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Saturation Throughput Analysis in IEEE 802.11 DCF using semi-Markov Model

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Abstract

A semi-Markov model to analyze the saturation throughput of the IEEE 802.11 Distributed Coordination Function (DCF) is proposed in this letter. Compared to the Markov model [2], the number of states is reduced and the backoff process is simplified by the semi-Markov model. Furthermore, this model considers both transmission errors and packet retry limit as specified in the 802.11 standard. This analytical model is validated by using the NS-2 simulator and more close to the simulation results.

Keywords: IEEE 802.11 DCF, saturation throughput analysis, semi-Markov model.

1 Introduction

The IEEE 802.11 [1] is the dominating standard for Wireless Local Area Networks (WLANs) and employs the DCF as the essential medium access protocol. In DCF, a station transmits if the medium is idle for a period of time. If the medium is sensed busy, the station waits until the current transmission is done. The station then defers for a randomly chosen backoff time interval to minimize the probability of collisions before transmitting. Each station maintains a retry count that indicates the number of retransmission of a packet. If the retry count reaches the pre-defined limit, the packet is discarded. Many research efforts have been conducted on modeling IEEE 802.11 DCF. Bianchi [2] used the Markov chain to model the idealistic assumption that packet retransmission is unlimited and a packet is being transmitted continuously until its successful reception. Wu [3] extended Bianchi's model to include the finite packet retry limit as defined in the IEEE 802.11 standard. In this letter, a semi-Markov model is proposed to simplify the backoff process. Other than the Markov models was proposed in [2], [3], the semi-Markov

Table 1: Summary of notations

W_i	Maximum backoff window size in the backoff stage i.
$m^{'}$	The maximum backoff stage (i.e. the maximum retransmission limit).
$\mu_{i,j}$	The expected amount of time the process spends in state (i, j).
$E[T]$	The average length of a slot time.
τ	The stationary probability that a WSTA transmitting in a randomly chosen slot.
P	The probability that a transmission fails.
P_{tr}	The probability that at least one transmission occurs in a randomly chosen slot.
P_{er}	The packet error rate.
P_s	The conditional probability that this transmission is without collision.
H	PHY header plus MAC header.
L	The packet payload length.
σ	The duration of an empty slot.
δ	The propagation time.
T_s	The average time the channel is sensed busy due to a success transmission.
T_c	The average time the channel is sensed busy due to a collision.
T_e	The average time the channel is sensed busy due to an error transmission.
S	The normalized system throughput within a WLAN cell.
R	The channel bit rate.

model is used in this letter since the duration time in each state is different. Furthermore, this model takes into account both the packet error rate (PER) which depend on the channel bit error rate and the packet retransmission limit. This analytical model is validated by using the NS-2 and simulation results show that the proposed model is more close to the simulation results than that in [2].

2 Saturation Throughput Analysis

Table 1 summaries the notations and definitions used in this letter and the semi-Markov model is depicted in Fig. 1.

State (i, j) of a WSTA is defined by the backoff stage i and the current value of its backoff window size is $j \in (W_i - 1)$. In the beginning $(i = 0)$, j is uniformly chosen between $(0, W_0 - 1)$, where W_0 is the initial window size. When the WSTA enters the backoff stage i, the backoff window size is reinitialized to a random value between $(0, W_i - 1)$. The backoff window size is increased by a factor of 2 until it reaches the maximum value (W_m) in the m-th retransmission, and then it is frozen until the m'-th retransmission. After the m'-th retransmission, the backoff window size is reinitialized to $(0, W_0 - 1)$ whether this transmission is successful or failure. Therefore, we have

$$W_i = \begin{cases} W_0 \cdot 2^i, & 0 \le i < m \\ W_0 \cdot 2^m, & m \le i \le m'. \end{cases} \tag{1}$$

A WSTA can only transmit packets when the backoff timer reaches 0 or it must wait. The backoff timer is decreased by 1 in each slot time. Note that the slot time is referred to as the constant σ or the variable time intervals T_s, T_c, and T_e. The key approximation in this model is that the probability p is independent of the states. Those states can be classified into three kinds:
(i, k_i): It denotes the waiting state in backoff stage i since a WSTA is waiting when its backoff timer $k_j \in (1, W_i - 1)$. Note that the backoff timer is decreased by 1 until it reaches 0, there are no other cases. So, the probability of (i, k_i) transits to $(i, 0)$ is $1/(W_i - 1)$. The duration time μ_{i,k_i} can be given as:
μ_{i,k_i} = E[number of slots waiting in (i, k_i)] $\cdot$ E[T]

$$= \sum_{k_i=1}^{W_i-1} \frac{k_i}{W_i - 1} \cdot E[T] = \frac{W_i}{2} E[T]. \tag{2}$$

