A Fuzzy Approach on Vendor Managed Inventory Policy

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Abstract
This paper deals with one-vendor, multi-buyer integrated production inventory model with consignment stock policy, a particular vendor managed inventory policy, in which inspection cost is included in the total cost to supply good quality items to the buyers. An analytical model is developed to obtain optimal replenishment decisions for both vendor and buyers. Fuzzy mathematical model is also developed in which setup costs, holding costs of vendor and buyers and inspection cost of the vendor are taken as triangular fuzzy numbers. Modified integration representation method is used for defuzzifying the total cost. Finally the benefit of CS policy is explained through a numerical example.

Mathematics Subject Classification: 03E72, 90B05

Keywords: Supply chain, consignment stock, inspection cost, triangular fuzzy numbers, defuzzification

1 Introduction
Supply chain management has enabled numerous firms to enjoy great advantages by integrating all activities associated with the flow of materials. The main goal of
supply chain management is to deliver superior quality goods at a lesser cost. For this, the companies have to strengthen their supply agreements and management of their inventories. Vendor managed inventory (VMI) represents an interesting approach to stock monitoring and control. Consignment Stock (CS) is a particular VMI policy which represents a successful strategy for both the buyer and the supplier. The most radical application of CS may lead to the suppression of the vendor inventory, as this actor uses the buyer’s warehouse to stock its finished products. As a counterpart the vendor with guarantee that the quantity stored in the buyer’s warehouse will be kept between a maximum level and a minimum one, also supporting the additional costs eventually induced by stock-out conditions. The buyer will pickup from its store the quantity of material needed to meet its production plans and the material itself will be paid to the buyer according to the agreement signed. This strategy spreads out rapidly in different manufacturing environments, confirming its strategic interest for companies and its positive attitude in being implemented in supply chains. When a large company interacts with small or medium sized vendors, the buyer may get maximum advantages from CS agreement. In case when a vendor sells the same device to different customers, he may be doubtful about the real advantages of CS agreement. Lucio Zavenella, Simone Zanoni [13] investigated single vendor-multibuyer environment where the CS policy may be implemented in supply chains. The ultimate end users will be highly satisfied if they receive all good quality items. This paper introduces inspection cost so that the customers satisfaction is guaranteed with an additional cost which maybe very small.

2. Literature Review


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2.2. Single-Vendor Multiple-Buyer Models

Joglekar [14] pointed out that purchase order sizes affect not only the vendor’s revenue stream but also the manufacturing cost stream. Joglekar and Tharthare [15] proposed an individually responsible and rational decision approach to lotsizes for one vendor and many buyers. Viswanathan and Piplani [21] proposed a model to study and analyse the benefit of coordinating supply chain by means of common replenishment epochs. Woo et al [22] considered an integrated inventory model where a single vendor purchases and processes raw materials and deliver finished items to multiple buyers. Boyaci and Gallego [3] analysed inventory and pricing policies that jointly maximize the profit of one wholesaler and multi retailers. Siajadi et al [19] proposed a multiple shipment policy for joint economic lotsize which allowed multiple shipments. Kim et al [16] considered three stage supply chain, the third level consists of multiple retailer who interact with the single manufacturer.

In this paper we have taken single vendor-two buyer production inventory model to determine the benefits of CS policy in which inspection cost is included with the total cost. Also setup costs, holding costs of both buyer and vendor and inspection cost are taken as triangular fuzzy numbers. The total cost is defuzzified using modified integration representation method.

