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An Approximation to the Solution of the Brusselator System by Adomian Decomposition Method and Comparing the Results with Runge-Kutta Method

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Abstract

Adomian decomposition method, as a convenience device has been used to solve many functional equations so far. In this manuscript, we consider a system of nonlinear ordinary differential equations, which governs on general reaction in biochemistry as a theoretical problem of concentration kinetics. These system, which is known as Brusselator system has been solved by applying Adomian decomposition method and the results are compared with the results of runge kutta method.

Keywords: decomposition method, Brusselator system, runge kutta method

1 Introduction

Brusselator system has the following general form:

$$\begin{aligned} \tfrac{\partial x}{\partial t} &= a + x^2 y - (b+1)x, \\ \tfrac{\partial y}{\partial t} &= bx - x^2 y. \end{aligned} \tag{1}$$

where $a > 0$ and $a > 0$ are constant(Google search). It is one of the simplest but most fundamental models which displays biological and chemical oscillations. Let have the following initial conditions:

$$\begin{aligned} x(0) &= N_1 \\ y(0) &= N_2 \end{aligned} \tag{2}$$

2 Solution of the system (1) by the Adomian decomposition method:

Let's consider the system (1) in the operator form:

$$
\begin{aligned}
L_t x &= a + x^2 y - (b+1)x, \\
L_t y &= bx - x^2 y.
\end{aligned}
\tag{3}
$$

where

$$L_t = \frac{\partial}{\partial t}$$

By applying the inverse operator $L_t^{-1} = \int_0^t (.)dt$ to each equation in the system (3) we derive:

$$
\begin{aligned}
x(t) &= N_1 + at + \textstyle\int_0^t (x^2 y - (b+1)x)dt \\
y(t) &= N_2 + \textstyle\int_0^t (bx - x^2 y)dt.
\end{aligned}
\tag{4}
$$

Adomian decomposition method, well addressed in [1, 2] considers x and y as the summations of two series, say

$$x = \Sigma_{n=0}^{\infty} x_n, \quad y = \Sigma_{n=0}^{\infty} y_n \tag{5}$$

And the nonlinear term $x^2 y$ is represented as:

$$x^2 y = \Sigma_{n=0}^{\infty} A_n(x_0, x_1, \ldots, x_n, y_0, y_1, \ldots, y_n). \tag{6}$$

Where A_n's are called Adomian polynomials and should be determined. Adomian polynomials have been calculated by using an alternate algorithm for calculating Adomian polynomial [3]. Therefore we have:

$$A_0 = x_0^2 y_0$$

$$A_1 = 2x_0 y_0 x_1 + x_0^2 y_1$$

$$A_2 = x_1^2 y_0 + 2x_0 y_1 x_1 + 2x_0 y_0 x_2 + x_0^2 y_2$$

$$A_3 = 2x_1 y_0 x_2 + x_1^2 y_1 + 2x_0 y_2 x_1 + 2x_0 y_1 x_2 + 2x_0 y_0 x_3 + x_0^2 y_3$$

$$A_4 = x_2^2 y_0 + 2x_1 y_1 x_2 + 2x_1 y_0 x_3 + x_1^2 y_2 + 2x_0 y_3 x_1 + 2x_0 y_2 x_2 + 2x_0 y_1 x_3 + 2x_0 y_0 x_4 + x_0^2 y_4$$

Substituting (5) and (6) into (4) leads to the following system,

$$
\begin{aligned}
\Sigma_{n=0}^{\infty} x_n &= N_1 + at + \Sigma_{n=0}^{\infty} \textstyle\int_0^t A_n dt - (b+1)\Sigma_{n=0}^{\infty} \textstyle\int_0^t x_n dt \\
\Sigma_{n=0}^{\infty} y_n &= N_2 + b\Sigma_{n=0}^{\infty} \textstyle\int_0^t x_n dt - \Sigma_{n=0}^{\infty} \textstyle\int_0^t A_n dt.
\end{aligned}
\tag{7}
$$

From which we defined

$$x_0 = N_1 + at, \quad y_0 = N_2$$

$$x_{n+1} = \int_0^t A_n dt - (b+1)\int_0^t x_n dt \qquad n = 0, 1, 2, \dots \tag{8}$$

$$y_{n+1} = b\int_0^t x_n dt - \int_0^t A_n dt \qquad n = 0, 1, 2, \dots$$

Knowing Adomian polynomial, few first terms of the series (5), are calculated using the scheme (9).