$(i, 0)$: It denotes the transmission state in backoff state i,$\forall i \in (0, m' - 1)$. If a failed transmission occurs in $(i - 1, 0)$, the process transits to $(i, 0)$ with a probability p/W_i or transits to (i, k_i) with a probability $p(W_i - 1)/W_i$. On the contrary, if this transmission is successful, it transits to $(0, 0)$ with a probability $(1 - p)/W_0$ or transits to $(0, k_0)$ with a probability $(1 - p)(W_0 - 1)/W_0$. Since a WSTA can transmit immediately in $(i, 0)$, and then wait one slot time to get ACK. So, the duration time $\mu_{i,0}$ can be given as:

$$\mu_{i,0} = 1 \cdot E[T] = E[T]. \tag{3}$$

$(m', 0)$: It denotes the transmission state in backoff state m'. In this state, whether the transmission is successful or not, the WSTA transits to $(0, 0)$ with a probability $1/W_0$ or transits to $(0, k_0)$ with a probability $(W_0 - 1)/W_0$. The duration time in $(m', 0)$ is equal to that in $(i, 0)$.
The transition probabilities can be summarized as

$$\begin{cases} P\{(i,0)|(i,k_i)\} = 1/(W_i - 1) & i \in (0, m') \\ P\{(0,0)|(i,0)\} = (1-p)/W_0 & i \in (0, m'-1) \\ P\{(0,k_0)|(i,0)\} = (1-p)(W_0-1)/W_0 & i \in (0, m'-1) \\ P\{(i,0)|(i-1,0)\} = p/(W_i) & i \in (0, m') \\ P\{(i,k_i)|(i-1,0)\} = p(W_i-1)/W_i & i \in (1, m') \\ P\{(0,0)|(m',0)\} = 1/W_0 & \\ P\{(0,k_0)|(m'0)\} = (W_0-1)/W_0 & \end{cases} \tag{4}$$

Let $\pi_{i,j}$ be the stationary distribution of the chain, denoting the limiting probability of the WSTA to be in state (i,j). Obviously, we have

$$\pi_{i,0} = \pi_{i-1,0} \cdot p, 0 < i \leq m' \tag{5}$$

, which yields the following:

$$\pi_{i,0} = \pi_{0,0} \cdot p^i, 0 < i \leq m' \tag{6}$$

The probability π_{i,k_i} for $0 < i < m'$, can be given as

$$\pi_{i,k_i} = \frac{p(W_i - 1)}{W_i} \cdot \pi_{i-1,0} = \frac{p^i(W_i - 1)}{W_i} \cdot \pi_{0,0} \tag{7}$$

,and for π_{0,k_0}:

$$\pi_{0,k_0} = \frac{(1-p)(W_0 - 1)}{W_0} \cdot \sum_{i=0}^{m'-1} \pi_{i,0} + \frac{W_0 - 1}{W_0} \cdot \pi_{m',0}$$

$$= \frac{(1-p)(W_0 - 1)}{W_0} \cdot \sum_{i=0}^{m'-1} p^i \cdot \pi_{i,0} + \frac{W_0 - 1}{W_0} \cdot p^{m'} \cdot \pi_{0,0}$$

$$= \frac{W_0 - 1}{W_0} \cdot \pi_{0,0}. \tag{8}$$

From (7) and (8), π_{i,k_i}can be generally expressed as a function of $\pi_{0,0}$:

$$\pi_{i,k_i} = \frac{(W_i - 1)}{W_i} \cdot p^i \cdot \pi_{i,0}, 0 \leq i \leq m' \tag{9}$$

Let $P_{i,j}$ be the proportion of time that the process is in state (i,j), we can get

$$P_{i,0} = \frac{\pi_{i,0}\mu_{i,0}}{\sum_{i=0}^{m'} \pi_{i,0}\mu_{i,0} + \sum_{i=0}^{m'} \pi_{i,k_i}\mu_{i,k_i}}$$

$$= \frac{p^i}{\frac{1-p^{m'+1}}{1-p} + \frac{1}{2}\left[W_0\left(\frac{1-(2p)^{m+1}}{1-2p} + \frac{2^m p^{m+1}\left(1-p^{m'-m}\right)}{1-p}\right) - \frac{1-p^{m'+1}}{1-p}\right]} \tag{10}$$