3. Notations and Assumptions

\[ A_1 : \text{ batch setup cost faced by the vendor (€/setup)} \]
\[ A_{2i} : \text{ order emission cost faced by the } i^{\text{th}} \text{ buyer (€/order)} \]
\[ h_1 : \text{ vendor holding cost per item and per time unit (€/item time unit)} \]
\[ h_{2i} : \text{ } i^{\text{th}} \text{ buyer holding cost per item and per time unit (€/item time unit)} \]
\[ P : \text{ vendor production rate (continuous) (item/time unit)} \]
\[ d_i : \text{ demand rate seen by the } i^{\text{th}} \text{ buyer (continuous) (item/time unit)} \]
\[ N : \text{ number of buyers} \]
\[ T : \text{ ordering or production cycle time (time unit)} \]
\[ n_{ii} : \text{ } i^{\text{th}} \text{ buyer number of transport operations per production cycle time} \]
\[ C : \text{ inspection cost incurred by the vendor (€/item/time unit)} \]
\[ \tilde{A}_1 : \text{ fuzzy batch setup cost faced by the vendor (€/setup)} \]
\[ \tilde{h}_1 : \text{ fuzzy vendor holding cost per item and per time unit (€/item time unit)} \]
\[ \tilde{A}_{2i} : \text{ fuzzy } i^{\text{th}} \text{ buyer holding cost per item and per time unit (€/item time unit)} \]
\[ \tilde{A}_{2i} : \text{ fuzzy order emission cost faced by the } i^{\text{th}} \text{ buyer (€/order)} \]
\[ \tilde{C} : \text{ fuzzy inspection cost incurred by the vendor (€/item/time unit)} \]
\[ TC : \text{ average total cost of the system per time unit, function of } n_i \text{ and } T (€/item time unit)} \]
\[ \tilde{TC} : \text{ fuzzy average total cost of the system per time unit, function of } n_i \text{ and } T (€/item time unit)} \]
A cycle is defined as the period during which the vendor incurs in one setup activity, thus producing the amount of components to be delivered to the buyers so as to allow them to satisfy the demand seen by the buyers themselves during the cycle. The cycle is replicated identically within the time horizon. It is also assumed that $P > D$, where $D = \sum_i^Y d_i$.

**Case $h_{2,i} > h_2 \forall i$**

This situation refers to the assumption of items increasing their value while descending the production-distribution chain. As a consequence, goods are preferably kept in the vendor’s warehouses until the buyer asks for a further shipment.

**Case $h_{2,i} < h_1 \forall i$**

The opposite situation can be found in practice, especially as a consequence of the CS inventory parameter settings.

![Fig.1. Vendor and buyer stocks against time, with a production cycle time length equal to $T$](image)

The aim is to minimize the stock held by the vendor, shipping all the stocks available whenever a delivery is ready for transportation. The shipment policy is based on making equal-sized shipments (possibly different for different buyers) while production is taking place, the last shipment being made as soon as production finishes. According to the given notations, the total cost of the vendor per unit time is the sum of setup cost, holding cost and inspection cost.
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\[ \therefore \text{TC}_{\text{vendor}} = \frac{A_1}{T} + \frac{h_i T}{2P} \sum_{i=1}^{N} \frac{d_i^2}{n_i} + \frac{CP}{T} \quad \ldots (3.1) \]

The total cost of all the buyers is the sum of order emission cost and holding cost.

\[ \text{TC}_{\text{buyer}} = \sum_{i=1}^{N} \left[ \frac{n_i A_{2i}}{T} + \frac{T}{2} h_{2i} \left( 1 - \frac{d_i}{P} + \frac{d_i}{n_i P} \right) \right] \quad \ldots (3.2) \]

The total average cost for the whole system is given by

\[ \text{TC} = \text{TC}_{\text{vendor}} + \text{TC}_{\text{buyer}} \]

\[ \text{TC} = \frac{1}{T} \left( A_1 + \sum_{i=1}^{N} n_i A_{2i} + CP \right) + h_i \frac{T}{2} \sum_{i=1}^{N} \left[ \frac{d_i^2}{n_i P} \right] + \frac{T}{2} \sum_{i=1}^{N} h_{2i} d_i \left( 1 - \frac{d_i}{P} + \frac{d_i}{n_i P} \right) \quad \ldots (3.3) \]

when \( N = 1 \), the cost function is equal to the cost function of single vendor – single buyer CS case (Braglia and Zavenella, 2003) where \( T_D = n q \).

**The Joint Optimum:** The total cost given in (3) is taken as a function of 2 decision variables \( T \) and \( n_i \), \( i = 1, 2, 3, \ldots, N \). Whatever the value of \( n_i \), \( T \) will be determined by equating \( \frac{\partial \text{TC}}{\partial T} = 0 \) which gives

\[ T^* = \frac{2 \left( A_1 + \sum_{i=1}^{N} n_i A_{2i} \right) + PC}{h_i \sum_{i=1}^{N} \frac{d_i^2}{n_i P} + \sum_{i=1}^{N} h_{2i} d_i \left( 1 - \frac{d_i}{P} + \frac{d_i}{n_i P} \right)} \quad \ldots (3.4) \]