$$x_1 = \frac{1}{3}a^2N_2t^3 + aN_1N_2t^2 - \frac{abt^2}{2} - \frac{at^2}{2} + N_1^2N_2t - N_1bt - N_1t$$

$$y_1 = \frac{-1}{3}a^2N_2t^3 - aN_1N_2t^2 + \frac{abt^2}{2} - N_1^2N_2t + N_1bt$$

$$\begin{aligned}x_2 = {} & \frac{-1}{18}a^4N_2t^6 + \frac{2}{15}a^3N_2^2t^5 - \frac{1}{3}a^3N_1N_2t^5 + \frac{1}{10}a^3bt^5 - \frac{1}{3}a^2N_2t^4 - \frac{1}{3}a^2bN_2t^4 \\ & + \frac{2}{3}a^2N_1N_2^2t^4 - \frac{5}{6}a^2N_1^2N_2t^4 + \frac{1}{2}a^2bN_1t^4 + \frac{1}{3}abt^3 + \frac{1}{6}ab^2t^3 + \frac{1}{6}at^3 - \frac{4}{3}aN_1N_2t^3 \\ & + \frac{4}{3}aN_1^2N_2^2t^3 + \frac{5}{6}abN_1^2t^3 - aN_1^3N_2t^3 - \frac{4}{3}abN_1N_2t^3 + N_1^3N_2^2t^2 - \frac{3}{2}N_1^2N_2t^2 \\ & + \frac{1}{2}b^2N_1t^2 + bN_1t^2 + \frac{1}{2}bN_1^3t^2 - \frac{1}{2}N_1^4N_2t^2 + \frac{1}{2}N_1t^2 - \frac{3}{2}bN_1^2N_2t^2\end{aligned}$$

$$\begin{aligned}y_2 = {} & \frac{1}{18}a^4N_2t^6 - \frac{2}{15}a^3N_2^2t^5 + \frac{1}{3}a^3N_1N_2t^5 - \frac{1}{10}a^3bt^5 + \frac{1}{4}a^2N_2t^4 + \frac{1}{3}a^2bN_2t^4 \\ & - \frac{2}{3}a^2N_1N_2^2t^4 + \frac{5}{6}a^2N_1^2N_2t^4 - \frac{1}{2}a^2bN_1t^4 - \frac{1}{6}abt^3 - \frac{1}{6}ab^2t^3 + \frac{4}{3}abN_1N_2t^3 \\ & - \frac{4}{3}aN_1^2N_2^2t^3 - \frac{5}{6}abN_1^2t^3 + aN_1^3N_2t^3 + aN_1N_2t^3 - N_1^3N_2^2t^2 + \frac{3}{2}bN_1^2N_2t^2 \\ & - \frac{1}{2}b^2N_1t^2 - \frac{1}{2}bN_1t^2 + N_1^2N_2t^2 - \frac{1}{2}bN_1^3t^2 + \frac{1}{2}N_1^4N_2t^2\end{aligned}$$

$$\vdots$$

We can determine the components $x_0, x_1, x_2, \dots$ and $y_0, y_1, y_2, \dots$ as many as is necessary to enhance the desired accuracy for the approximations. So the approximations $x^{(n)} = \Sigma_{i=0}^{n-1} x_i$ and $y^{(n)} = \Sigma_{i=0}^{n-1} y_i$, determine n-terms approximation to the solutions.

3 Solution of the system (1) by the runge kutta method

To solve the Brusselator system, (1), by Runge-Kutta method, let's define the following functions:

$$f(t,x,y)=a+x^2(t)y(t)-(b+1)x(t)$$

$$g(t,x,y)=bx(t)-x^2(t)y(t)$$

Runge-Kutta method computes the approximations by the following iterative equations

$$x_{n+1}=x_n+\frac{h}{6}(m_1+2m_2+2m_3+m_4)$$

$$y_{n+1}=y_n+\frac{h}{6}(n_1+2n_2+2n_3+n_4)$$

Where

$$m_1=f(t_n,x_n,y_n),\quad n_1=g(t_n,x_n,y_n)$$

$$m_2=f(t_n+\frac{h}{2},x_n+m_1\frac{h}{2},y_n+n_1\frac{h}{2})$$

$$n_2=g(t_n+\frac{h}{2},x_n+m_1\frac{h}{2},y_n+n_1\frac{h}{2})$$

$$m_3=f(t_n+\frac{h}{2},x_n+m_2\frac{h}{2},y_n+n_2\frac{h}{2})$$

$$n_3=g(t_n+\frac{h}{2},x_n+m_2\frac{h}{2},y_n+n_2\frac{h}{2})$$

$$m_4=f(t_n+h,x_n+m_3h,y_n+n_3h)$$

$$n_4=g(t_n+h,x_n+m_3h,y_n+n_3h)$$

4 Numerical results

To illustrate Adomian decomposition method and comparing the results of this method with the results of Runge-Kutta method, here are two examples.

