition } T_1 \text{ and } \ell = 2 \text{ for coalition } T_2 \}.$$

(1.13)

For coalition $T_1$ this system represents $n + r$ linear equations in $m + n + q$ unknowns $\eta_i, i = 1, \ldots, m$, $\lambda_j, j = 1, \ldots, n$, and $\phi_k, k = 1, \ldots, q$ which can be solved
and for coalition $T_2$ this system represents $n + r$ linear equations in $s \cdot m + n + q$ unknowns $\eta_i,$ $i = m + 1, \ldots, r,$ $\lambda_j,$ $j = 1, \ldots, n,$ and $\phi_k,$ $k = 1, \ldots, q$ Which can be solved. The stability set of the first kind for coalition $T_1$ and for coalition $T_2$ is

\begin{equation}
S_{i_1} (x^*, u^*, \nu^*) = \{ \eta \in R^m | (\eta, \lambda, \phi) \in D_{i_1} \},
\end{equation}

\begin{equation}
S_{i_2} (x^*, u^*, \nu^*) = \{ \eta \in R^{s \cdot m} | (\eta, \lambda, \phi) \in D_{i_2} \},
\end{equation}

respectively.

5. Nash cooperative solution for coalition $T_1$ and coalition $T_2$

After obtaining the compromise weights $\eta_i^*, i = 1, \ldots, m$ which obtain the best compromise solution for coalition $T_1$ and $\eta_i^*, i = m+1, \ldots, r$ which obtain the best compromise solution for coalition $T_2$, the two coalition are playing according to the Nash equilibrium solutions concept by solving:

\begin{equation}
\text{Min} \quad G_{i_1} (x, u, \nu) = \sum_{i \in i_1} \eta_i^* G_i (x, u, \nu), \quad \ell = 1, 2
\end{equation}

S.T

\begin{equation}
g(x, u, \nu) = 0
\end{equation}

\begin{equation}
\Omega = \{ u \in R^t | h (x, u, \nu) \geq 0 \}
\end{equation}

Definition 4

A point $\hat{u} \in \Omega$ is a Nash cooperative point for problems (1.16) if and only if,

\begin{equation}
G_{i_1} (\xi (\hat{u} ), \hat{u} ) \leq G_{i_1} (\xi (u, \nu), u, \nu) \quad \text{for coalition } T_1 \text{, and}
\end{equation}

\begin{equation}
G_{i_2} (\xi (\hat{u} ), \hat{u} ) \leq G_{i_2} (\xi (u, \nu), \hat{u}, \nu) \quad \text{for coalition } T_2 \text{, where } \hat{u} = (\hat{u}, \nu) \in \Omega
\end{equation}

Theorem 2

If $\hat{u} \in \Omega$ is a completely regular local Nash cooperative solution for the game (1.16) and \( \hat{x} = \xi (\hat{u}) \) is the solution to $g(x, \hat{u}) = 0$, then for each coalition
There exists a vector $\lambda(t_\ell) \in \mathbb{R}^n$ and a vector $\mu(t_\ell) \in \mathbb{R}^q$, $\ell = 1, 2$ such that

$$\frac{\partial L_{\ell}}{\partial x} [\hat{x}, \hat{u}, \lambda(t_\ell), \mu(t_\ell)] = 0, \quad \ell = 1, 2$$

$$\frac{\partial L_{\ell}}{\partial u} [\hat{x}, \hat{u}, \lambda(t_\ell), \mu(t_\ell)] = 0,$$

$$\frac{\partial L_{\ell}}{\partial v} [\hat{x}, \hat{u}, \lambda(t_\ell), \mu(t_\ell)] = 0,$$

$$g(\hat{x}, \hat{u}) = 0$$

$$\mu^T(t_\ell) \ h((\hat{x}, \hat{u})) = 0$$

$$h((\hat{x}, \hat{u})) \geq 0$$

$$\mu(t_\ell) \geq 0$$

where

$$L_{\ell} [x,u, \lambda(t_\ell), \mu(t_\ell)] = G_{t_\ell}(x,u) - \lambda^T(t_\ell) g(x,u) -$$

$$\mu^T(t_\ell) h(x,u), \quad \ell = 1, 2.$$

### 6. The algorithm of interactive stability compromise programming (ISCP) for solving Nash cooperative continuous static games (NCCSG)

The steps of the algorithm can be summarized as follows:

**Step 1:** A coalition $T_1 = \{1, \ldots, m\} \subset \{1, 2, \ldots, r\}$ (the set of all players) is formed and another coalition $T_2 = \{m+1, \ldots, r\}$ is formed by the other players where cooperation among all of the players is possible in each coalition.

