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Finding Suitable Benchmark for Inefficient Commercial Bank Branches, Application of DEA

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Abstract

In the most applications of Data Envelopment Analysis (DEA), the presented models are designed to obtain a measure of efficiency. Therefore, DEA is a mathematical programming approach that uses the production frontiers to assess relative efficiency. It is well known that the measurement of efficiency for each Decision Making Unit (DMU) depends on itself and its projection point on the efficient frontier. In this paper, we assess the efficiency of under evaluation unit, when, inputs/outputs of its efficient projection point to lie within bounded intervals. Also, in this paper, the commercial banks of Iran are evaluated with DEA technique. Hence, we present suitable pattern for inefficient banks with respect to the exist indexes.

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1. INTRODUCTION

Data Envelopment Analysis (DEA) is a non-parametric method for evaluating the relative efficiency of Decision Making Units (DMUs) of multiple inputs and outputs. The original DEA models (Charnes et al (1978)[1], Banker et. al (1984)[2], Charnes et. al (1985)[3]) assume that inputs and outputs are measured by exact values and the value of efficiency be assessed base on relationship between the evaluated unit and its projection point on efficient frontier.

There are several methods for the inefficient units, so that, they are projected on the efficient frontier. Decreasing input or increasing output is base on all methods.

In practically, it is possible, inputs/outputs of efficient projection point have limited with bounded intervals. For instance, in evaluating the performance of banks, it is possible, the age of staff is as i-th input index with amount 30, for the evaluate under bank, and its efficient score in the input-oriented be 0.1, that is, this unit is inefficient. Then, the efficient projection point of evaluate under unit (bank) have 3 in i-th index. It seems that, it is impossible.

For this purpose, we introduce constrains with lower and upper bounds on the inputs/outputs of efficient projection point from an inefficient unit, on the DEA models that computes efficient.

The paper is organized as follows. Section 2 introduces the Data Envelopment Analysis model (CCR model) and proposed method on base the CCR model. In section 3, we explain the concept of suitable benchmark. In section 4 are given illustrative numerical examples. Finally, conclusions are given in section 5.

2. DATA ENVELOPMENT ANALYSIS

Assume that n units, each using m inputs produce s outputs. Also assume that$X_p = (x_{1p}, ..., x_{mp})$ and $Y_p = (y_{1p}, ..., y_{sp})$ be inputs and outputs vectors DMU_p , where, $X_p \geq 0, X_p \neq 0, Y_p \geq 0, Y_p \neq 0$. The efficiency DMU_p (here, we consider the CCR model in input-oriented) is obtained as follow:

$$Min \quad \theta - \varepsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+)$$

$$\begin{aligned} s.t.\ & \Sigma_{j=1}^{n}\lambda_j x_{ij} + s_i^- = \theta x_{ip}, \qquad i = 1,...,m \\ & \Sigma_{j=1}^{n}\lambda_j y_{rj} - s_i^+ = y_{rp}, \qquad r = 1,...,s \\ & \lambda_j,\ s_i^-,\ s_r^+ \geq 0, \quad j = 1,...,n,\ i = 1,...,m,\ r = 1,...,s. \end{aligned} \tag{1}$$

Which, it is known to CCR model in the input-oriented, and $\epsilon > 0$ is a "non-Archimedean element" defined to be smaller than any positive real number. This means that ϵ is not a real number. Assume that θ^* be the optimal value of unit DMU_p . If $\theta^* = 1$, then DMU_p is called efficient and if all slack variables be zero in all optimal solutions of model (1) then unit is called power efficient. For an inefficient unit in CCR model, the projection point, that, it does not usually external exists, show the inefficient factor. Also, it will be a benchmark for evaluated DMU (Fig. 2.1). The virtual DMU_C is a benchmark for the inefficient DMU_A , but, it may be not suitable benchmark in practically applications, that is, it may be input of DMU_C not be acceptable for evaluating input of DMU_A .(Note that DMU_B and virtual DMU_C have the same nature in CCR model). The output-oriented CCR model (1) is given below:

$$\begin{aligned} Max \quad & \varphi + \varepsilon(\textstyle\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+) \\ s.t.\ & \Sigma_{j=1}^{n}\lambda_j x_{ij} + s_i^- = x_{ip}, \qquad i = 1,...,m \\ & \Sigma_{j=1}^{n}\lambda_j y_{rj} - s_i^+ = \phi y_{rp}, \qquad r = 1,...,s \\ & \lambda_j,\ s_i^-,\ s_r^+ \geq 0, \quad j = 1,...,n,\ i = 1,...,m,\ r = 1,...,s. \end{aligned} \tag{2}$$

