Inventory System with Defective Supplies and Financial Support for Customers

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Abstract

Inventory model of \((s, S)\) type is considered and realized orders with more than \((s + 1)\) defective items are rejected. Two models in which (i) all demands require financial support and (ii) a demand requires finance with a probability are treated. Steady state inventory level probabilities are presented. Numerical cases are studied.

Mathematics Subject Classifications: 30C45, 30C80, 90B05

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1 INTRODUCTION

Several researchers have studied single commodity inventory systems of \((s, S)\) type. Arrow, Karlin and Scrat [1] first analyzed such inventory systems. Danial and Ramanarayanan [2] discussed \((s, S)\) inventory system with random lead times and unit demand. Models with bulk demands were treated by Ramanarayanan and Jacob [9]. Murthy and Ramanarayanan [4, 5, 6, 7] considered several \((s, S)\) inventory systems. Kun-Shan Wu and Liang-Yuh Ouyang [3] have studied \((Q, r, L)\) inventory model with defective items. Murthy and Ramanarayanan [7] have discussed the \((s, S)\) inventory system with defective supplies where the supply is accepted only when at least \(s + 1\) units are good.
The whole lot is rejected otherwise and fresh order is made. In these models demands occur for one unit at a time and a unit is sold whenever a demand occurs. So far no models in these areas have been studied with requirement of finance for the purchase of a unit by a customer. It has been noticed that when the units are costly many customers require financial support for the purchase. These are widely felt in automobile and housing sectors where many banking institutions provide required finance for the purchase. In this paper we consider two models. In model A we consider the maximum of two random times one for the occurrence of a demand and one for the time to get finance for the purchase. The sale of a unit is made only when a demand and finance are both available. In model B we discuss the case in which with probability $\alpha$ the arriving demand requires finance and does not require finance with probability $1-\alpha$ and finance is made available after a random time. We treat these models under Markovian assumptions by identifying the infinitesimal generator and use block partitioning methods, Neuts [8]. We calculate the inventory level probabilities. Numerical examples are also presented.

2 Main Models

The main assumptions are given below.

1. Let $S$ be the maximum capacity of a warehouse. In the beginning the inventory is full and the stock level falls by one when a unit is sold.

2. When the inventory level falls to $s$ an order for $S-s$ units are made. The lead time distribution for the supply of $S-s$ units is exponential with parameter $\mu$.

3. If the number of perfect units supplied is less than $s+1$, the whole lot is rejected and fresh order is made for the supply of $S-s$ units. The supply is accepted only when at least $(s+1)$ units are good.

Let $p_i$ be the probability that $i$ units are defective out of $S-s$ units where $p_i > 0$ and $\sum_{i=0}^{S-s} p_i = 1$ and the rejection probability of a lot is $q = \sum_{j=S-2s}^{S-s} p_j$ and $\bar{q} = 1 - q$.

In section 2 we consider the model in which the finance is required by all demands and a unit is sold only when both of them occur.

In section 3 we treat the model when finance is required only with a probability $\alpha$. In section 4 we consider numerical cases.

2.1 Model A: Maximum of random times.

The following are the assumptions of the model A.
1. The demand occurs in a random time with exponential distribution with parameter $\lambda$ and the finance availability takes a random time with exponential distribution with parameter $a$. The time required to sell a unit has distribution function $F(x) = (1 - e^{-\lambda x})(1 - e^{-ax})$. The inter occurrence times of sales are independent and identically distributed random variables with cdf $F(x)$.

2. When a demand or a financial support is available the next demand or financial support can occur only after a sale is made.

3. When the inventory is dry arriving demand waits for finance and gets a unit of the inventory on the supply of finance if order has already been realized. If finance is made available when the inventory is dry, finance waits for demand and a sale occurs if order is realized before the arrival of demand. If order for the supply of units is not realized arriving demand is lost on getting finance and arriving finance is lost on getting demand.