The \( T^* \) value leads to the minimum total cost (with respect to \( T \) alone)

\[ \text{TC}^* = \sqrt{2 \left( \sum_{i=1}^{N} (h_1 + h_{2i}) \frac{d_i^2}{n_i P} + h_{2i} d_i \left( 1 - \frac{d_i}{P} + \frac{d_i}{n_i P} \right) \right) \times \left( A_1 + \sum_{j=1}^{N} (n_j A_{2j}) + PC \right)} \quad \ldots (3.5) \]

However minimizing \( \text{TC}^* \) function requires that

\[ \frac{\partial}{\partial n_i} \left[ h_i \sum_{i=1}^{N} \frac{d_i^2}{n_i P} + \sum_{i=1}^{N} h_{2i} d_i \left( 1 - \frac{d_i}{P} + \frac{d_i}{n_i P} \right) \right] \left( A_1 + \sum_{j=1}^{N} (n_j A_{2j}) + PC \right) = 0 \]

where \( i = 1, 2, \ldots, N \).

\[ (\text{ie}) \left( -h_i \frac{d_i^2}{n_i^2 P} - h_{2i} \frac{d_i}{n_i P} \right) \left( A_1 + \sum_{j=1}^{N} (n_j A_{2j}) + PC \right) \]

\[ + A_{2i} \sum_{j=1}^{N} \left( h_i \frac{d_j^2}{n_j P} + h_{2j} d_j \left( 1 - \frac{d_j}{P} + \frac{d_j}{n_j P} \right) \right) = 0 \]
\[
\frac{(A_{2i})n_i^2p}{(h_i + h_{2i})d_i^2} = \frac{A_i + \sum_{j=1}^{N}(n_jA_{2j}) + PC}{\sum_{j=1}^{N}\left(\frac{h_jd_j}{n_jp} + h_{2j}d_j\left(1 - \frac{d_j}{p} + \frac{d_j}{n_jp}\right)\right)} = K
\]

and \(n_i = \frac{\sqrt{K}\sqrt{(h_i + h_{2i})d_i^2}}{\sqrt{(A_{2i})p}}\) where \(K\) is independent of individual \(n_i\).

Then \(K = \frac{A_i + \sum_{j=1}^{N}(n_jA_{2j}) + PC}{\sum_{j=1}^{N}\left(\frac{h_jd_j}{n_jp} + h_{2j}d_j\left(1 - \frac{d_j}{p} + \frac{d_j}{n_jp}\right)\right)}\)

After some algebraic steps, we obtain
\[
\sqrt{K} = \frac{A_i + PC}{\sqrt{\sum_{j=1}^{N}h_{2j}d_j\left(1 - \frac{d_j}{p}\right)}}
\]

and so \(n_i^* = \frac{(h_i + h_{2i})d_i^2(A_i + PC)}{\sqrt{(A_{2i})p\sum_{j=1}^{N}h_{2j}d_j\left(1 - \frac{d_j}{p}\right)}}\) \(\ldots (3.7)\)

The values determined for \(n_i^*\) and \(T^*\) minimizes the total cost \(TC^*\).

### 3.1. The Sequential Solution

Here the total cost of vendor present \(T\) as the decision variable while the \(n_i\) values are given and buyer’s present \(n_i\) as decision variable while \(T\) is given. Therefore the decisions are taken sequentially so that vendor’s optimal choice about \(T\) depends on \(n_i\) values which are got by differentiating \(TC_{vendor}\) with respect to \(T\).

\[T^{**} = \frac{2P(A_i + PC)}{h_i\sum_{j=1}^{N}\frac{d_j}{n_j}}\] \(\ldots (3.1.1)\)

where the \(n_i\) values are chosen by each buyer so as to minimize \(TC_{buyer}\) function. Differentiating \(TC_{buyer}\) with respect to \(n_i\) and equating to zero we have

\[n_i^{**} = Td_i\frac{h_{2i}}{2P(A_{2i})} i = 1, 2, \ldots N\] \(\ldots (3.1.2)\)

and \(n_i^{**}\) depends on \(T\).