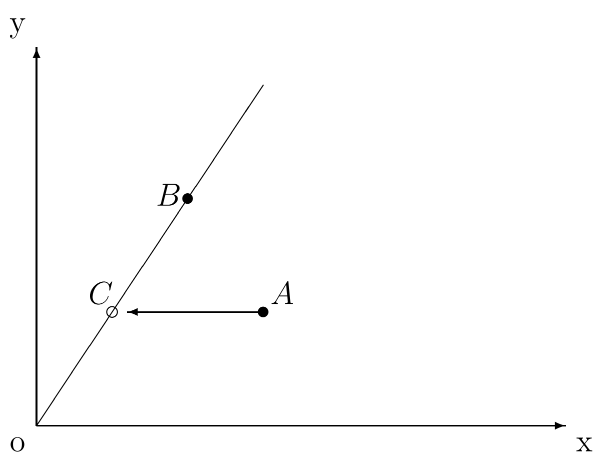

Fig. 2.1. Illustration of projection point in the CCR

3. SUITABLE BENCHMARK

An important aspect of DEA efficiency assessment is the correct selection of set of input/output that an inefficient DMU is efficient. Therefore, selection of appropriate benchmark for evaluating of an inefficient DMU is favorable for us. For this purpose, consider evaluating of bank branches that the set of input/output are given as follows:

Inputs:
* Number of employees in the branch.
* Floor space of the branch (in m).
* Operation costs.
* Number of computer terminals.
* Mean of the age of staff.
Outputs:
* Number of general service transactions performed by branch staff.
* Number of all types of accounts at the branch.
* Value of loans.
* Value of savings.
* Deposits (resource-consuming).

Suppose that, in evaluating one of bank branches, using the input-oriented CCR model, the efficiency score is 0.01, also, assume that, one of its input index, say, mean of the age of staff (input index) is 30. Then, the score of this input index of the projection point on the efficient frontier, for evaluated unit will be 0.3, also, assume that,say, value of deposits (output index) be 40% and the efficient scale, using the output-oriented CCR model is 3. Then efficient scale of projection point on the efficient frontier, for this output index of evaluated unit will be 120%. It seems that, the obtained results are impossible and erroneous in both of cases.

Now, consider the following question: how can find the projection point from an inefficient DMU, on the efficient frontier (not essentially), so that inputs/outputs of the projection point are acceptable and logically? In order to answer to above question, we need to add two sets of below constraints to model (1).

$a^i \leq \sum_{j=1}^{n} \lambda_j x_{ij} \leq b^i, \quad i = 1, ..., m$ and $c^r \leq \sum_{j=1}^{n} \lambda_j y_{rj} \leq d^r, \quad r = 1, ..., s.$

Where, $\sum_{j=1}^{n} \lambda_j x_{ij}$ and $\sum_{j=1}^{n} \lambda_j y_{rj}$ show the coordinates a point as nonnegative linear combination of inputs and outputs of all DMU_s, respectively.
With respect to model (1), the projection point will be ($\sum_{j=1}^{n} \lambda_j x_{ij} = \theta x_{ip} - s_i^-, \sum_{j=1}^{n} \lambda_j y_{rj} = y_{rp} + s_r^+$). Also, $a^i, b^i, c^r, d^r, i = 1, ..., m, r = 1, ..., s$ are exactly definite, that can be selected with the discretion of manager or other users. The model used in this form will be as follows:

$$\begin{array}{lll} Min & \theta - \varepsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+) & \\ s.t. & \Sigma_{j=1}^{n}\lambda_j x_{ij} + s_i^- = \theta x_{ip}, & i = 1, ..., m \\ & \Sigma_{j=1}^{n}\lambda_j y_{rj} - s_i^+ = y_{rp}, & r = 1, ..., s \\ & a^i \leq \sum_{j=1}^{n} \lambda_j x_{ij} \leq b^i, & i = 1, ..., m \\ & c^r \leq \sum_{j=1}^{n} \lambda_j y_{rj} \leq d^r, & r = 1, ..., s \\ & \lambda_j,\ s_i^-,\ s_r^+ \ \geq 0, \quad j = 1, ..., n,\ i = 1, ..., m,\ r = 1, ..., s. & \end{array} \tag{3}$$