It may be noted that the sale process has the following infinitesimal generator given by

$$
S = \begin{bmatrix}
-(\lambda + a) & \lambda & a \\
 a & -a & 0 \\
 \lambda & 0 & -\lambda 
\end{bmatrix}
$$

We list the various states of the continuous time Markov chain for the inventory system as follows.

$$
\Omega = \{(i, 0, 0), (i, 1, 0), (i, 0, 1) : \text{for } i = 0, 1, 2, \cdots S\}
$$

The Markov chain is in the state $(i, 0, 0)$ where $i$ units are available in the inventory the demand and the financial support are awaited. The chain is in the state $(i, 1, 0)$ when $i$ units are in the inventory $0 \leq i \leq S$, the demand has occurred and the financial support is awaited. The chain is in the state $(i, 0, 1)$ when $i$ units are in the inventory, the demand has to occur and the financial support is available. The infinitesimal generator of the continuous time Markov chain can be seen as follows.
The infinitesimal generator $Q$ is a square matrix of order $3(S + 1)$ in which all the unmarked entries are 0 and the steady state probability vector $\pi$ of the system satisfies the following.

$$\pi Q = 0 \quad \text{and} \quad \pi e = 1$$

where

$$\pi = (\pi_S, \pi_{S-1}, \ldots, \pi_2, \pi_1, \pi_0).$$

$$\pi_i = (\pi_{i(0,0)}, \pi_{i(1,0)}, \pi_{i(0,1)}) \quad \text{for} \quad 0 \leq i \leq S.$$ (3)

$\pi_{i,j,k}$ is the steady state probability that the state of the system is $(i, j, k)$ in the steady state for $0 \leq i \leq S$, $(j, k) = (1, 0)$ or $(0, 1)$ and $\overline{e} = (1, 1, \ldots, 1)^t$ is column vector. The matrices $T, A, S - \mu q I$, are as follows.

$$T = \begin{bmatrix} -(\lambda + a) & \lambda & a \\ 0 & -a & 0 \\ 0 & 0 & -\lambda \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ a & 0 & 0 \\ \lambda & 0 & 0 \end{bmatrix}$$

The matrices $T, A, S - \mu q I$, are as follows.
Inventory system with defective supplies

\[ S - \mu q I = \begin{bmatrix} -(\lambda + a + \mu q) & \lambda & a \\ a & -(a + \mu q) & 0 \\ \lambda & 0 & -(\lambda + \mu q) \end{bmatrix} \]

\[ \mu d I = \begin{bmatrix} \mu d & 0 & 0 \\ 0 & \mu d & 0 \\ 0 & 0 & \mu d \end{bmatrix} \]

for \( d = p_i, \) for \( 0 \leq i \leq S - 2s - 1 \) and \( d = q. \) \( (7) \)

We use the first equation of (4) corresponding the multiplication of steady state probability vector and the matrix \( Q \) and present the vectors \( \pi_s \) for \( i \neq s \) in terms of \( \pi_s \) and use the second total probability equation for finding \( \pi_s. \) The vector \( \pi \) multiplication with matrix \( Q \) for the block columns, \( s - 1, s - 2, \ldots, 2, 1 \) presents

\[ \pi_i = \pi_s B^{s-i} \quad \text{for} \quad 1 \leq i \leq s - 1 \] \( (8) \)

We set for convenience \( A(\mu q I - T)^{-1} = B \) and \( A(\mu q I - S)^{-1} = C. \) \( (9) \)

Using the last block column \( 0 \) and equation (8) we get

\[ \pi_0 = \pi_s B^{s-1} C. \] \( (10) \)

Using (4) and first block column of \( Q \) we get

\[ \pi_S = \pi_s (\mu p_0 I)(-T)^{-1}. \] \( (11) \)

Using the second block column

\[ \pi_S A + \pi_{S-1} T + \pi_s \mu p_1 I + \pi_{s-1} \mu p_0 I = 0. \]

\[ \pi_{s-1} = \pi_S A(-T)^{-1} + \pi_s (\mu p_1 I)(-T)^{-1} + \pi_{s-1} (\mu p_0 I)(-T)^{-1} \]

\[ = \pi_S A(-T)^{-1} + \pi_s \sum_{j=0}^{1} B^j (\mu p_{1-j} I)(-T)^{-1}. \]

