A Numerical Solution of the Lax’s 7th-order KdV Equation by Pseudospectral Method and Darvish’s Preconditioning

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Abstract

In this paper, we solve the Lax’s seventh-order Korteweg-de Vries (KdV) equation by pseudospectral method. To enhance the accuracy in matrix vector multiplication method we use Darvish’s preconditioning. We compare the numerical results with exact solution of the equation. They are in a good agreement with each other because absolute errors are very small.

Mathematics Subject Classification: 65N55

Keywords: Lax’s seventh-order KdV equation; Pseudospectral method; Darvish’s preconditioning

1 Introduction

The Lax’s seventh-order KdV equation is taken as [14]

\[ u_t + (35u^4 + 70(u^2u_{xx} + uu_x^2) + 7(2uu_{xxxx} + 3u^2_{xx} + 4u_xu_{xxx}) + u_{xxxxx})_x = 0. \] (1)

Equation (1) is a nonlinear problem. The study of nonlinear problems is of crucial importance in mathematics and physics. Many authors had paid attention to study the solution of nonlinear equations using different schemes. Among these methods, we can point to Backlund transformation [1], inverse scattering scheme [10], the tanh-function [13], Hirota’s bilinear method [12] and the homogeneous method [18]. Fan [9] described an extended tanh-function method and symbolic computation to solve the new coupled modified KdV

The aim of this study is to solve the Lax’s seventh-order KdV equation with known initial and boundary conditions by pseudospectral method using Darvishi’s preconditioning.

2 Pseudospectral method

We can approximate the arbitrary function \( f(x) \) by polynomials in \( x \). However, it is well known that the Lagrange interpolation polynomial based on equally spaced points does not give a satisfactory approximation to general smooth \( f \). In fact, it converges for analytic functions with poles far enough from the interval of interpolation. This poor behavior of polynomial interpolation can be avoided for smoothly differentiable functions by removing the restriction to equally spaced collocation points. Good results are obtained by relating the collocation points to the structure of classical orthogonal polynomials, like Chebyshev and Legendre polynomials. In the most common pseudospectral Chebyshev method, the interpolation points in the interval \([-1, 1]\) are the Chebyshev-Gauss-Lobatto collocation points \( x_j = \cos\left(\frac{j\pi}{N}\right) \) for \( j = 0, \ldots, N \), which are the extreme of the \( N \)th order Chebyshev polynomials \( T_N(x) = \cos(N\cos^{-1}x) \). In order to construct the interpolant of \( f(x) \) at the point \( x \) the following polynomials are defined

\[
g_j(x) = \frac{(-1)^{j+1}(1-x^2)T''_N(x)}{c_jN^2(x-x_j)}, \quad j = 0, \ldots, N, \tag{2}
\]

where \( c_0 = c_N = 2 \) and \( c_j = 1 \) for \( j = 1, 2, \ldots, N-1 \). The interpolation polynomial, \( P_N(x) \), to \( f(x) \) is given by

\[
P_N(x) = \sum_{j=0}^{N} f(x_j)g_j(x). \tag{3}
\]

Then to obtain a pseudospectral approximation we have to express the derivatives of \( P_N(x) \) in terms of \( f(x) \) at collocation points \( x_j \). This can be done by differentiating (3), that is

\[
\frac{d^r P_N(x)}{dx^r} = \sum_{j=0}^{N} f(x_j)g_j^{(r)}(x), \tag{4}
\]

so that

\[
\frac{d^r P_N(x_k)}{dx^r} = \sum_{j=0}^{N} f(x_j)d_k^{(r)}, \tag{5}
\]
where

\[ d_{kj}^{(r)} = g_j^{(r)}(x_k), \]

are the elements of differentiation matrix \( D_r \). The elements of \( D_r \) can be obtained analytically. For example if \( r = 1 \) and \( j, k = 0, \ldots, N \) elements of \( D \) are [11]

\[
\begin{align*}
  d_{kj} &= \frac{a_k}{c_j} \frac{(-1)^{j+k}}{(x_k - x_j)}, & k \neq j \\
  d_{kk} &= -\frac{1}{2} \frac{(1-x_k^2)}{x_k}, & k \neq 0, N \\
  d_{00} &= \frac{2N^2+1}{6}, \\
  d_{NN} &= -\frac{2N^2-1}{6}.
\end{align*}
\]

One of the basic steps in pseudospectral methods involves finding an approximation for the differential operator in terms of the grid point values of \( u_N \). The derivative of \( u(x) \) at collocation points \( x_j \) can be computed using the matrix-vector multiplication method.