So that, if in model (3), $a^i = 0,\ (i = 1, ..., m),\ c^r = 0,\ (r = 1, ..., s)$ and $b^i,\ (i = 1, ..., m),\ d^r,\ (r = 1, ..., s)$, are very big positive values, then model (3) will be the same input oriented CCR model (model (1)). The standard form of model (3) is given in below.

$$\begin{array}{lll} Min & \theta - \varepsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+) & \\ s.t. & \Sigma_{j=1}^{n}\lambda_j x_{ij} + s_i^- = \theta x_{ip}, & i = 1, ..., m \\ & \Sigma_{j=1}^{n}\lambda_j y_{rj} - s_i^+ = y_{rp}, & r = 1, ..., s \\ & \sum_{j=1}^{n} \lambda_j x_{ij} - \alpha_i = a^i, & i = 1, ..., m \\ & \sum_{j=1}^{n} \lambda_j x_{ij} + \beta_i = b^i, & i = 1, ..., m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - \gamma_r = c^r, & r = 1, ..., s \\ & \sum_{j=1}^{n} \lambda_j y_{rj} + \delta_r = d^r, & r = 1, ..., s \\ & \lambda_j \geq 0, & j = 1, ..., n \\ & \alpha_i, \beta_i, s_i^- \geq 0, & i = 1, ..., m \\ & \gamma_r, \delta_r, s_r^+ \geq 0, & r = 1, ..., s. \end{array} \tag{4}$$

Notice that, in practical problems, the set of constraints $\sum_{j=1}^{n} \lambda_j x_{ij} + \beta_i = b^i,\ i = 1, ..., m$ and $\sum_{j=1}^{n} \lambda_j y_{rj} - \gamma_r = c^r,\ r = 1, ..., s$ are redundant and can be thrown away. Therefore, we have:

$$\begin{array}{lll} Min & \theta - \varepsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+) & \\ s.t. & \Sigma_{j=1}^{n}\lambda_j x_{ij} + s_i^- = \theta x_{ip}, & i = 1, ..., m \\ & \Sigma_{j=1}^{n}\lambda_j y_{rj} - s_i^+ = y_{rp}, & r = 1, ..., s \\ & \sum_{j=1}^{n} \lambda_j x_{ij} - \alpha_i = a^i, & i = 1, ..., m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} + \delta_r = d^r, & r = 1, ..., s \\ & \lambda_j \geq 0, & j = 1, ..., n \\ & \alpha_i, \beta_i, s_i^- \geq 0, & i = 1, ..., m \\ & \gamma_r, \delta_r, s_r^+ \geq 0, & r = 1, ..., s. \end{array} \tag{5}$$

The output-oriented CCR model is given as follows:

$$\begin{array}{lll} Max & \varphi + \varepsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+) & \\ s.t. & \Sigma_{j=1}^{n}\lambda_j x_{ij} + s_i^- = x_{ip}, & i = 1, ..., m \\ & \Sigma_{j=1}^{n}\lambda_j y_{rj} - s_i^+ = \varphi y_{rp}, & r = 1, ..., s \\ & \sum_{j=1}^{n} \lambda_j x_{ij} - \alpha_i = a^i, & i = 1, ..., m \end{array} \tag{6}$$

$$
\begin{array}{ll}
\sum_{j=1}^{n} \lambda_j y_{rj} + \delta_r = d^r, & r = 1, ..., s \\
\lambda_j \geq 0, & j = 1, ..., n \\
\alpha_i, \beta_i, s_i^- \geq 0, & i = 1, ..., m \\
\gamma_r, \delta_r, s_r^+ \geq 0, & r = 1, ..., s.
\end{array}
$$

Figure 3.2 shows the output-oriented CCR model with one input and one output, where it can help us more understand the treatment in model (6). Notice that, with respect to discretionary of manager, the benchmark point (projection point) can be an inefficient point (Fig. 3.3). This point is utility point for manager person.