**Step 2:** Construct problem (1.5) and (1.6) for coalitions $T_1$ and $T_2$.

**Step 3:** An interactive stability compromise method is used for solving (NCCSG) problems as follows:

for coalition $T_\ell$, set $\ell = 1$, Determine $G_i^U$, $G_i^L$ for all $i \in T_\ell$ as follows:

(i) Max $G_i (\xi(u, v), u, v)$, $i \in T_\ell$
S.T

\[ g(x, u, \nu) = 0 \]
\[ \Omega = \{ u \in R^r \mid h(x, u, \nu) \geq 0 \} \]

The solutions of this problem are \( u^U_i \), \( \nu^U_i \) and \( G_i^U \).

(ii) Min \( G_i (\xi(u, \nu), u, \nu) \), \( i \in T_\ell \)

S.T

\[ g(x, u, \nu) = 0 \]
\[ \Omega = \{ u \in R^r \mid h(x, u, \nu) \geq 0 \} \]

The solution are \( u^{IL}_i \), \( \nu^{IL}_i \) and \( G_i^L \).

**Step 4:** Determine the membership functions corresponding the solution \( (u^{IL}_i, \nu^{IL}_i) \), \( i \in T_\ell \) as in relation (1.7). The pay off table can be arranged for coalition \( T_\ell \) (for coalition \( T_i \) using Table (1) and for coalition \( T_2 \) using Table (2)) as follows:-

<table>
<thead>
<tr>
<th>( G_1 )</th>
<th>( u^I ), ( \nu^I )</th>
<th>( u^2 ), ( \nu^2 )</th>
<th>( u^m ), ( \nu^m )</th>
<th>( G_i^L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_{G_1} )</td>
<td>( \mu_{G_1}^1 )</td>
<td>( \mu_{G_1}^2 )</td>
<td>( \mu_{G_1}^m )</td>
<td>( G_1^L )</td>
</tr>
<tr>
<td>( \mu_{G_2} )</td>
<td>( \mu_{G_2}^1 )</td>
<td>( \mu_{G_2}^2 )</td>
<td>( \mu_{G_2}^m )</td>
<td>( G_2^L )</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( \mu_{G_m} )</td>
<td>( \mu_{G_m}^1 )</td>
<td>( \mu_{G_m}^2 )</td>
<td>( \mu_{G_m}^m )</td>
<td>( G_m^L )</td>
</tr>
</tbody>
</table>

**Table(1) Pay-off table for coalition \( T_i \)**
where $\mu_{G_{i}}$ is the value of $\mu_{G_{i}}$ in $u^{i}$, $v^{i}$, $i \in T_{\ell}$.

**Step 5:** The compromise weights $\eta_{i}$, $i \in T_{\ell}$ can be found from the relation (1.10). If $(p_{i}^{l})^{-1}$ does not exist, the set of normalized weights $\eta_{i}$, $i \in T_{\ell}$ is computed from (1.11) if $\ell = l$ or from (1.12) if $\ell = 2$.

**Step 6:** By using these weights, we establish the new composite function to obtain the new alternative compromise solution, $(u^{m+1}, v^{m+1})$ for coalition $T_{I}$ from problem (1.8), $(u^{r+1}, v^{r+1})$ for coalition $T_{2}$ from problem (1.9).

**Step 7:** Determine the stability set of the first kind corresponding to this solution as in relations (1.13) and (1.14) for coalition $T_{I}$ and in relations (1.13) and (1.15) for coalition $T_{2}$.

**Step 8:** Determine the membership cost functions of the new solution of problem in step 6, $\mu_{G_{m+1}}$ for coalitions $T_{I}$ or $\mu_{G_{r+1}}$ for coalitions $T_{2}$. Add this column to the table in step 4.

**Step 9:** Ask the players in coalition $T_{\ell}$ whether he prefers one solution strictly over all the other $m$- solutions for coalition $T_{I}$ or $r$- solutions for coalition $T_{2}$. If he does, go to step 11. Otherwise ask him his least preferred solution among all the others. Then replace this preferred solution by the new found in step 8 and go to step 5.
**Step 10:** set \( \ell = 2 \) and repeat the steps from step 3 to step 9, and go to step 11.

**Step 11:** After obtaining the compromise weights for each coalition which obtain the best compromise solution for coalitions \( T_1 \) and \( T_2 \), the two coalitions are playing according to the Nash equilibrium solutions concept by solving problems (1.16). The solution is called Nash cooperative solution.

**Step 12:** Stop.

**7. Conclusion**

This paper proposed a method called interactive stability compromise programming for solving Nash cooperative continuous static games by using the method of compromise programming and the method of compromise weights from the pay-off table of membership function for each cost function in each coalition. Also we obtain the stability set of the first kind for the solution in each coalition.
References


Received: March 28, 2007