Finding Suitable Benchmark for Inefficient Commercial Bank Branches, Application of DEA

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Abstract

In the most applications of Data Envelopment Analysis (DEA), the presented models are designed to obtain a measure of efficiency. Therefore, DEA is a mathematical programming approach that uses the production frontiers to assess relative efficiency. It is well known that the measurement of efficiency for each Decision Making Unit (DMU) depends on itself and its projection point on the efficient frontier. In this paper, we assess the efficiency of under evaluation unit, when, inputs/outputs of its efficient projection point to lie within bounded intervals. Also, in this paper, the commercial banks of Iran are evaluated with DEA technique. Hence, we present suitable pattern for inefficient banks with respect to the exist indexes.

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1. INTRODUCTION

Data Envelopment Analysis (DEA) is a non-parametric method for evaluating the relative efficiency of Decision Making Units (DMUs) of multiple inputs and outputs. The original DEA models (Charnes et al (1978)[1], Banker et. al (1984)[2], Charnes et. al (1985)[3]) assume that inputs and outputs are measured by exact values and the value of efficiency be assessed base on relationship between the evaluated unit and its projection point on efficient frontier.

There are several methods for the inefficient units, so that, they are projected on the efficient frontier. Decreasing input or increasing output is base on all methods.

In practically, it is possible, inputs/outputs of efficient projection point have limited with bounded intervals. For instance, in evaluating the performance of banks, it is possible, the age of staff is as i-th input index with amount 30, for the evaluate under bank, and its efficient score in the input-oriented be 0.1, that is, this unit is inefficient. Then, the efficient projection point of evaluate under unit (bank) have 3 in i-th index. It seems that, it is impossible.

For this purpose, we introduce constrains with lower and upper bounds on the inputs/outputs of efficient projection point from an inefficient unit, on the DEA models that computes efficient.

The paper is organized as follows. Section 2 introduces the Data Envelopment Analysis model (CCR model) and proposed method on base the CCR model. In section 3, we explain the concept of suitable benchmark. In section 4 are given illustrative numerical examples. Finally, conclusions are given in section 5.

2. DATA ENVELOPMENT ANALYSIS

Assume that n units, each using m inputs produce s outputs. Also assume that $X_p = (x_{1p}, ..., x_{mp})$ and $Y_p = (y_{1p}, ..., y_{sp})$ be inputs and outputs vectors $DMU_p$, where, $X_p \geq 0, X_p \neq 0, Y_p \geq 0, Y_p \neq 0$. The efficiency $DMU_p$ (here, we consider the CCR model in input-oriented) is obtained as follow:

$$Min \quad \theta - \varepsilon (\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+)$$
Application of DEA

\[ \begin{align*}
\text{s.t. } & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{ip}, & i = 1, ..., m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_i^+ = y_{rp}, & r = 1, ..., s \\
& \lambda_j, s_i^-, s_i^+ \geq 0, & j = 1, ..., n, i = 1, ..., m, r = 1, ..., s.
\end{align*} \] (1)

Which, it is known to CCR model in the input-oriented, and \( \epsilon > 0 \) is a "non-Archimedean element" defined to be smaller than any positive real number. This means that \( \epsilon \) is not a real number. Assume that \( \theta^* \) be the optimal value of unit \( DMU_p \). If \( \theta^* = 1 \), then \( DMU_p \) is called efficient and if all slack variables be zero in all optimal solutions of model (1) then unit is called power efficient. For an inefficient unit in CCR model, the projection point, that, it does not usually external exists, show the inefficient factor. Also, it will be a benchmark for evaluated DMU (Fig. 2.1). The virtual \( DMU_C \) is a benchmark for the inefficient \( DMU_A \), but, it may be not suitable benchmark in practically applications, that is, it may be input of \( DMU_C \) not be acceptable for evaluating input of \( DMU_A \). (Note that \( DMU_B \) and virtual \( DMU_C \) have the same nature in CCR model). The output-oriented CCR model (1) is given below:

\[ \begin{align*}
\text{Max } & \varphi + \epsilon (\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+) \\
\text{s.t. } & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{ip}, & i = 1, ..., m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_i^+ = \phi y_{rp}, & r = 1, ..., s \\
& \lambda_j, s_i^-, s_i^+ \geq 0, & j = 1, ..., n, i = 1, ..., m, r = 1, ..., s.
\end{align*} \] (2)

