Generalization of Newsboy Problem with Demand Distribution Satisfying the SCBZ Property

Dowlath Fathima¹, P.S. Sheik Uduman and S. Srinivasan

Mathematics Department, BS Abdur Rahman University, Chennai, India
¹dow_lath@yahoo.co.in

Abstract

In certain situations the inventory process gets terminated after a finite duration which implies that, the entire inventory on hand has no value after a finite duration. In order to control this, an inventory model is developed. The demand distributions which satisfy the so called setting the clock back to zero property is used to depict the several individual demands for a single product. The aim of this paper is to show how SCBZ property is applied in case of single period, single product inventory model with several individual source of demand. The objective is to derive the optimal stock level or the optimal reorder level. Hence the optimal order quantity is derived. Numerical illustrations are also provided as an example for the validation of our model.

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1. Introduction

The Newsboy /Newsvendor problem is under the category of finite inventory process. There is a onetime supply of the newspapers per day and the demand is probabilistic. The classical Newsboy problem is to find order quantity for the products by which the expected profit is maximized in a single period probabilistic demand framework. Stochastic initial inventory may also be applied to the situation where decision about the order quantity must be made before the start of the period and the available inventory at the decision time may decrease stochastically due to the several factors.
The classical model assumes that if the order quantity is larger than the realised demand, the items which are left over at the end of period are sold at a salvage value or are disposed of. Further in cases of shortages unsatisfied demand is lost.

The Newsboy problem is one in which the situation is such that the newspaper should be sold the same day itself. Every unit of the newspaper sold gives some profit, but at the same time, if it is not sold at the same day it has only negligible value. Hence excess supply will result in loss, which is equivalent to the inventory holding cost. If the supply is not adequate then shortage occurs which in turn will result in shortage loss. Under these conditions how many units of the newspaper to be ordered is the prime problem of interest. It is interesting to note that the conventional inventory holding cost is introduced in a terms of a new cost called the salvage loss.

The basic Newsboy model has been discussed in Hansmann.F [5]. Also a partial review of the Newsboy problem has been conducted in a textbook by Silver.et al [12]. According to the review the Researchers have followed two approaches to solve the newsboy problems. In the first approaches the expected costs was overestimating and demand was underestimating. In the second approaches the expected profit was maximized. But both the approaches yield the same results. In this paper we use the first approaches to solve the newsboy problem where the demand is considered and it is shown that how by a right decision, the expected cost due to lost sale could be minimised.

Lau [7] studied the price dependent demand in the Newsboy problem, Chin Tsai [3] studied the generalisation of Chang and Lin’s model in a multi location Newsboy problem in which the actual model of Chang and Lin’s model was extended by adding the delay supply product cost. In chin Tsai [3], the Newsboy problem was solved in the centralised and decentralised system. Nicholas A. Nechval [8] showed how the statistical inference equivalence principle could be employed in a particular case of finding the effective statistical solution for the multiproduct Newsboy problem with constraints.

Traditional Newsboy models focus on risk neutral decisions makers (ie) optimizing the expected profit or cost. But experimental finding states that the actual quantity ordered derivatives from the optimal quantity is derived from the classical newsboy model. In view of this a number of papers have been devoted to risk analysis of newsboy problems [6, 7, 8 and 9].

Guinquing zhang and Yin Feng xu [4] considered the Newsboy problem with range information. In Jixan Xiao, Fangling Lu [6] a stochastic Newsboy inventory control model was considered and it was solved on multivariate product order and pricing. Also in this paper how much order of inventory should be purchased and maintained was suggested.
In this paper a generalisation of the actual problem in P.S Sheik uduman [9] is derived using the demand distribution which satisfies the SCBZ property. The problem is verified using the numerical illustration.

2. Assumption and Notations

\( C_1(X_i) \) = The cost of each unit of Newspapers purchased for several individual demand but not sold called salvage loss.

\( C_2(X_i) \) = The shortage cost arising due to each unit of unsatisfied individual demand of Newspapers.

\( X_i \) = Random variable denoting the several individual demands at the \( i^{th} \) location where \( i=1,2,3,...,n \).

\( f_i(X_i) \) = The probability density function

\( S_i \) = Supply level and \( \hat{S}_i \) is the optimal value of \( S_i \).