The two equations maybe combined thus obtaining the sequential solution.
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\[ T^* = \sqrt{2P} \frac{(A_1 + PC)}{h_{2j}} \]\[ \sum_{j=1}^{N} d_j \sqrt{\frac{A_{2j}}{h_{2j}}} \] \[ n_i^* = \frac{(A_1 + PC)\sqrt{\frac{h_{2j}}{A_{2i}}}}{h_{1i}} \sum_{j=1}^{N} d_j \sqrt{\frac{A_{2j}}{h_{2j}}} \] \[ \ldots (3.1.3) \]

\[ n_i^* = \frac{(A_1 + PC)\sqrt{\frac{h_{2j}}{A_{2i}}}}{h_{1i}} \sum_{j=1}^{N} d_j \sqrt{\frac{A_{2j}}{h_{2j}}} \] \[ \ldots (3.1.4) \]

3.2. Numerical Illustration

We consider the situation with two buyers with different demands, holding and ordering costs.

\[ P = 3200 \text{ item/year} \quad d_1 = 500 \text{ item/year} \quad D = 1500 \text{ item/year} \quad d_2 = 1000 \text{ item/year} \]
\[ A_1 = 400 \ \epsilon \text{/setup} \quad A_{21} = 75 \ \epsilon \text{/order} \quad A_2 = 25 \ \epsilon \text{/order} \quad h_1 = 5 \ \epsilon \text{/item/year} \]
\[ h_{21} = 4 \ \epsilon \text{/item/year} \quad h_{22} = 4 \ \epsilon \text{/item/year} \quad C = 0.1 \ \epsilon \text{/item} \]

The application of joint optimum model leads to the following results.

<table>
<thead>
<tr>
<th>( n_1 )</th>
<th>( n_2 )</th>
<th>( T^* )</th>
<th>( TC_{\text{vendor}} )</th>
<th>( TC_{\text{buyer},1} )</th>
<th>( TC_{\text{buyer},2} )</th>
<th>( TC^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>0.535</td>
<td>1589.61</td>
<td>675.18</td>
<td>987.26</td>
<td>3252.05</td>
</tr>
</tbody>
</table>

The application of sequential solution model leads to the following results.

<table>
<thead>
<tr>
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<th>( TC^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>12</td>
<td>2.46</td>
<td>352.18</td>
<td>2293.67</td>
<td>3632.58</td>
<td>6278.43</td>
</tr>
</tbody>
</table>

From the above table, it is obvious that the buyers are in advantageous position and vendor is in a disadvantageous position when joint optimum policy is adopted instead of sequential solution.

4. Fuzzy Mathematical Model

Let \( \tilde{A}_1 = (A_{11}, A_{12}, A_{13}) \), \( \tilde{A}_{2i} = (\tilde{A}_{2i1}, \tilde{A}_{2i2}, \tilde{A}_{2i3}) \), \( \tilde{h}_1 = (h_{11}, h_{12}, h_{13}) \), \( \tilde{h}_{2i} = (\tilde{h}_{2i1}, \tilde{h}_{2i2}, \tilde{h}_{2i3}) \), \( \tilde{C} = (C_1, C_2, C_3) \) be triangular fuzzy numbers.

4.1. Joint Optimum:

The total average cost of the whole system is given by

\[ \tilde{T}C = \tilde{T}C_{\text{vendor}} + \tilde{T}C_{\text{buyer}} \] \[ \ldots (4.1.1) \]

\[ \tilde{T}C = \frac{T}{T} (\tilde{A}_1 + \sum n_i \tilde{A}_{2i} + \tilde{C}P) + \tilde{h}_1 \frac{T}{2} \sum_{i=1}^{N} \frac{d_i^2}{n_iP} + \frac{T}{2} \sum \left( \tilde{h}_{2i}d_i \left( 1 - \frac{d_i}{P} + \frac{d_i}{n_iP} \right) \right) \]