Since transmission occurs only in states $(i,0)$, the probability τ can expressed as

$$\tau = \sum_{i=0}^{m'-1} P_{i,0} = \frac{\frac{1-p^{m'+1}}{1-p}}{\frac{1-p^{m'+1}}{1-p} + \frac{1}{2}\left[W_0\left(\frac{1-(2p)^{m+1}}{1-2p} + \frac{2^m p^{m+1}\left(1-p^{m'-m}\right)}{1-p}\right) - \frac{1-p^{m'+1}}{1-p}\right]} \tag{11}$$

Since there are stations, a failed transmission of a station can happen due to the collision with the remaining stations or an error packet. Since both events are independent, the probability can be the expressed as

$$p = 1 - (1-\tau)^{n-1} \cdot (1 - p_{er}) \tag{12}$$

Equations (13) and (14) represent a non-linear system in the two unknowns p and τ, which can be solved using numerical techniques. Since n stations contending in the network and each transmits with probability τ. We can get

$$p = 1 - (1-\tau)^n \tag{13}$$

Since p_s is given by the probability that exactly one WSTA transmit on channel, conditioned on p_{tr}. So, it can be given in the following:

$$p_s = \frac{C_1^n \tau \cdot (1-\tau)^{n-1}}{p_{tr}} = \frac{n \cdot \tau \cdot (1-\tau)^{n-1}}{1-(1-\tau)^n} \tag{14}$$

The normalized system throughput S is defined as the fraction of time the channel is used to transmit the packet payload successfully. We can express S as the ratio

$$S = \frac{\text{E[payload transmitted in a slot time]}}{E[T]} \tag{15}$$

A slot time can be idle or sensed busy due to (1) a successful transmission, (2) a collision, and (3) an error frame. The probability of an empty slot is $(1-p_{tr})$, and the probabilities, $p_{tr} \cdot p_s \cdot (1-p_{tr}), p_{tr} \cdot (1-p_s)$ and correspond to a successful transmission, a collision, and an error frame, respectively. Assuming that all packets are of the same size, we can now express S as

$$S = \frac{p_{tr} \cdot p_s \cdot (1-p_{er}) \cdot L}{(1-p_{tr}) \cdot \sigma + p_{tr} \cdot p_s \cdot (1-p_{er}) \cdot T_s + p_{tr} \cdot (1-p_s) \cdot T_c + p_{tr} \cdot (p_s) \cdot P_{er} \cdot T_e} \tag{16}$$

3 Numerical Results

The model is validated and compared based on the simulator NS-2 [4], and assumes each station has enough data to be sent to obtain the saturated throughput. All the system parameters used in model and simulations are based on the parameters in [2]. Note that we assume that the payload size is fixed with 1024 bytes and the channel bit rate is 1 Mbps. In the model validation, we will vary the number of stations to see the effect of throughput degradation due to the increased collision probability for both of basic mode and RTS/CTS mode. The results for both basic and RTS/CTS cases shown in Fig. 2 are obtained with a 95% confidence level in a less than 2% variation from the mean. The deviation between our analysis and simulation results is less than 5% and is more close to the simulation results than the Markov model in [2] since our model considers the retransmission limit and the PER. From Fig. 2, we can conclude that RTS/CTS access method is useful to reduce the performance degradation which is insensitive the number of stations. Note that our analysis still overestimates the throughput in the RTS/CTS mode. It may due to the fact that there are some routing packets, which are transmitted by broadcast and RTS/CTS handshaking is ignored.

4 Conclusion

In this letter, a semi-Markov model is proposed to evaluate the saturation throughput of the IEEE 802.11 DCF. The semi-Markov model can simplify the 802.11 backoff process and can reduce the states in Markov model. Furthermore, we consider both packet retransmission limit and transmission errors which make the analysis more accurate. This model can be used for both the basic access mode and the RTS/CTS mode in DCF. We validate this model by using the NS-2 simulator and the validation showed that this model is more close to the simulation results than the Markov model. From the analysis and simulation results, we conclude that the performance of DCF is highly dependent on the number of stations, and increases the number of stations will degrade the system throughput. Moreover, the RTS/CTS access method is useful to reduce the performance degradation which is insensitive the number of stations.

References

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Figure captions:

Figure 1. The semi-Markov model for the 802.11 backoff process.

Figure 2. Saturation throughput evaluation for $W_0 = 32$,$m^{'} = 7$ and $P_{er} = 0.05$.