Fig. 2.1. Illustration of projection point in the CCR

3. SUITABLE BENCHMARK

An important aspect of DEA efficiency assessment is the correct selection
of set of input/output that an inefficient DMU is efficient. Therefore, selection of appropriate benchmark for evaluating of an inefficient DMU is favorable for us. For this purpose, consider evaluating of bank branches that the set of input/output are given as follows:

Inputs:
* Number of employees in the branch.
* Floor space of the branch (in m).
* Operation costs.
* Number of computer terminals.
* Mean of the age of staff.

Outputs:
* Number of general service transactions performed by branch staff.
* Number of all types of accounts at the branch.
* Value of loans.
* Value of savings.
* Deposits (resource-consuming).

Suppose that, in evaluating one of bank branches, using the input-oriented CCR model, the efficiency score is 0.01, also, assume that, one of its input index, say, mean of the age of staff (input index) is 30. Then, the score of this input index of the projection point on the efficient frontier, for evaluated unit will be 0.3, also, assume that, say, value of deposits (output index) be 40% and the efficient scale, using the output-oriented CCR model is 3. Then efficient scale of projection point on the efficient frontier, for this output index of evaluated unit will be 120%. It seems that, the obtained results are impossible and erroneous in both of cases.

Now, consider the following question: how can find the projection point from an inefficient DMU, on the efficient frontier (not essentially), so that inputs/outputs of the projection point are acceptable and logically? In order to answer to above question, we need to add two sets of below constraints to model (1).

\[
a^i \leq \sum_{j=1}^{n} \lambda_j x_{ij} \leq b^i, \quad i = 1, \ldots, m \quad \text{and} \quad c^r \leq \sum_{j=1}^{n} \lambda_j y_{rj} \leq d^r, \quad r = 1, \ldots, s.
\]

Where, \(\sum_{j=1}^{n} \lambda_j x_{ij}\) and \(\sum_{j=1}^{n} \lambda_j y_{rj}\) show the coordinates a point as nonnegative linear combination of inputs and outputs of all DMUs, respectively.

With respect to model (1), the projection point will be \((\sum_{j=1}^{n} \lambda_j x_{ij} = \theta x_{ip} - s^-_i, \sum_{j=1}^{n} \lambda_j y_{rj} = y_{rp} + s^+_r). Also, \(a^i, b^i, c^r, d^r, i = 1, \ldots, m, r = 1, \ldots, s\) are exactly definite, that can be selected with the discretion of manager or other users. The model used in this form will be as follows:
Min $\theta - \varepsilon (\sum_{i=1}^{m} s_i^{-} + \sum_{r=1}^{s} s_r^{+})$

s.t. $\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^{-} = \theta x_{ip}$, $i = 1, ..., m$

$\sum_{j=1}^{n} \lambda_j y_{ip} - s_i^{+} = y_{rp}$, $r = 1, ..., s$

$\alpha^i \leq \sum_{j=1}^{n} \lambda_j x_{ij} \leq \beta^i$, $i = 1, ..., m$

$\gamma^r \leq \sum_{j=1}^{n} \lambda_j y_{ij} \leq d^r$, $r = 1, ..., s$

$\lambda_j, s_i^{-}, s_r^{+} \geq 0$, $j = 1, ..., n$, $i = 1, ..., m$, $r = 1, ..., s$.

So that, if in model (3), $a^i = 0$, $(i = 1, ..., m)$, $c^r = 0$, $(r = 1, ..., s)$ and $b^i$, $(i = 1, ..., m)$, $d^r$, $(r = 1, ..., s)$, are very big positive values, then model (3) will be the same input oriented CCR model (model (1)). The standard form of model (3) is given in below.