Example1 Consider the following system:

$$\frac{\partial x}{\partial t}=-2x+x^2y$$

$$\frac{\partial y}{\partial t}=x-x^2y.$$

where

$$x(0) = 1, \quad y(0) = 1$$

By using Adomian decomposition method we have

$$x_0 = 1, \quad y_0 = 1$$

$$x_{n+1} = \int_0^t A_n dt - 2\int_0^t x_n dt \qquad n = 0, 1, 2, \ldots$$

$$y_{n+1} = \int_0^t x_n dt - \int_0^t A_n dt \qquad n = 0, 1, 2, \ldots$$

Ten-terms approximations to the solutions by using this scheme, the following are computed.

$$x = 1 - t + \frac{1}{2}t^3 - \frac{3}{8}t^4 + \frac{3}{20}t^5 + \frac{7}{240}t^6 - \frac{37}{240}t^7 + \frac{7}{40}t^8 - \frac{1813}{17280}t^9$$

$$y = 1 + \frac{1}{2}t^2 - \frac{1}{2}t^3 + \frac{1}{4}t^4 - \frac{3}{40}t^5 - \frac{13}{240}t^6 + \frac{3}{20}t^7 - \frac{299}{1920}t^8 + \frac{1477}{17280}t^9$$

For some specified values of , the results of Adomian method and Runge-Kutta method are compared in table 1.

*table*1

A comparison between the solutions of Ex. 1 *by Adomian method and Runge Kutta method*

t	A.D.M		Runge-Kutta method	
t	x(t)	y(t)	x(t)	y(t)
0.1	0.9004640155	1.004524209	0.9004639852	1.004524129
0.2	0.8034482877	1.016374098	0.8034482895	1.016373909
0.3	0.7108239629	1.033327532	0.7108241556	1.033327161
0.4	0.6238900641	1.053576241	0.6238926950	1.053574362
0.5	0.5434799759	1.075665396	0.5435048184	1.075649444
0.6	0.4699911064	1.098485574	0.4701502231	1.098381159
0.7	0.4032626837	1.121371970	0.4040245971	1.120858494

Example2 Consider system (1), by the following coefficients:

$$N_1 = 2, \quad N_2 = 3, \quad a = 0.02, \quad b = 0.1$$

By using (8), five-terms approximation to the solutions would be as:

$$x = 2 + 9.82t + 29.919t^2 + 4.1981t^3 - 541.5649675t^4 - 18.86059890t^5 - 0.2482093102t^6$$
$$-0.001544185238t^7 - 0.3663433205 \times 10^{-5}t^8 + 0.7957421519 \times 10^{-8}t^9$$

$$+0.6417701588 \times 10^{-10}t^{10} + 0.1043340067 \times 10^{-12}t^{11} - 0.3950617283 \times 10^{-16}t^{12}$$

$$y = 3 - 11.8t - 34.829t^2 - 14.1711t^3 + 540.5154425t^4 + 19.12049407t^5 + 0.252410192t^6$$

$$+0.001571419079t^7 + 0.3742804634 \times 10^{-5}t^8 - 0.7888828926 \times 10^{-8}t^9$$

$$-0.6429553440 \times 10^{-10}t^{10} - 0.1043340067 \times 10^{-12}t^{11} + 0.3950617283 \times 10^{-16}t^{12}$$

In table 2 the results of Adomian decomposition method and Runge-Kutta method, for some value of t, are presented for comparison.

table2

A comparison between the solutions of Ex. 2 by Adomian method

and Runge Kutta method

t	A.D.M		Runge-Kutta method	
t	x(t)	y(t)	x(t)	y(t)
0.01	2.101190680	2.878508336	2.101190332	2.878508706
0.02	2.208314475	2.750041574	2.208303814	2.750052616
0.03	2.321201323	2.614709563	2.321119682	2.614794237
0.04	2.439550740	2.472752329	2.439203761	2.473112684
0.05	2.562931584	2.324540313	2.561868382	2.325646157
0.06	2.690781830	2.170574602	2.688140848	2.173326164
0.07	2.822408345	2.011487155	2.816748613	2.017395286

5 Conclusion

The main goal of this work has been to derive an approximation for the solutions Brusselator system. We have achieved this goal by applying Adomian decomposition method. The results of applying A.D.M have been compared by the results of Runge-Kutta method for different coefficients of the system. we have derived different agreements for the results of two methods. As the results in the tables show in the first example we have achieved better agreement. The computations are done using Maple 9.

6 Reference

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