\[ \tilde{T}C = \]
\[
\begin{align*}
\left\{ \frac{1}{T} \left( A_{11} + \sum n_i A_{2i} + C_i P \right) + h_{11} \frac{T}{2} \sum_{i=1}^{N} \frac{d^2_{ii}}{n_i P} + \frac{T}{2} \sum \left( h_{2i} d_i \left( 1 - \frac{d_i}{P} + \frac{d_i}{n_i P} \right) \right) \right\}, \\
\left\{ \frac{1}{T} \left( A_{12} + \sum n_i A_{2i} + C_2 P \right) + h_{12} \frac{T}{2} \sum_{i=1}^{N} \frac{d^2_{ii}}{n_i P} + \frac{T}{2} \sum \left( h_{2i} d_i \left( 1 - \frac{d_i}{P} + \frac{d_i}{n_i P} \right) \right) \right\}, \\
\left\{ \frac{1}{T} \left( A_{13} + \sum n_i A_{2i} + C_3 P \right) + h_{13} \frac{T}{2} \sum_{i=1}^{N} \frac{d^2_{ii}}{n_i P} + \frac{T}{2} \sum \left( h_{2i} d_i \left( 1 - \frac{d_i}{P} + \frac{d_i}{n_i P} \right) \right) \right\}
\end{align*}
\]

\[\ldots (4.1.2)\]

\[P(\bar{T} C) = \frac{1}{6} \left\{ \frac{1}{T} \left( A_{11} + \sum n_i A_{2i} + C_i P \right) + h_{11} \frac{T}{2} \sum_{i=1}^{N} \frac{d^2_{ii}}{n_i P} + \frac{T}{2} \sum \left( h_{2i} d_i \left( 1 - \frac{d_i}{P} + \frac{d_i}{n_i P} \right) \right) \right\} + 4 \left\{ \frac{1}{T} \left( A_{12} + \sum n_i A_{2i} + C_2 P \right) + h_{12} \frac{T}{2} \sum_{i=1}^{N} \frac{d^2_{ii}}{n_i P} + \frac{T}{2} \sum \left( h_{2i} d_i \left( 1 - \frac{d_i}{P} + \frac{d_i}{n_i P} \right) \right) \right\} + \left\{ \frac{1}{T} \left( A_{13} + \sum n_i A_{2i} + C_3 P \right) + h_{13} \frac{T}{2} \sum_{i=1}^{N} \frac{d^2_{ii}}{n_i P} + \frac{T}{2} \sum \left( h_{2i} d_i \left( 1 - \frac{d_i}{P} + \frac{d_i}{n_i P} \right) \right) \right\} \right\} \]

\[\frac{\partial}{\partial r} P(\bar{T} C) = 0 \text{ gives} \]

\[T^* = \sqrt{\frac{2 \left\{ A_{11} + 4A_{12} + A_{13} + \sum (n_i (A_{2i_i} + 4A_{2i_2} + A_{2i_3})) + P(C_1 + 4C_2 + C_3) \right\}}{(h_{11} + 4h_{12} + h_{13}) \sum \left( \frac{d^2_{ii}}{n_i P} \right) + \sum \left( h_{2i} d_i \left( 1 - \frac{d_i}{P} + \frac{d_i}{n_i P} \right) \right) \left( h_{2i} + 4h_{2i} + h_{2i} \right)}} \]

\[\ldots (4.1.3)\]

\[TC^* = \frac{2 \left\{ \frac{1}{6} \sum (h_{11} + 4h_{12} + h_{13}) \frac{d^2_{ii}}{n_i P} + 4 \left( h_{1i} + h_{2i} A_{11} \right) \frac{d^2_{ii}}{n_i P} + \left( h_{13} + h_{2i} \right) \frac{d^2_{ii}}{n_i P} + \frac{1}{6} \left( d_i \left( 1 - \frac{d_i}{P} \right) \left( h_{2i} + 4h_{2i} + h_{2i} \right) \right) \right\}}{\left\{ \frac{A_{11} + 4A_{12} + A_{13}}{6} \right\} + \frac{1}{6} \sum n_j \left( A_{2j_i} + 4A_{2j_2} + A_{2j_3} \right) + P(C_1 + 4C_2 + C_3) \right\}} \]

\[\ldots (4.1.4)\]
The \( T^* \) value leads to the minimum total cost (with respect to \( T \) alone). But Total cost depends on \( n_i \) also. So if we differentiate \( P(\tilde{T}C) \) with respect to \( n_i \) and equating to zero we get after some algebraic steps:

\[
n_i^* = \sqrt{\frac{(h_{i1} + 4h_{i2} + h_{i3}) + (h_{2i1} + 4h_{2i2} + h_{2i3})d_i^2(A_{i1} + 4A_{i2} + A_{i3})}{(A_{2i1} + 4A_{2i2} + A_{2i3})P\sum_{j=1}^{N}\left(h_{2j1} + 4h_{2j2} + h_{2j3}\right)d_j\left(1 - \frac{d_j}{P}\right)}}.
\] \[4.1.6\]