Min $\theta - \varepsilon (\sum_{i=1}^{m} s_i^{-} + \sum_{r=1}^{s} s_r^{+})$

s.t. $\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^{-} = \theta x_{ip}$, $i = 1, ..., m$

$\sum_{j=1}^{n} \lambda_j y_{ip} - s_i^{+} = y_{rp}$, $r = 1, ..., s$

$\sum_{j=1}^{n} \lambda_j x_{ij} - \alpha_i = a^i$, $i = 1, ..., m$

$\sum_{j=1}^{n} \lambda_j y_{ij} + \beta_i = b^i$, $i = 1, ..., m$

$\sum_{j=1}^{n} \lambda_j y_{ip} - \gamma_r = c^r$, $r = 1, ..., s$

$\sum_{j=1}^{n} \lambda_j y_{ij} + \delta_r = d^r$, $r = 1, ..., s$

$\lambda_j \geq 0$, $j = 1, ..., n$

$\alpha_i, \beta_i, s_i^{-} \geq 0$, $i = 1, ..., m$

$\gamma_r, \delta_r, s_r^{+} \geq 0$, $r = 1, ..., s$.

Notice that, in practical problems, the set of constraints $\sum_{j=1}^{n} \lambda_j x_{ij} + \beta_i = b^i$, $i = 1, ..., m$ and $\sum_{j=1}^{n} \lambda_j y_{ip} - \gamma_r = c^r$, $r = 1, ..., s$ are redundant and can be thrown away. Therefore, we have:

Min $\theta - \varepsilon (\sum_{i=1}^{m} s_i^{-} + \sum_{r=1}^{s} s_r^{+})$

s.t. $\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^{-} = \theta x_{ip}$, $i = 1, ..., m$

$\sum_{j=1}^{n} \lambda_j y_{ip} - s_i^{+} = y_{rp}$, $r = 1, ..., s$

$\sum_{j=1}^{n} \lambda_j x_{ij} - \alpha_i = a^i$, $i = 1, ..., m$

$\sum_{j=1}^{n} \lambda_j y_{ij} + \delta_r = d^r$, $r = 1, ..., s$

$\lambda_j \geq 0$, $j = 1, ..., n$

$\alpha_i, \beta_i, s_i^{-} \geq 0$, $i = 1, ..., m$

$\gamma_r, \delta_r, s_r^{+} \geq 0$, $r = 1, ..., s$.

The output-oriented CCR model is given as follows:

Max $\varphi + \varepsilon (\sum_{i=1}^{m} s_i^{-} + \sum_{r=1}^{s} s_r^{+})$

s.t. $\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^{-} = x_{ip}$, $i = 1, ..., m$

$\sum_{j=1}^{n} \lambda_j y_{ip} - s_i^{+} = \varphi y_{rp}$, $r = 1, ..., s$

$\sum_{j=1}^{n} \lambda_j x_{ij} - \alpha_i = a^i$, $i = 1, ..., m$
\[
\sum_{j=1}^{n} \lambda_j y_{rj} + \delta_r = d^r, \quad r = 1, \ldots, s
\]

\[
\lambda_j \geq 0, \quad j = 1, \ldots, n
\]

\[
\alpha_i, \beta_i, s_i^- \geq 0, \quad i = 1, \ldots, m
\]

\[
\gamma_r, \delta_r, s_r^+ \geq 0, \quad r = 1, \ldots, s.
\]

Figure 3.2 shows the output-oriented CCR model with one input and one output, where it can help us more understand the treatment in model (6). Notice that, with respect to discretionary of manager, the benchmark point (projection point) can be an inefficient point (Fig. 3.3). This point is utility point for manager person.