This reduces using equation (11) to

\[ \pi_{S-1} = \pi_s \sum_{k=0}^{1} \sum_{j=0}^{k} B^j (\mu p_{k-j} I)(-T)^{-1}[A(-T)^{-1}]^{1-k}. \]
Proceeding in this manner, we get for \(1 \leq i \leq s - 1\)

\[
\pi_{S-i} = \pi_s \sum_{k=0}^{i} \sum_{j=0}^{k} B^j(\mu p_{k-j} I)(-T)^{-1}[A(-T)^{-1}]^{i-k}. \tag{12}
\]

Similarly we get

\[
\pi_{s-s+1}A + \pi_{S-s}T + \pi_s \mu p_s I + \pi_{s-1} \mu p_{s-1} I + \cdots + \pi_s \mu p_0 I = 0.
\]

This gives

\[
\pi_{s-s} = \pi_{s-s+1}[A(-T)^{-1}] + \sum_{j=0}^{s-1} \pi_{s-j}(\mu p_{s-j} I)(-T)^{-1} + \pi_0(\mu p_0 I)(-T)^{-1}.
\]

Using this and equations (8 to 12) we get

\[
\pi_{S-s} = \pi_s \left\{ \sum_{k=0}^{s-1} \sum_{j=0}^{k} B^j(\mu p_{k-j} I)(-T)^{-1}[A(-T)^{-1}]^{s-k} \right.
\]

\[
+ \sum_{j=0}^{s-1} B^j(\mu p_{s-j} I)(-T)^{-1} + B^{s-1}C(\mu p_0 I)(-T)^{-1} \right\}.
\]

Proceeding in this manner, we obtain for \(1 \leq i \leq S - 3s - 1\)

\[
\pi_{s-s-i} = \pi_s \left\{ \sum_{k=0}^{s-1} \sum_{j=0}^{k} B^j(\mu p_{k-j} I)(-T)^{-1}[A(-T)^{-1}]^{s-k+i} \right.
\]

\[
+ \sum_{k=0}^{i} \sum_{j=0}^{s-1} B^j(\mu p_{s-j+k} I)(-T)^{-1}[A(-T)^{-1}]^{i-k} \right.
\]

\[
+ \sum_{k=0}^{i} B^{s-1}C(\mu p_{0+k} I)(-T)^{-1}[A(-T)^{-1}]^{i-k} \right\}. \tag{13}
\]

Again following the above multiplication method we get

\[
\pi_{2s} = \pi_{2s+1}[A(-T)^{-1}] + \sum_{j=1}^{s-i} \pi_{s-j}((\mu p_{S-2s-j} I)(-T)^{-1} + \pi_0(\mu p_{S-3s} I)(-T)^{-1}.
\]
Using the above method after simplification we get

\[
\pi_{2s} = \pi_{s} \left\{ \sum_{k=0}^{s-1} \sum_{j=0}^{k} B^j(\mu p_{k-j}I)(-T)^{-1}[A(-T)^{-1}]^{S-2s-k} \right. \\
+ \sum_{k=0}^{S-3s-1} \sum_{j=0}^{s-1} B^j(\mu p_{s-j+k}I)(-T)^{-1}[A(-T)^{-1}]^{S-3s-k} \\
+ \sum_{k=0}^{s-3s} B^{s-1}C(\mu p_{0+k}I)(-T)^{-1}[A(-T)^{-1}]^{S-3s-k} \\
+ \left. \sum_{j=0}^{s-1} B^j(\mu p_{S-2s-j}I)(-T)^{-1} \right\}.
\]

