Rational curves in grassmannians and 
their Plücker embeddings: an application

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Abstract. Here we prove a corollary of [1, Thm. 1], providing sufficient condition 
for the existence of α-stable coherent systems of type \((n, d, k)\) for some \(k > n\).

Mathematics Subject Classification: 14H60

Keywords: stable vector bundles on curves; coherent system; Grassmannian; spanned vector bundle

1. Introduction

Let \(X\) be a smooth and connected projective curve. A coherent system on \(X\) 
is a pair \((E, V)\) such that \(E\) is a vector bundle on \(X\) and \(V \subseteq H^0(X, E)\) 
is a linear subspace. The pair \((E, V)\) is of type \((n, d, k)\) if \(\text{rank}(E) = n\), \(\text{deg}(E) = d\) 
and \(\text{dim}(V) = k\). Fix \(\alpha \in \mathbb{R}\). Let \(\mu(E) := d/n\) denote the slope of \(E\). Set 
\(\mu_\alpha(E, V) := \mu(E) + \alpha k/n\). The real number \(\mu_\alpha\) is called the α-slope of the pair 
\((E, V)\). A coherent subsystem \((F, W) \subseteq (E, V)\) is a coherent system such that 
\(F \subseteq E\) and \(W \subseteq V \cap H^0(X, F)\). The pair \((E, V)\) is said to be α-stable (resp. 
α-semistable) if \(\mu_\alpha(F, W) < \mu_\alpha(E, V)\) (resp. \(\mu_\alpha(F, W) \leq \mu_\alpha(E, V)\)) for all proper 
coherent subsystems \((F, W)\) of \((E, V)\). For the general theory of coherent systems 
and several results on the moduli schemes of α-stable coherent systems see [7], [4], 
[2], [5], [6] and [3]. Here we prove a corollary of [1, Thm. 1], providing sufficient 
conditions for the existence of α-stable coherent systems of type \((n, d, k)\) for some \(k > n\).

**Proposition 1.** Fix \(\alpha \in \mathbb{R}\) and integers integers \(n \geq 2\), \(a_1 \geq \cdots \geq a_n > 0\) and \(k\) 
such that \(\binom{k}{n} \leq 1 + n a_n\) and \(\alpha > (na_1 - \sum_{i=1}^{n} a_i)/(k - n)\). Set \(E := \oplus_{i=1}^{n} \mathcal{O}_{\mathbb{P}^1}(a_i)\) 
and take a general \(k\)-dimensional linear subspace \(V\) of \(H^0(\mathbb{P}^1, E)\). Then the coherent 
system \((E, V)\) is α-stable. Furthermore, for all coherent subsystems \((F, W)\) of \((E, V)\)

\textsuperscript{1}The author was partially supported by MIUR and GNSAGA of INdAM (Italy).
\textsuperscript{2}The author was partially supported by MIUR and GNSAGA of INdAM (Italy) and HPMT-CT-2001-00277.
\textsuperscript{3}The author was partially supported by MIUR and GNSAGA of INdAM (Italy) and HPMT-CT-2001-00277.
such that $1 \leq \text{rank}(F) < n$ we have $\mu_\alpha(E,V) - \mu_\alpha(F,W) \geq (\sum_{i=1}^{n} a_i)/n + (k - n)\alpha/n - a_n$

Proof. By [1] for all integers $r$ such that $1 \leq r < n$ and all rank $r$ subsheaves $F$ of $E$ we have $\dim(V) \cap H^0(P^1, F)) \leq r$. Since $\mu_+(E) = a_1$, we have $\mu(F) \leq a_1$. Thus $\mu_\alpha(F, W) \leq a_1 + \alpha < (\sum_{i=1}^{n} a_i)/n + (k/n)\alpha$, concluding the proof.

\begin{thebibliography}{9}

Received: March 1, 2006