Fig. 3.2. Illustration of desirable benchmark for inefficient DMU

Fig. 3.3. Illustration of utility point for manager that can be inefficient

4. EXAMPLES

4.1 A NUMERICAL EXAMPLE
Consider 7 DMUs with two inputs and one output (Table 1).

Table 1. The value of inputs and output

<table>
<thead>
<tr>
<th>DMUs</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input I</td>
<td>$X_1$</td>
<td>4</td>
<td>7</td>
<td>8</td>
<td>4</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>Input II</td>
<td>$X_2$</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Output</td>
<td>$Y$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 3.4 shows a typical production possibility set in two dimension space for the two inputs and single output case, so that $m=2$ and $s=1$, respectively.

In Figure 3.4 the units C, D and E are strongly efficient, the unit F is weakly efficient and the units A, B and G are inefficient.

Consider $DMU_B$. We evaluate it by solving model (1), that is, (the input-oriented CCR model). We have:

$\theta^* = 0.6316, \lambda_C^* = 0.1053, \lambda_D^* = 0.8947, \text{ other } \lambda_j^* = 0, j = A, B, E, F, G$

$S_1^{-*} = S_2^{-*} = S^{++} = 0$

Note that, in Figure 3.4, the $B'(4.4211, 1.8974)$ point is the efficient projection point of B.

Suppose that in evaluating $DMU_B$, the first input of its efficient projection point could not be less than 5. Then, by using model (5), we again evaluate $DMU_B$. We have:

$\theta^* = 0.7143, \lambda_C^* = 0.25, \lambda_D^* = 0.75, \text{ other } \lambda_j^* = 0, j = A, B, E, F, G$

$S_1^{-*} = 0, S_2^{-*} = 0.3929, S^{++} = 0$

From Figure 3.4, $B''(5.0000, 1.75)$ point is the efficient projection point of B, when it is used model (5). With comparing these results, we see, although, the coordinates of $B'$ and $B''$ are fully efficient, but, the activity $B''$ is not input proportional to $B(7, 3)$ and therefore, the unit $B$ is not radially efficient when
it evaluated in model (5). Note that, the reference set for $B$ is $E_B = \{C, D\}$ in both of cases.

Now, if the second input of the efficient projection point of $DMU_B$, restricted in interval $[2.5, 3]$, then, with using the model (5), we have:

$\theta^* = 0.8333$, $\lambda^*_D = 0.75$, $\lambda^*_E = 0.25$, other $\lambda^*_j = 0$, $j = A, B, C, F, G$

$S_1^* = 2.3331$, $S_2^* = 0$, $S_3^* = 0$

From Figure 3.4, $B''(3.5, 2.5)$ point is the efficient projection point of $B$, when it is used model (5). In this case, the reference set for $B$ will be $E_B = \{D, E\}$.

### 4.2 AN APPLICATION EXAMPLE

The real data set, documented in Table 1, contains 20 banks with four inputs and five outputs, where the set of inputs/outputs are given as follows:

**Inputs:**
* Payable interest, Personnel, Non-performing loans, Number of branch.

**Outputs:**
* The total sum of four main deposits, Other deposits, Loans granted, Received interest, Fee.

In Table 2 the lower bound inputs and the upper bound outputs for the inefficient units are given. Also the results obtained from above example by solving the model (1) and (6) are summarized in Table 3. Note, the units 3, 4, 5, 6, 7, 10, 12, and 19 are efficient. Meanwhile, the units 1, 2, 8, 9, 11, 13, 14, 15, 16, 17, 18 and 20 are inefficient.
Table 2. The inputs lower bound and the outputs upper bound of projection point for the inefficient units

<table>
<thead>
<tr>
<th>DMUs</th>
<th>$L_1$</th>
<th>$L_2$</th>
<th>$L_3$</th>
<th>$L_4$</th>
<th>$U_1$</th>
<th>$U_2$</th>
<th>$U_3$</th>
<th>$U_4$</th>
<th>$U_5$</th>
</tr>
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<tr>
<td>1</td>
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<td>150</td>
<td>55000</td>
<td>25</td>
<td>110000</td>
<td>47000</td>
<td>68000</td>
<td>35000</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
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<td>250000</td>
<td>50</td>
<td>560000</td>
<td>108000</td>
<td>560000007</td>
<td>120000</td>
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<tr>
<td>11</td>
<td>8394.97</td>
<td>529.64</td>
<td>64750</td>
<td>98</td>
<td>1437663</td>
<td>60187</td>
<td>929473</td>
<td>20853.48</td>
<td>281.64</td>
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</table>