### 2.1 Matrix-Vector multiplication method

If \( u = \{u(x_i)\} \) is the vector consisting of values of \( u(x) \) at the \( N+1 \) collocation points and \( u' = \{u'(x_i)\} \) consists of values of the derivative at the collocation points, then the collocation derivative matrix \( D \) is the matrix mapping \( u \mapsto u' \). Two matrix multiplications yield \( u'' \), where \( u'' \) is the vector containing the second derivatives evaluated at the collocation points. More efficiently, the matrix \( D^2 = D_2 \) maps \( u \mapsto u'' \), and so on. Collocation matrix methods are asymptotically less efficient than the Chebyshev transform method (see [16]), because it needs \( O(N^2) \) operations. It can be shown that the Chebyshev derivative, when computing the derivative using the matrix vector multiplication method, is a rather ill-conditioned operator, and inaccuracies in the function can be magnified by as much as \( O(N^4) \) where \( N \) is the number of collocation points. Hence attempts have been made to improve the method. These studies have concentrated on the problem of the roundoff error in Chebyshev collocation methods and various algorithms have been suggested to reduce it. The best result for the matrix vector multiplication algorithm managed to reduce the roundoff error from \( O(N^4\varepsilon) \) to \( O(N^3\varepsilon) \), where \( \varepsilon \) is the machine precision [7, 8].

Some researchers have studied the problem of reducing roundoff errors in Chebyshev collocation derivative methods. Baltensperger and Trummer [3] demonstrated that naive algorithms for computing these matrices suffer from severe loss of accuracy due to roundoff errors. Breuer and Everson [5] introduced a preconditioning to reduce roundoff error by making the value of the function on the boundaries vanish. Tang and Trummer [17] used trigonometric identities and a flipping trick to reduce roundoff errors. A different approach was suggested in [2, 4].
Don and Solomonoff attempted to reduce the roundoff error using trigonometric identities for rewriting components of the derivative matrix as follows [7]:

\[
\begin{align*}
    d_{kj} &= -\frac{1}{2} \frac{c_k}{c_j} \sin\left(\frac{(k+j)}{2N}\right) \sin\left(\frac{(k-j)}{2N}\right), \\
    d_{kk} &= -\frac{1}{2} \frac{x_k}{\sin^2\left(\frac{k\pi}{2N}\right)}, \\
    d_{00} &= \frac{2N^2+1}{6}, \\
    d_{NN} &= -\frac{2N^2+1}{6}.
\end{align*}
\]

Formula (8), which avoids the differencing of nearly equal numbers was introduced to reduce this source of error from \(O(N^4\epsilon)\) to \(O(N^3\epsilon)\).

**Preconditioning.** From (8) for \(k \neq j\) we have

\[
|d_{kj}| = \frac{c_k}{2c_j} \frac{1}{|\sin((k+j)\frac{\pi}{2N})\sin((k-j)\frac{\pi}{2N})|}
\]

since the value of \(\sin((k-j)\frac{\pi}{2N})\) is near zero when \(k\) is near \(j\), hence this causes \(|d_{kj}|\) to become large for \(k\) near \(j\). This means that the entries of derivative matrix \(D\) with large absolute values are on a band near the main diagonal. That is, large values of \(|d_{kj}|\) correspond to values of \(k\) near \(j\). Therefore, the matrix vector multiplication method causes large roundoff errors. In [6], to reduce roundoff error in the \(k\)th node, Darvishi and Ghoreishi defined \(h_k(x)\) as follows:

\[
h_k(x) = u(x) - u(x_k),
\]

where \(x_k = \cos(\frac{k\pi}{2N})\). We can interpolate \(h_k(x)\) as

\[
h_k(x) = \sum_{j=0}^{N} g_j(x) h_k(x_j)
\]

hence from (4)-(6) the derivative of \(h_k\) at \(x_k\) is

\[
h'_k(x_k) = \sum_{j=0}^{N} d_{kj} h_k(x_j)
\]

or as \(h' = u'\) we have

\[
u'(x_k) = \sum_{j=0}^{N} d_{kj} (u(x_j) - u(x_k)).
\]

Therefore, using this preconditioning, we can vanish the influence of large values of \(|d_{kj}|\), that is \(|d_{kk}|\) for all \(k\) in the matrix vector multiplication method.
3 Governing equation