\( E(X_i) \) = The expected several individual demand before the truncation point \( x_0 \) is \( E(X_i) = 1/\theta_1 \) and after the truncation point \( x_0 \) is

\[ E(X_i) = \frac{1}{2\theta_2}. \]

\( f_1(X_i, \theta_1) \) = The probability distribution function when \( X_i \leq x_0 \)

\( f_2(X_i, \theta_2) \) = The probability distribution function when \( X_i > x_0 \)

\( \theta_1 \) = The parameter of the demand distribution prior to the truncation point \( x_0 \)

\( \theta_2 \) = The parameter of the demand distribution posterior to the truncation point \( x_0 \)

2.1. Basic Inventory Models:

➢ The decentralized inventory system:

The decentralized inventory system is a system in which a separate inventory is kept to satisfy the several individual source of demand and there is no reinforcement between locations of demands. The aim is to minimize the expected total cost \( E(C) \):

The expected total cost function is given in the form
In Hansmann,F [1] the basic Newsboy problem is derived as follows

The expected total cost function is given by

\[
E(C) = \int_0^s C_1 (S - x_i) f_i(x_i) dx_i + \int_s^\infty C_2 (x_i - S_i) f_i(x_i) dx_i
\]

(1)

Where \( i = 1, 2, 3, \ldots, n \)

To find optimal \( \hat{S} \) we have

\[
\frac{dE(C)}{dS} = 0
\]

(3)

\[
\frac{dE(C)}{dS} = C_1 \left( \int_0^s (S - x) f(x) dx \right) + C_2 \left( \int_s^\infty (x - S) f(x) dx \right)
\]

(4)

Since the integrals involve ‘S’ both in integrand and in the limits here the differentiation of integral formula was used to solve the problem using \( \frac{dE(C)}{dS} = 0 \)

Hence it is proved to result in the following equation.

\[
F(\hat{S}) = P(X \leq \hat{S}) = \frac{C_2}{C_1 + C_2}
\]

(5)

Given the probability distribution of the demand \( X \) using the expression for \( F(\hat{S}) \), the optimal \( \hat{S} \) can be determined. This was the basic Newsboy problem discussed by Hanssmann [5].

3. Main result

In this paper a different version of Newsboy problem is discussed and the optimal \( S_i \) is derived by assuming several individual demand for the product namely newspaper which undergoes a seasonal change.

The demand distribution in this paper is considered in a decentralised inventory system in which the expected cost is minimized and the quantity of the optimal stock level should be ordered is suggested. The Newsboy problem in the case of single demand was studied by P.S Sheik Uduman[9]. In the present paper the Newsboy problem is studied in case of several individual sources of demand. The
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Demand distribution here undergoes the parametric change and it satisfies the SCBZ property discussed by Raja Rao and Talwakar [10].

A random variable $X$ is said to satisfy the SCBZ property if

\[
S_i(x_i + x, \theta_1, \theta_2) = S_i(x_i, \theta_2) \quad i=1,2,3,...,n \tag{6}
\]

where $S_i(x_i, \theta_1)$ denotes the survivor function

which is $S(x_i, \theta_1) = 1 - F(x_i, \theta_1)$. Here $x_0$ is called the truncation point of the random variable $x_i$. The meaning of the SCBZ property is that the probability distribution of the random variable undergoes a parametric change after the truncation point $x_0$. The several individual demands for the single product is assumed to be a random variable $x_i$ which satisfies SCBZ property.

Accordingly we define the pdf as

\[
f_i(x_i) = \begin{cases} \theta_1 e^{-\theta_1 x_i} & \text{if } x_i \leq x_0 \\ \theta_2 e^{-\theta_2 x_i} & \text{if } x_i > x_0 \end{cases} \tag{7}
\]

where $x_0$ is constant denoting truncation point. The probability distribution function is denoted as

\[
f_1(x_i, \theta_1) \quad \text{if } X_i \leq x_0
\]
\[
f_2(x_i, \theta_2) \quad \text{if } X_i > x_0
\]

such a model has been discussed by Sathiyamoorthy and Parthasarthy [11].

The probability distribution function defined above satisfies the SCBZ property under the above assumptions and the optimal $S_i$ is to be derived. Now, the total expected cost is given by

\[
E(C) = C_i(X_i) \left[ \int_0^{S_i} (S_i - X_i) f_1(X_i, \theta_1) \, dx_i + \int_{S_i}^\infty (S_i - X_i) f_2(X_i, \theta_2) \, dx_i \right] \\
+ C_2(X_i) \left[ \int_0^{S_i} (X_i - S_i) f_1(X_i, \theta_1) \, dx_i + \int_{S_i}^\infty (X_i - S_i) f_2(X_i, \theta_2) \, dx_i \right] \tag{9}
\]

Using the p.d.f. given in (7) for $f_1(X_i, \theta_0)$ and $f_2(X_i, \theta_0)$ in (9) we get

\[
E(C) = C_i(X_i) \left[ \frac{d}{dx_i} \left[ \int_0^{S_i} (S_i - X_i) 0, e^{-\theta_1 x_i} \, dx_i + \int_{S_i}^\infty (S_i - X_i) e^{s_i(\theta_1 - \theta_0)} 0, e^{-\theta_2 x_i} \, dx_i \right] \right]
\]
\[
\frac{dE}{dS_i} = 0
\]  