### 4.2. Sequential Solution in Fuzzy Sense

In this case the vendor’s total costs present \( T \) as the decision variable while \( n_i \) values are given. Each buyer’s total costs present \( n_i \) as the decision variable while \( T \) is given. Hence differentiating \( \tilde{T}C \) vendor with respect to \( T \) and equating to zero we get

\[
T^{**} = \sqrt{2P(A_{11} + 4A_{12} + A_{13} + C_jP + 4C_2P + C_3P)}
\] \[\ldots (4.2.1)\]

where the \( n_i \) values are chosen by each buyer so as to minimize \( \tilde{T}C \) buyer function.

Differentiating \( \tilde{T}C \) buyer with respect to \( n_i \), the optimal value is given by

\[
n_i^{**} = T_d\sqrt{\frac{(h_{i1} + 4h_{i2} + h_{i3})}{2P(A_{2i1} + 4A_{2i2} + A_{2i3})}} \quad \text{where } i = 1, 2, \ldots, N. \ldots (4.2.2)
\]

and \( n_i^{**} \) depends on \( T \).

The two equations may be combined thus obtaining the sequential solution which is

\[
T^{**} = \frac{(A_{11} + 4A_{12} + A_{13} + C_jP + 4C_2P + C_3P)\sqrt{2P}}{(h_{i1} + 4h_{i2} + h_{i3})\sum_{j=1}^{N}d_j\left(A_{2j1} + 4A_{2j2} + A_{2j3}\right)}
\] \[\ldots (4.2.3)\]

\[
n_i^{**} = \frac{(A_{11} + 4A_{12} + A_{13})d_i}{(h_{i1} + 4h_{i2} + h_{i3})\sum_{j=1}^{N}d_j\left(A_{2j1} + 4A_{2j2} + A_{2j3}\right)}
\] \[\ldots (4.2.4)\]

### 4.3. Numerical Illustration

Let \( \tilde{A}_{11} = (390, 400, 410) \quad \tilde{A}_{21} = (74, 75, 76) \quad \tilde{A}_{22} = (24, 25, 26) \)
\[ \tilde{h}_1 = (4.9, 5, 5.1) \quad \tilde{h}_{21} = (3.9, 4, 4.1) \quad \tilde{h}_{22} = (3.9, 4, 4.1) \quad \tilde{C} = (0.09, 0.1, 0.11) \]

The application of joint optimum model leads to the following result.

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<th>( TC_{\text{buyer,2}} )</th>
<th>( TC^{**} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
<td>2.469</td>
<td>808.33</td>
<td>2337.24</td>
<td>3686.8</td>
<td>6832.37</td>
</tr>
</tbody>
</table>

The adoption of Joint optimum policy instead of sequential solution results in the following economic impact.

<table>
<thead>
<tr>
<th>VENDOR SAVINGS</th>
<th>BUYER 1 SAVINGS</th>
<th>BUYER 2 SAVINGS</th>
<th>2 TC SAVINGS</th>
</tr>
</thead>
<tbody>
<tr>
<td>-96.64 %</td>
<td>71.11 %</td>
<td>73.22 %</td>
<td>52.4 %</td>
</tr>
</tbody>
</table>