The Lax’s seventh-order equation is defined as

\[ u_t + (35u^4 + 70(u^2_{xx} + uu_x^2) + 7(2uu_{xxxx} + 3u_{xx}^2 + 4u_xu_{xxx}) + u_{xxxxxx})_x = 0, \]

where \( a \leq x \leq b \), \( t \geq 0 \) \hspace{1cm} (14)

where \( a \) and \( b \) are real numbers. The initial and boundary conditions of equation (14) are taken as

\[ u(x, 0) = 2k^2 \text{sech}^2(kx) = H(x) \]
\[ u(a, t) = 2k^2 \text{sech}^2(k(a - 64k^6t)) = g_1(t) \]
\[ u(b, t) = 2k^2 \text{sech}^2(k(b - 64k^6t)) = g_2(t). \]

The exact solution of equation (14) is taken as

\[ u(x, t) = 2k^2 \text{sech}^2(k(x - 64k^6t)), \]

where \( k \) is a known constant [15]. To solve equation (14) by pseudospectral method we discretize the equation in space

\[ \frac{\partial u}{\partial t} = -(70D^3u(D^2u + u^2) + 140uD(2D^2u + u^2) + 42DuD^4u - 70(Du)^3 + 14uD^5u + D^7u) \]

where \( D \) is the differentiation matrix and if \( Dr \) is the matrix mapping \( u \mapsto u^{(r)} \), then we have \( Dr = D^r \), [11]. From equation (19) in the \( k \)th collocation point we have

\[ \frac{\partial u}{\partial t}(x_k, t) = -(70 \sum_{j=0}^{N} d_{kj}^{(3)} u(x_j, t) \{ \sum_{j=0}^{N} d_{kj}^{(2)} u(x_j, t) + u^2(x_k, t) \} + 140u(x_k, t) \sum_{j=0}^{N} d_{kj} u(x_j, t) \{ 2 \sum_{j=0}^{N} d_{kj}^{(2)} u(x_j, t) + u^2(x_k, t) \} + 42 \sum_{j=0}^{N} d_{kj} u(x_j, t) \sum_{j=0}^{N} d_{kj}^{(4)} u(x_j, t) + 70 \{ \sum_{j=0}^{N} d_{kj} u(x_j, t) \}^3 + 14u(x_k, t) \sum_{j=0}^{N} d_{kj}^{(7)} u(x_j, t) + \sum_{j=0}^{N} d_{kj}^{(5)} u(x_j, t) \} \}
\]

where \( D^r = (d_{kj}^{(r)}) \) for \( r = 2, 3, \ldots, 7 \) and \( D = D_1 \). For simplicity we set \( u(x_k, t) = v_k(t), \ k = 0, \ldots, N \). Hence \( v_k(t) \) is a function of \( t \), where \( v_0(t) \) and \( v_N(t) \) are boundary functions, namely \( g_1(t) \) and \( g_2(t) \). Equation (20) can be rewritten as

\[ v'_k(t) = -(70 \sum_{j=0}^{N} d_{kj}^{(3)} v_j(t) \{ \sum_{j=0}^{N} d_{kj}^{(2)} v_j(t) + v_k^2(t) \} + 140v_k(t) \sum_{j=0}^{N} d_{kj} v_j(t) \{ 2 \sum_{j=0}^{N} d_{kj}^{(2)} v_j(t) + v_k^2(t) \} + 42 \sum_{j=0}^{N} d_{kj} v_j(t) \sum_{j=0}^{N} d_{kj}^{(4)} v_j(t) + 70 \{ \sum_{j=0}^{N} d_{kj} v_j(t) \}^3 + 14v_k(t) \sum_{j=0}^{N} d_{kj}^{(5)} v_j(t) + \sum_{j=0}^{N} d_{kj}^{(7)} v_j(t) \}, \ k = 1, \ldots, N - 1 \]

(21)
Preconditioned system. The $k$th row of the preconditioned system of equation (20) by Darvishi’s preconditioning is as follows