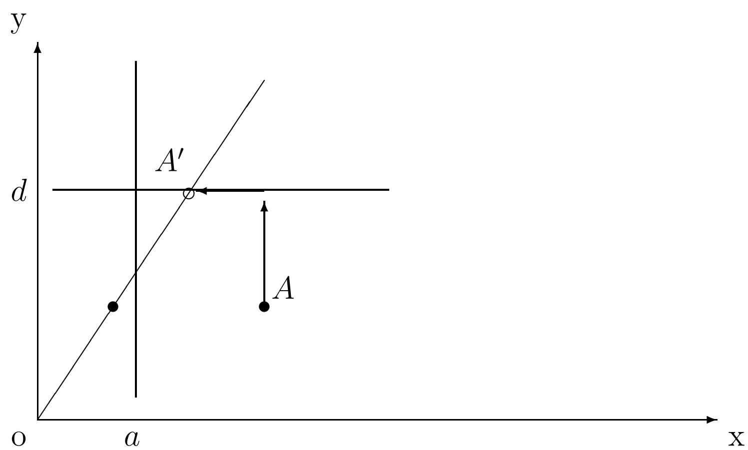

Fig. 3.2. Illustration of desirable benchmark for inefficient DMU_A

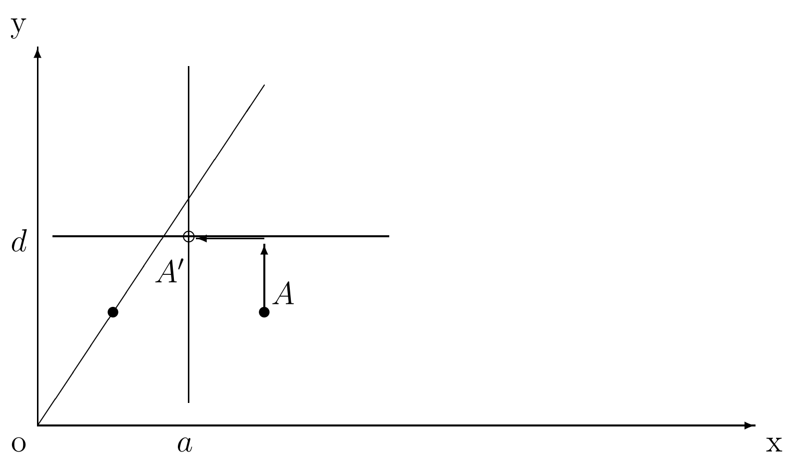

Fig. 3.3. Illustration of utility point for manager that can be inefficient

4. EXAMPLES

4.1 A NUMERICAL EXAMPLE

Consider 7 DMUs with two inputs and one output (Table 1).

Table 1. The value of inputs and output

	DMUs	A	B	C	D	E	F	G
Input I	X_1	4	7	8	4	2	10	3
Input II	X_2	3	3	1	2	4	1	7
Output	Y	1	1	1	1	1	1	1

Figure 3.4 shows a typical production possibility set in two dimension space for the two inputs and single output case, so that m=2 and s=1, respectively.

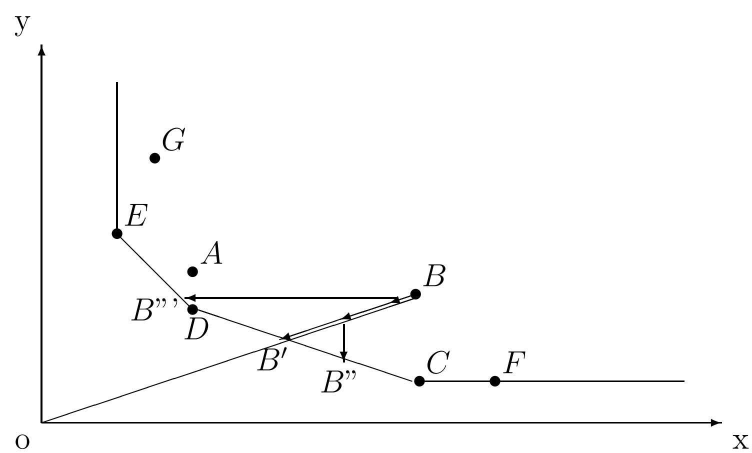

Fig. 3.4. A typically production possibility set for example section 4

In Figure 3.4 the units C, D and E are strongly efficient, the unit F is weakly efficient and the units A, B and G are inefficient.
Consider DMU_B . We evaluate it by solving model (1), that is, (the input-oriented CCR model). We have:
$\theta^* = 0.6316,\ \lambda_C^* = 0.1053,\ \lambda_D^* = 0.8947,\ \text{other } \lambda_j^* = 0,\ j = A, B, E, F, G$
$S_1^{-*} = S_2^{-*} = S^{+*} = 0$
Note that, in Figure 3.4, the $B^{'}(4.4211, 1.8974)$ point is the efficient projection point of B.
Suppose that in evaluating DMU_B , the first input of its efficient projection point could not be less than 5. Then, by using model (5), we again evaluate DMU_B . We have:

$\theta^* = 0.7143,\ \lambda_C^* = 0.25,\ \lambda_D^* = 0.75,\ \text{other } \lambda_j^* = 0,\ j = A, B, E, F, G$
$S_1^{-*} = 0,\ S_2^{-*} = 0.3929,\ S^{+*} = 0$
From Figure 3.4, $B^{"}(5.0000, 1.75)$ point is the efficient projection point of B, when it is used model (5). With comparing these results, we see, although, the coordinates of $B^{'}$ and $B^{"}$ are fully efficient, but, the activity $B^{"}$ is not input proportional to $B(7, 3)$ and therefore, the unit B is not radially efficient when

it evaluated in model (5). Note that, the reference set for B is $E_B = \{C, D\}$ in both of cases.
Now, if the second input of the efficient projection point of DMU_B , restricted in interval [2.5, 3], then, with using the model (5), we have:
$\theta^* = 0.8333,\ \lambda^*_D = 0.75,\ \lambda^*_E = 0.25,$ other $\lambda^*_j = 0,\ j = A, B, C, F, G$
$S^{-*}_1 = 2.3331,\ S^{-*}_2 = 0,\ S^{+*} = 0$
From Figure 3.4, B"'$(3.5, 2.5)$ point is the efficient projection point of B, when it is used model (5). In this case, the reference set for B will be $E_B = \{D, E\}$.

4.2 AN APPLICATION EXAMPLE

The real data set, documented in Table 1, contains 20 banks with four inputs and five outputs, where the set of inputs/outputs are given as follows:
Inputs:
* Payable interest , Personnel, Non-performing loans, Number of branch.

Outputs:
* The total sum of four main deposits, Other deposits, Loans granted, Received interest, Fee.

Table 1. Data contains 20 banks with four inputs and five outputs

DMU_s	I_1	I_2	I_3	I_4	O_1	O_2	O_3	O_4	O_5
1	4707.86	175.8	60801	31	1033890	42954	611224	31671.6	189.17
2	32641.23	477.94	264991	52	5398005	966040	5090776	108826.19	2328.4
3	24603.99	511.76	238510	53	5795565	871880	4839322	131011.56	2335.87
4	9097.12	348.65	85897	48	2332104	815245	3284772	65056.46	2936.8
5	34766.12	276.55	402614	36	4313779	539228	7878616	231066.45	2306.15
6	41239.42	408.88	105778	46	6136069	298420	5115135	29197.01	1838.93
7	24978.41	459.78	321776	49	4923925	1802130	4887652	123469.12	3580.4
8	4902.54	254.34	110543	46	1097316	122046	1127011	12581.5	306.16
9	2278.13	142.75	30084	34	555997	22165	168786	3672.26	137.19
10	23642.26	736.26	58238	141	3736368	190077	1353879	23249.96	512.91
11	8394.97	529.64	64750	98	1437663	60187	929473	20853.48	281.64
12	411.48	28.16	2059	6	125767	11638	66532	5208.43	62.1
13	2698.51	175.95	26732	45	524945	21484	277671	5134.55	162.29
14	3490.86	181.79	22065	48	568498	86932	495530	5618.5	109.64
15	18372.59	681.88	186281	144	2866310	245966	2055363	34231.45	586.61
16	1551.69	132.14	24805	33	415291	22353	339450	8397.85	157.32
17	1862.82	46.98	27059	14	245523	7189	269819	3189.24	142.58
18	7887.42	415.75	111632	89	1898925	115275	1222240	24371.74	744.78
19	1658.79	101.68	21245	28	467922	22728	446906	7491.01	193.06
20	2293.71	146.92	24579	34	513104	36438	502190	8826.21	191.12

In Table 2 the lower bound inputs and the upper bound outputs for the inefficient units are given. Also the results obtained from above example by solving the model (1) and (6) are summarized in Table 3. Note, the units 3, 4, 5, 6, 7, 10, 12, and 19 are efficient. Meanwhile, the units 1, 2, 8, 9, 11, 13, 14, 15, 16, 17, 18 and 20 are inefficient.