Using similar arguments we obtain, for \(1 \leq i \leq s-1\),

\[
\pi_{2s-i} = \pi_{s} \left\{ \sum_{k=0}^{s-1} \sum_{j=0}^{k} B^j(\mu p_{k-j}I)(-T)^{-1}[A(-T)^{-1}]^{S-2s-k+i} \\
+ \sum_{k=0}^{S-3s-i} \sum_{j=0}^{s-1} B^j(\mu p_{s-j+k}I)(-T)^{-1}[A(-T)^{-1}]^{S-3s-k+i} \\
+ \sum_{k=0}^{s-3s+i} B^{s-1}C(\mu p_{0+k}I)(-T)^{-1}[A(-T)^{-1}]^{S-3s-k+i} \\
+ \left. \sum_{j=0}^{i} \sum_{k=0}^{s-1} B^j(\mu p_{S-2s-j+k}I)(-T)^{-1}[A(-T)^{-1}]^{i-k} \right\}.
\] \quad (14)

Using the column of block \(s\), \(\pi_{s+1}A + \pi_{s}(T - \mu \overline{q}I) = 0\). From this we can find easily

\[
\pi_{s} = \Pi_{(s,0,0)} \left[ 1, \frac{\lambda}{a + \mu \overline{q}}, \frac{a}{a + \mu \overline{q}} \right].
\] \quad (15)
Using the total probability law (4) we get

\[
\Pi_{(s,0,0)} = \left\{ \begin{array}{l}
1, \frac{\lambda a}{a + \mu a} \alpha \\
\frac{\lambda a}{a + \mu a} \alpha [\mu p_0 I (T)^{-1}] \\
\sum_{i=1}^{s-1} \sum_{k=0}^{s-1} B^i (\mu p_{k-j} I) (T)^{-1} [A (T)^{-1}]^{i-k} \\
\sum_{i=0}^{s-1} \left[ \sum_{k=0}^{s-1} \sum_{j=0}^{s-1} B^i (\mu p_{k-j} I) (T)^{-1} [A (T)^{-1}]^{s-k+i} \right] \\
\sum_{k=0}^{s-1} \sum_{j=0}^{s-1} B^i (\mu p_{s-j+k} I) (T)^{-1} [A (T)^{-1}]^{s-3s-k+i} \\
\sum_{k=0}^{s-1} \sum_{j=k+1}^{s-1} B^i (\mu p_{s-2s-j+k} I) [A (T)^{-1}]^{i-k} \\
I + \sum_{i=1}^{s-1} B^{s-i} + B^{s-1} C \right\}^{-1} (16)
\]

All inventory level steady state probabilities are given by equations (8 to 16). When the steady state probabilities are known we can obtain mean of the inventory size. We treat them in section 4.

3 Model B. Finance required in certain cases.

The following is the assumption of the model B.

1. The demand occurs in a random time with exponential distribution with parameter \( \lambda \). The arriving demand requires finance with probability \( \alpha \).
and does not require finance with probability $\beta = 1 - \alpha$. It takes a random
time with exponential distribution with parameter $a$ for providing finance
only when demand needs the same.

2. When a demand waits for financial support the next demand can occur
only after a sale is made.

3. When the inventory is dry arriving demand is lost when it does not
require finance. It waits for finance and gets a unit of the inventory on
the supply of finance if order of units has already been realized. If order
for the supply of units is not realized arriving demand is lost on getting
finance.

The time required to sell a unit has pdf $f(x)$ and cdf $F(x)$ respectively
as follows.

We get

$$f(x) = \beta \lambda e^{-\lambda x} + \alpha \int_0^x \lambda e^{-\lambda a} a e^{-a(x-u)} du$$

and it reduces to

$$f(x) = \beta \lambda e^{-\lambda x} + \left[ \frac{\alpha \lambda a}{(\lambda - a)} \right] (e^{-ax} - e^{-\lambda x})$$

$$F(x) = \beta (1 - e^{-\lambda x}) + \left[ \frac{\alpha \lambda}{(\lambda - a)} \right] (1 - e^{-\lambda x}) - \left[ \frac{\alpha \lambda}{(\lambda - a)} \right] (1 - e^{-\lambda x}).$$  \(17\)

We list the various states of the continuous time Markov chain as follows.

$$\Omega = \{(i, 0), (i, 1) : \text{for}, \; i = 0, 1, 2, \ldots S\}$$  \(18\)