To find \( \frac{dE}{dS_i} \), we have

\[
I_1 = C_1(X_i) \left\{ \theta_1 \int_0^{s_i} (S_i - x_i) e^{-\theta_1 x_i} dx_i + \theta_2 \int_0^{s_i} (S_i - x_i) e^{\theta_2(\theta_2 - \theta_1)} e^{-\theta_1 x_i} dx_i \right\}
\]

And

\[
I_2 = C_2(X_i) \left\{ \theta_1 \int_0^{s_i} (X_i - S_i) e^{-\theta_1 x_i} dx_i + \theta_2 \int_0^{s_i} (X_i - S_i) e^{\theta_2(\theta_2 - \theta_1)} e^{-\theta_1 x_i} dx_i \right\}
\]  

To find \( \frac{dI}{dS_i} \), we have

\[
C_1(X_i) \left\{ \theta_1 \int_0^{s_i} (S_i - x_i) e^{-\theta_1 x_i} dx_i \right\} = C_1(X_i) \theta_1 \left\{ \left[ -\frac{(S_i - x_i) e^{-\theta_1 x_i}}{\theta_1} \right]_0^{s_i} + \left[ \frac{e^{-\theta_1 x_i}}{\theta_1^2} \right]_0^{s_i} \right\} = C_1(X_i) \theta_1 \left\{ \left[ -\frac{e^{-\theta_1 x_i}}{\theta_1^2} \right]_0^{s_i} + \frac{1}{\theta_1^2} \right\}
\]

\[
C_1(X_i) \left\{ \theta_1 \int_0^{s_i} (S_i - x_i) e^{-\theta_1 x_i} dx_i \right\} = C_1(X_i) \left[ 1 - e^{-\theta_1 s_i} \right]
\]

\[
C_1(X_i) \left\{ \theta_2 \int_0^{s_i} (S_i - x_i) e^{\theta_2(\theta_2 - \theta_1)} e^{-\theta_1 x_i} dx_i \right\} = C_1(X_i) \left\{ \theta_2 \int_0^{s_i} e^{\theta_2(\theta_2 - \theta_1)} e^{-\theta_1 x_i} dx_i \right\} = C_1(X_i) \left\{ \theta_2 e^{\theta_2(\theta_2 - \theta_1)} \left[ e^\theta \right]_0^{s_i} \right\}
\]

\[
C_1(X_i) \left\{ \theta_2 \int_0^{s_i} (S_i - x_i) e^{\theta_2(\theta_2 - \theta_1)} e^{-\theta_1 x_i} dx_i \right\} = C_1(X_i) \left\{ \theta_2 e^{\theta_2(\theta_2 - \theta_1)} \left[ e^\theta \right]_0^{s_i} \right\}
\]

\[
= C_1(X_i) \left\{ \theta_2 e^{\theta_2(\theta_2 - \theta_1)} \left[ e^{-\theta_1 s_i} - e^{-\theta_1 s_i} \right] \right\}
\]

\[
(13)
\]

\[
\text{(14)}
\]
Adding (13) + (14) we get
\[ \frac{dl_i}{dS_i} = C_i(X_i) \left[ 1 - e^{-\theta_1 s_i} - e^{\theta_1 s_i} \right] \] (15)

Similarly to find \( \frac{dl_j}{dS_j} \) we have
\[
\begin{align*}
C_2(X_i) \left\{ \theta_1 \int_{s_i}^{x_i} (x_i - s_i) e^{-\theta_1 x_i} dx_i \right\} &= -C_2(X_i) \theta_1 \int_{s_i}^{x_i} e^{-\theta_1 x_i} dx_i \\
&= -C_2(X_i) \theta_1 \left[ \frac{e^{-\theta_1 x_i}}{-\theta_1} \right]_{s_i}^{x_i} \\
C_2(X_i) \left\{ \theta_1 \int_{s_i}^{x_i} (x_i - S_i) e^{-\theta_1 x_i} dx_i \right\} &= C_2(X_i) \left[ e^{-\theta_1 x_i} - e^{\theta_1 x_i} \right] \\
\end{align*}
\]
(16)
\[
\begin{align*}
C_2(X_i) \left\{ \theta_2 e^{\theta_2 \theta_1} \right\} \left\{ \left\{ (x_0 - S_i) e^{\theta_2 x_i} \right\} - \left\{ e^{\theta_2 x_i} - \theta_2 \right\} \right\} \\
C_2(X_i) \left\{ \theta_2 \int_{s_i}^{x_i} (x_i - S_i) e^{\theta_2 x_i} dx_i \right\} &= C_2(X_i) \left\{ \theta_2 e^{\theta_2 \theta_1} \int_{s_i}^{x_i} (x_i - S_i) e^{\theta_2 x_i} dx_i \right\} \\
C_2(X_i) \left\{ \theta_1 \int_{s_i}^{x_i} (x_i - S_i) e^{-\theta_1 x_i} dx_i \right\} + \theta_2 \int_{s_i}^{x_i} (x_i - S_i) e^{\theta_2 x_i} dx_i &= -C_2(X_i) e^{-\theta_1 x_i} \\
\end{align*}
\]
(17)