Table 3. The results of example 4.1

<table>
<thead>
<tr>
<th>DMUs</th>
<th>Input-Oriented</th>
<th>Output-Oriented</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\theta^*$</td>
<td>$\theta^*_N$</td>
</tr>
<tr>
<td>1</td>
<td>0.85</td>
<td>0.968</td>
</tr>
<tr>
<td>2</td>
<td>0.92</td>
<td>0.962</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
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<td>6</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
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<td>1</td>
</tr>
<tr>
<td>8</td>
<td>0.81</td>
<td>0.975</td>
</tr>
<tr>
<td>9</td>
<td>0.82</td>
<td>0.897</td>
</tr>
<tr>
<td>10</td>
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<td>1</td>
</tr>
<tr>
<td>11</td>
<td>0.59</td>
<td>0.714</td>
</tr>
<tr>
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<td>1</td>
</tr>
<tr>
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</tr>
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<td>14</td>
<td>0.61</td>
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<tr>
<td>20</td>
<td>0.80</td>
<td>0.872</td>
</tr>
</tbody>
</table>

Where, $\theta^*$ and $\varphi^*$ are the optimal values from the models (1) and (2), and also $\theta^*_N$ and $\varphi^*_N$ are the optimal values from models (5) and (6). Table 1 shows
the original data contains 20 banks with four inputs and five outputs. Table 2 reports present the lower bound and the upper bound on inputs and outputs, respectively. Table 3 shows the optimal values in input-oriented and output-oriented base on the models (1), (2), that is, \( \theta^* \) and \( \varphi^* \) and the models (5), (6), that is, \( \theta^*_N \) and \( \varphi^*_N \), respectively. For example, in evaluating of \( DMU_1 \), before of accomplish of restrictions over the inputs lower bound and the outputs upper bound, we obtain \( \theta^* = 0.85 \) and \( \varphi^* = 1.177 \). Meanwhile, after achievement of restrictions over the inputs lower bound (the first input until the forth input, that is, 4300, 150, 55000, 25), and also over the outputs upper bound (the first output until the fifth output, that is, 1100000, 47000, 680000, 35000 and 200), respectively. Then, we obtain \( \theta^*_N = 0.968 \) and \( \varphi^*_N = 1.006 \). Similar to the survey of the other \( DMU_s \), we result \( \theta^* \leq \theta^*_N \) and \( \varphi^* \geq \varphi^*_N \) are correct for all \( DMU_s \). The obtained results tell to us that the efficiency measure doesn’t improve in both of cases input and output.

5. CONCLUSION

In most practical applications of Data Envelopment Analysis (DEA), some or all of input/output indexes of the efficient projection point from inefficient DMU, can not get any value. Therefore, it is needed that the presented models are revised in order to obtain the exact measure of efficiency. For example, if in evaluating an inefficient unit of bank branches, one of its input indexes, say, means of the age of staff (input index) is 30, and the efficient obtained, using the input-oriented CCR model is 0.01. Then, the efficient scale of projection point on the efficient frontier will be 0.3 for this input index of the evaluated unit. Also, assume that, say, value of deposits (output index) is 40% and the obtained efficient, using the output-oriented CCR model is 3. Then, the efficient scale of projection point on the efficient frontier, for this output index of evaluated unit will be 120%. It seems that, results are impossible and erroneous in the both from cases. Hence, in this paper, we have presented confines for projection point obtained an inefficient unit as acceptable interval, that it is applied empirically. In continue, we attend that the components of inputs/outputs indexes for the projection point from an inefficient DMU to be as positive rational, positive irrational and special data, so that, are selected at the discretions of management or other users.

REFERENCES


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