**SENSITIVITY ANALYSIS**

The aim is to identify the influence of parameters in various costs and savings in costs of members in supply chain.
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<table>
<thead>
<tr>
<th>h222</th>
<th>h223</th>
<th>d1</th>
<th>d2</th>
<th>P</th>
<th>T*</th>
<th>TC</th>
<th>N1</th>
<th>N2</th>
<th>T**</th>
<th>N1**</th>
<th>N2**</th>
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TC | SEQUENTIAL TC OF VENDOR | SEQUENTIAL TC OF BUYER1 | SEQUENTIAL TC OF BUYER2 | % OF SAVINGS IN VENDOR TC | % OF SAVING S FOR BUYER1 TC | % OF SAVING S FOR BUYER2 TC | % OF TC SAV
---|----------------------|------------------------|------------------------|------------------------|----------------------------|----------------------------|------------------------|
3252.07 | 808.33 | 2237.24 | 3686.8 | -96.64 | 71.11 | 73.22 | 52.40206 |
3276.36 | 883.42 | 2136.36 | 3367.44 | -83.60 | 68.53 | 71.01 | 48.86453 |
3344.21 | 807.37 | 2491.39 | 3613.3 | -104.39 | 72.44 | 74.26 | 55.63025 |
3804.41 | 1038 | 3330.33 | 5272.08 | -90.67 | 77.95 | 79.13 | 60.5386 |
3261.4 | 805.02 | 2306.15 | 3633.67 | -97.06 | 70.29 | 72.77 | 51.6603 |
3318.27 | 814.68 | 2690.02 | 3783.52 | -97.88 | 72.75 | 74.28 | 54.74767 |
3279.98 | 841.79 | 2229.57 | 3551.56 | -97.71 | 69.56 | 71.25 | 50.4735 |
3356.29 | 790.19 | 2407.36 | 4243.06 | -106.29 | 72.46 | 74.83 | 54.83157 |
4284.38 | 1267.67 | 4332.21 | 6867.62 | -77.54 | 81.27 | 82.2 | 65.63557 |
3533.45 | 999.99 | 2310.8 | 3436.4 | -75.90 | 67.28 | 68.83 | 47.39964 |
3369.19 | 873.37 | 2275.74 | 3588.39 | -92.78 | 69.97 | 72.08 | 49.90337 |
3459.57 | 807.13 | 2812.49 | 4131.65 | -102.42 | 71.89 | 73.98 | 54.18551 |
3579.44 | 872.14 | 2543.99 | 4021.9 | -98.28 | 70.78 | 72.48 | 51.87256 |
4141.21 | 1164.25 | 2930.92 | 3988.91 | -77.34 | 69 | 70.57 | 48.64625 |
4237.83 | 1133.87 | 3353.78 | 4705.59 | -84.04 | 72.44 | 73.93 | 53.92019 |
5963.82 | 2030.17 | 7139.16 | 10106.32 | -66.77 | 83.49 | 84.20 | 69.12441 |
The following are the inferences from Sensitivity analysis

- When holding cost of vendor increases $T^*, T^{**}, N_2^{**}$ decrease. There is an increase in % of savings of vendor, but % of savings for buyers and total cost increase.

- When ordering cost of vendor increases $T^*, T^{**}, N_2^{**}$ increase. There is an increase in % of savings of vendor decreases, but % of savings for buyers and total cost increase.

- When Inspection cost increases, $T^*, T^{**}$ increase and % of savings of vendor, buyers and total cost increase.

- When holding cost of buyer 1 increases $T^*$ decreases, $T^{**}$ increases. There is decrease in % of savings of vendor, but % of savings for buyers and total cost increase.

- When holding cost of buyer 2 increases $T^*$ decreases, $N_2^{**}$ increases. There is decrease in % of savings of vendor, but % of savings for buyers and total cost increase.

- When ordering cost of buyer 1 increases $T^*$ increases. There is decrease in % of savings of vendor, buyers and total cost.

- When ordering cost of buyer 2 increases $T^*, T^{**}$ increase. There is an increase in % of savings of vendor, but % of savings for buyers and total cost decrease.

- When holding cost ordering cost of vendor increase $T^*$, increases. There is an increase in % of savings of vendor, but % of savings for buyers and total cost decrease.

- When holding cost ordering cost of buyers 1 & 2 increase $T^*$ decreases. There is decrease in % of savings of vendor, but % of savings for buyers and total cost increase.

5. Conclusion

The present study aimed at proposing a model for a single-vendor. Multi-buyer integrated production inventory with adoption of CS vendor managed inventory where the inspection cost is included with the total system cost to ensure the supply of good quality items to the buyers. It is proved that total cost is slightly higher in this model than the model given in [13] but quality is guaranteed, which is an essential feature in supply chain. A fuzzy mathematical model is also formulated to find optimal inventory replenishment decisions by taking setup costs, inventory holding cost of buyer and vendors and inspection cost as triangular fuzzy numbers and modified integration representation method is used to defuzzify the total cost. It is observed that there is no difference between the crisp solution and defuzzified solution. The results of sensitivity analysis are summarized above. The cycle time has increased as compared to cycle time in [13] due to the screening time.
References


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