(16) + (17) give
\[
\frac{dl_j}{dS_j} = -C_2(X_i) e^{-\theta_1 x_i}
\]

Hence \( \frac{dE(C_i)}{dS_i} = 0 \Rightarrow \frac{dl_i}{dS_i} + \frac{dl_j}{dS_j} = 0 \)

Therefore
\[
C_i(X_i) \left( 1 - e^{-\theta_1 s_i} - e^{\theta_1 s_i} \right) - C_2(X_i) \left( e^{-\theta_1 s_i} \right) = 0
\]
\[ C_1(x_i) - C_1(x_i)(e^{-\theta_1}x_0e^{\theta_2-x_0}) - C_2(x_i)(e^{-\theta_2}x_0) = 0 \]
\[ C_1(x_i) = C_1(x_i)(e^{-\theta_1}x_0e^{\theta_2-x_0}) + C_2(x_i)(e^{-\theta_2}x_0) \]

Taking log on both sides we get
\[
\log C_1(x_i) = \log C_1(x_i) - \theta_2 S_i + x_0(\theta_2 - \theta_1) + \log C_2(x_i) - \theta_2 S_i \\
\log C_1(x_i) - \log C_1(x_i) + x_0(\theta_2 - \theta_1) + \log C_2(x_i) = \theta_2 S_i + \theta_2 S_i \\
\log C_2(x_i) + x_0(\theta_2 - \theta_1) = 2\theta_2 S_i \\
S_i = \frac{\log C_1(x_i) + x_0(\theta_2 - \theta_1)}{2\theta_2} \quad (18)
\]

By substituting the values of \( C_2(x_i), x_0, \theta_1 \) and \( \theta_2 \) we obtain the value of \( S_i \) which satisfies equation (18). The value of \( S_i \) can also be evaluated using a suitable computer program.

**4. Numerical Illustration**

In the numerical example \( C_2(x_i) = 5 \) is fixed and \( \theta_1, \theta_2, x_0 \) is varied accordingly and these value are substituted in equation (18) to obtain the value of \( S_i \).

**Case i)** \( C_2(x_i) = 5, \theta_2 = 2.0, x_0 = 20 \)

<table>
<thead>
<tr>
<th>( \theta_1 )</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_i )</td>
<td>5.7</td>
<td>2.6</td>
<td>0.17</td>
</tr>
</tbody>
</table>

**Case ii)** \( C_2(x_i) = 5, \theta_1 = 1.5, x_0 = 20 \)
5. Numerical Inference

From the numerical illustrations and corresponding graphs the following conclusions may be drawn.
Case i): If the parameter $\theta_1$ of the demand distribution prior to the truncation point $x_0$ is varied exponentially then the expected demand will decrease because $E(X_i) = 1/\theta_1$ this implies that whenever $\theta_1$ increases the demand will decreases and hence a smaller supply size $S_i$ for several individual demand is suggested.

Case ii): When the parameter $\theta_1$ is fixed and $\theta_2$ which denotes the parameter of the demand distribution posterior to the truncation point $x_0$ is increased, then a corresponding increase in supply size $S_i$ is suggested.

Case iii): If both $\theta_1$ and $\theta_2$ are fixed and if the truncation point $x_0$ increases then there will be an increase in the supply size. This is due to the fact that the expected individual demand before the truncation point $x_0$ is $E(X_i) = 1/\theta_1$ and after the truncation $x_0$ it is $E(X_i) = 1/\theta_2$. Therefore the demand after $x_0$ is smaller. Hence as $x_0$ increases the truncation point $x_0$ increases. Therefore an increased inventory is suggested.

6. Conclusions

In this paper the Newsboy problem is considered under the several individual demands which are satisfied using the SCBZ property. Also in the numerical illustration, the comparative statics shows that the more risk averse the Newsvendor if he orders a less quantity. Hence the optimal order level/ Supply $S_i$ is less than the risk neutral counterpart. Moreover, the optimal order quantity is increasing in the emergency purchase price (salvage value).

Our model framework can be extended in several ways. An obvious extension would be to consider this as Newsvendor model for two product or multiproduct, in which case the expression would be more complex and it will have complex probability functions and integrations. In such a condition search method and computer programming are adequate to solve the problem.

References


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