$$
\frac{d}{dt}v_k(t) = -(70 \sum_{j=1}^{N-1} d^{(3)}_{kj}(v_j(t) - v_k(t))\{\sum_{j=1}^{N-1} d^{(2)}_{kj}(v_j(t) - v_k(t)) + v_k^2(t)\}
+ 140v_k(t)\sum_{j=1}^{N-1} d_{kj}(v_j(t) - v_k(t))\{2 \sum_{j=1}^{N-1} d^{(2)}_{kj}(v_j(t) - v_k(t)) + v_k^2(t)\}
+ 42 \sum_{j=1}^{N-1} d_{kj}(v_j(t) - v_k(t))\sum_{j=1}^{N-1} d^{(4)}_{kj}(v_j(t) - v_k(t)) + 70\{\sum_{j=1}^{N-1} d_{kj}(v_j(t) - v_k(t))
- v_k(t)\}\}^3 + 14v_k(t)\sum_{j=1}^{N-1} d^{(5)}_{kj}(v_j(t) - v_k(t)) + \sum_{j=1}^{N-1} d^{(7)}_{kj}(v_j(t) - v_k(t)) -
(70\{d^{(3)}_{k0}(g_1(t) - v_k(t)) + d^{(2)}_{kN}(g_2(t) - v_k(t))\} + \{d^{(2)}_{k0}(g_1(t) - v_k(t)) + d^{(2)}_{kN}(g_2(t) - v_k(t))\})2d^{(2)}_{k0}
(g_1(t) - v_k(t)) + 2d^{(2)}_{kN}(g_2(t) - v_k(t)) + v_k^2(t)\} + 42\{d^{(2)}_{k0}(g_1(t) - v_k(t)) + d^{(4)}_{kN}(g_2(t) - v_k(t))\} + 70\{d_{k0}
(g_1(t) - v_k(t)) + d_{kN}(g_2(t) - v_k(t))\}\}^3 + 14v_k(t)\{d^{(5)}_{k0}(g_1(t) - v_k(t)) + d^{(5)}_{kN}(g_2(t) - v_k(t))\}
+ \{d^{(5)}_{k0}(g_1(t) - v_k(t)) + d^{(5)}_{kN}(g_2(t) - v_k(t))\}
\}^3 + 14v_k(t)\{d^{(5)}_{k0}(g_1(t) - v_k(t)) + d^{(5)}_{kN}(g_2(t) - v_k(t))\}
\} + \{d^{(5)}_{k0}(g_1(t) - v_k(t)) + d^{(5)}_{kN}(g_2(t) - v_k(t))\}
\} \quad k = 1, \ldots, N - 1.
$$

(22)

Note that system (22) is a system of ordinary differential equations (ODEs). Hence we can use any ODE solver to solve it. In this paper we use the standard fourth-order Runge-Kutta method.

4 Numerical results

In this section we solve equation (14) for different values of $a$ and $b$. Note that as Chebyshev-Gauss-Lobatto collocation points are in interval $[-1, 1]$, therefore we have to map interval $[a, b]$ to $[-1, 1]$ by a linear mapping. To demonstrate the efficiency of our method we report the absolute errors in some arbitrary points in tables 1 and 2. To obtain the numerical results we used MATLAB 7.0 software.

Problem 1. We solve equation (14) for $-a = b = 100$ and $N = 8$ collocation points. Numerical results are shown in Table 1.

Problem 2. We set $b = -a = 200$ and $N = 8$ collocation points. The results are shown in Table 2.
Table 1. Comparison of the exact and numerical solutions by pseudospectral method using Darvishi’s preconditioning for the Lax’s seventh-order KdV equation with $t = 2$, $N = 8$ and $\Delta t = 0.0001$.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$x$</th>
<th>Exact solution</th>
<th>Numerical solution</th>
<th>Abs. error</th>
</tr>
</thead>
<tbody>
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<td>0.05</td>
<td>-92.388</td>
<td>9.4082083361$E - 05$</td>
<td>9.4082521546$E - 05$</td>
<td>4.38184$E - 10$</td>
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<td>-38.268</td>
<td>1.7334939645$E - 04$</td>
<td>1.7334980144$E - 04$</td>
<td>4.04986$E - 10$</td>
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Table 2. Comparison of the exact and numerical solutions by pseudospectral method using Darvishi’s preconditioning for the Lax’s seventh-order KdV equation with \( t = 15 \), \( N = 8 \) and \( \Delta t = 0.01 \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>Exact solution</th>
<th>Numerical solution</th>
<th>Abs. error</th>
</tr>
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<td>1.1699424120, (-04)</td>
<td>1.44738, (-07)</td>
</tr>
<tr>
<td></td>
<td>(184.78)</td>
<td>1.8951025056, (-05)</td>
<td>1.8916435829, (-05)</td>
<td>3.45892, (-08)</td>
</tr>
</tbody>
</table>

5 Conclusion

In this study, the Lax’s seventh-order Korteweg-de Vries equation is solved by pseudospectral method using a preconditioning scheme. As numerical results show, the errors are very small.

References


Numerical solution of the Lax’s 7th-order KdV equation


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