Table 2. The inputs lower bound and the outputs upper bound of projection point for the inefficient units

DMU_s	L_1	L_2	L_3	L_4	U_1	U_2	U_3	U_4	U_5
1	4300	150	55000	25	1100000	47000	680000	35000	200
2	31000	400	250000	50	5600000	1080000	56000007	120000	2600
3	-	-	-	-	-	-	-	-	-
4	-	-	-	-	-	-	-	-	-
5	-	-	-	-	-	-	-	-	-
6	-	-	-	-	-	-	-	-	-
7	-	-	-	-	-	-	-	-	-
8	4500	200	100000	40	1250000	140000	1300000	14000	350
9	2000	120	27000	30	650000	24000	190000	4200	155
10	-	-	-	-	-	-	-	-	-
11	8394.97	529.64	64750	98	1437663	60187	929473	20853.48	281.64
12	-	-	-	-	-	-	-	-	-
13	2000	130	2000	40	700000	31000	370000	6000	250
14	2000	150	2000	40	800000	120000	700000	80000	200
15	15000	500	150000	100	4000000	300000	3000000	50000	700
16	1000	100	20000	30	500000	25000	400000	10000	250
17	1000	30	15000	9	400000	10000	400000	5000	250
18	6000	350	100000	70	3000000	130000	2000000	35000	800
19	-	-	-	-	-	-	-	-	-
20	2000	100	20000	25	560000	40000	550000	100007	230

Table 3. The results of example 4.1

	Input-Oriented		Output-Oriented	
DMU_s	θ^*	θ^*_N	φ^*	φ^*_N
1	0.85	0.968	1.177	1.006
2	0.92	0.962	1.09	1.037
3	1	1	1	1
4	1	1	1	1
5	1	1	1	1
6	1	1	1	1
7	1	1	1	1
8	0.81	0.975	1.232	1.015
9	0.82	0.897	1.217	1.008
10	1	1	1	1
11	0.59	0.714	1.694	1.17
12	1	1	1	1
13	0.64	0.895	1.546	1.168
14	0.61	0.906	1.627	1.1
15	0.60	0.816	1.663	1.154
16	0.92	0.977	1.080	1.068
17	0.57	0.784	1.751	1.034
18	0.85	0.922	1.172	1.04
19	1	1	1	1
20	0.80	0.872	1.25	1.079

Where, θ^* and φ^* are the optimal values from the models (1) and (2), and also θ^*_N and φ^*_N are the optimal values from models (5) and (6). Table 1 shows the original data contains 20 banks with four inputs and five outputs. Table 2 reports present the lower bound and the upper bound on inputs and outputs, respectively. Table 3 shows the optimal values in input-oriented and output-oriented base on the models (1), (2), that is, θ^* and φ^* and the models (5), (6), that is, θ_N^* and φ_N^*, respectively. For example, in evaluating of DMU_1 , before of accomplish of restrictions over the inputs lower bound and the outputs upper bound, we obtain $\theta^* = 0.85$ and $\varphi^* = 1.177$. Mean while, after achievement of restrictions over the inputs lower bound (the first input until the forth input, that is, 4300, 150, 55000, 25), and also over the outputs upper bound (the first output until the fifth output, that is, 1100000, 47000, 680000, 35000 and 200), respectively. Then, we obtain $\theta_N^* = 0.968$ and $\varphi_N^* = 1.006$. Similar to the survey of the other DMU_s, we result $\theta^* \leq \theta_N^*$ and $\varphi^* \geq \varphi_N^*$ are correct for all DMU_s. The obtained results tell to us that the efficiency measure dosn't improve in both of cases input and output.

5. CONCLUSION

In most practical applications of Data Envelopment Analysis (DEA), some or all of input/output indexes of the efficient projection point from inefficient DMU, can not get any value. Therefore, it is needed that the presented models are revised in order to obtain the exact measure of efficiency. For example, if in evaluating an inefficient unit of bank branches, one of its input indexes, say, means of the age of staff (input index) is 30, and the efficient obtained, using the input-oriented CCR model is 0.01. Then, the efficient scale of projection point on the efficient frontier will be 0.3 for this input index of the evaluated unit.Also, assume that, say, value of deposits (output index) is 40% and the obtained efficient, using the output-oriented CCR model is 3. Then, the efficient scale of projection point on the efficient frontier, for this output index of evaluated unit will be 120%. It seems that, results are impossible and erroneous in the both from cases. Hence, in this paper, we have presented confines for projection point obtained an inefficient unit as acceptable interval, that it is applied empirically. In continue, we attend that the components of inputs/outputs indexes for the projection point from an inefficient DMU to be as positive rational, positive irrational and special data, so that, are selected at the discretions of management or other users.

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