The Markov chain is in the state $(i, 0)$ when $i$ units are available in the
inventory $0 \leq i \leq S$ and the demand is awaited. The chain is in the state
$(i, 1)$ when $i$ units are in the inventory $0 \leq i \leq S$, the demand has occurred and
the financial support is awaited. The infinitesimal generator of the continuous
time Markov chain can be seen with same structure as that of (3) with blocks
are second order matrices as follows.

$$T = \begin{bmatrix} -\lambda & \alpha \lambda \\ 0 & -a \end{bmatrix}, \quad A = \begin{bmatrix} \beta \lambda & 0 \\ a & 0 \end{bmatrix}$$

$$S - \mu q I = \begin{bmatrix} -(\lambda a + \mu q) & \alpha \lambda \\ a & -(a + \mu q) \end{bmatrix}, \quad \mu d I = \begin{bmatrix} \mu d & 0 \\ 0 & \mu d \end{bmatrix}$$

for $d = p_i$, for $0 \leq i \leq S - 2s - 1$ and $d = \overline{q}$.  \(19\)
All the equations (8) to (14) are valid. We may note using the column of block 

\[ \pi_{s+1} A + \pi_A (T - \mu q I) = 0. \]

This gives

\[ \Pi_{s,0} \alpha \lambda - \Pi_{s,1} (a + \mu q) = 0 \quad \text{and} \quad \pi_A = \Pi_{s,0} (1, \frac{\alpha \lambda}{a + \mu q}) \] (20)

Using the total probability law (4) we get

\[ \Pi_{(s,0)} = \left\{ \left[ 1, \left( \frac{\alpha \lambda}{a + \mu q} \right) \right] (\mu p_0 I) ((-T)^{-1}) \right. \]

\[ + \sum_{i=1}^{s-1} \sum_{k=0}^{i} \sum_{j=0}^{k} B^j (\mu p_{k-j} I)(-T)^{-1} [A(-T)^{-1}]^{i-k} \]

\[ + \sum_{i=0}^{S-3s-1} \sum_{k=0}^{i} \sum_{j=0}^{k} B^j (\mu p_{k-j} I)(-T)^{-1} [A(-T)^{-1}]^{s-k+i} \]

\[ + \sum_{k=0}^{s-1} \sum_{j=0}^{s-1} B^j (\mu p_{s-j+k} I)(-T)^{-1} [A(-T)^{-1}]^{i-k} \]

\[ + \sum_{k=0}^{S-3s-1} \sum_{j=0}^{s-1} B^j (\mu p_{s-j+k} I)(-T)^{-1} [A(-T)^{-1}]^{S-2s-k+i} \]

\[ + \sum_{k=0}^{S-3s-1} \sum_{j=0}^{s-1} B^j (\mu p_{s-j+k} I)(-T)^{-1} [A(-T)^{-1}]^{S-3s-k+i} \]

\[ + \sum_{k=0}^{S-3s+i} B^j (\mu p_{s-j+k} I)(-T)^{-1} [A(-T)^{-1}]^{S-3s-k+i} \]

\[ + \sum_{k=0}^{i} \sum_{j=k+1}^{s-1} B^j (\mu p_{s-2s-j+k} I)[A(-T)^{-1}]^{i-k} \]

\[ + I + \sum_{i=1}^{s-1} B^{s-i} + B^{s-1} C \ (1, 1)^t \}^{-1}. \] (21)

From the equations (8 to 14) and (18 to 21) we get all steady state probabilities of system.
4 NUMERICAL CASES

We consider a numerical example for model $A$ with $S = 10$, $s = 3$, $\lambda = 3$, $a = 4$, $\mu = 2$, $p_0 = .1$, $p_1 = .15$, $p_2 = .2$, $p_3 = .15$. We may note that the matrix $Q$ is of order 33 and the steady state probability vector of type $1 \times 33$. Since it satisfies the equation (4) we find its values are as follows. The following table gives its values approximated to 9 decimals.

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<th>10,1,0</th>
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Combining state $\Pi_{i,0,0} + \Pi_{i,1,0} + \Pi_{i,0,1}$ level probabilities together we can get inventory level probabilities for state $i$ for $0 \leq i \leq 10$ which are useful for finding expected inventory levels in the steady state.

Following the above method keeping all parameters constants except values of parameter $a$ we give below the summed up probabilities of inventory levels
for different values of parameter $a = 4, 5, 6, 7$.

<table>
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<th>a=6</th>
<th>a=7</th>
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</table>

It may be noted that when expected time to supply finance decreases ($a$ increases) then expected inventory level also decreases which is required for reducing storage costs.

We consider a numerical example for model $B$ with $S = 10, s = 3, \lambda = 3, a = 4, \mu = 2, p_0 = .1, p_1 = .15, p_2 = .2, p_3 = .15, \alpha = .7$. We may note that the matrix $Q$ is of order $22$ and the steady state probability vector of type $1 \times 22$. Since it satisfies the equation (4) we find its values are as follows. The following table gives values approximated to 9 decimals.

<table>
<thead>
<tr>
<th>State</th>
<th>10,0</th>
<th>10,1</th>
<th>9,0</th>
<th>9,1</th>
<th>8,0</th>
<th>8,1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>.005522663</td>
<td>.004572128</td>
<td>.019344492</td>
<td>.013666752</td>
<td>.042013185</td>
<td>.0275050506</td>
</tr>
<tr>
<td>State</td>
<td>7,0</td>
<td>7,1</td>
<td>6,0</td>
<td>6,1</td>
<td>5,0</td>
<td>5,1</td>
</tr>
<tr>
<td>Probability</td>
<td>.070105501</td>
<td>.04311391</td>
<td>.091876554</td>
<td>.052281649</td>
<td>.106158652</td>
<td>.058424978</td>
</tr>
<tr>
<td>State</td>
<td>4,0</td>
<td>4,1</td>
<td>3,0</td>
<td>3,1</td>
<td>2,0</td>
<td>2,1</td>
</tr>
<tr>
<td>Probability</td>
<td>.1141842</td>
<td>.06129049</td>
<td>.082839942</td>
<td>.033454592</td>
<td>.049612932</td>
<td>.020035992</td>
</tr>
<tr>
<td>State</td>
<td>1,0</td>
<td>1,1</td>
<td>0,0</td>
<td>0,1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Probability</td>
<td>.029713244</td>
<td>.011999578</td>
<td>.044366344</td>
<td>.017917177</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Combining state $\pi_{i,0} + \pi_{i,1}$ level probabilities together we can get inventory level probabilities for state $i$ for $0 \leq i \leq 10$ which are useful for finding expected inventory levels in the steady state.

Following the above method keeping all parameters constants except values of parameter $a$ we give below the summed up probabilities of inventory levels.
for different values of parameter $a = 4, 5, 6$ and 7.

<table>
<thead>
<tr>
<th>State \ probability</th>
<th>a=4</th>
<th>a=5</th>
<th>a=6</th>
<th>a=7</th>
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<td></td>
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<td>.009527301</td>
<td>.009155846</td>
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<td></td>
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<td>.031331638</td>
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<td></td>
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<td>.06629521</td>
<td>.064039654</td>
<td>.062378089</td>
</tr>
<tr>
<td></td>
<td>.113219409</td>
<td>.109059917</td>
<td>.10610579</td>
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<td></td>
<td>.144158191</td>
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<td></td>
<td>.16458363</td>
<td>.16178158</td>
<td>.159658921</td>
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<td></td>
<td>.17547469</td>
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<tr>
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<td>.069648924</td>
<td>.07278027</td>
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<td>.062283521</td>
<td>.072558022</td>
<td>.080716876</td>
<td>.0873114</td>
</tr>
<tr>
<td>E(X)</td>
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<td>4.560171987</td>
<td>4.482298684</td>
<td>4.42290706</td>
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<tr>
<td>Var(X)</td>
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<td>5.55230292</td>
<td>5.675216764</td>
<td>5.768757405</td>
</tr>
</tbody>
</table>

It may be noted here also that when expected time to supply finance decreases ($a$ increases) then expected inventory level also decreases.

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References


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