A Study of the Series Solutions of Three Species of Lotka Volterra Food Web Model Using Adomian Decomposition and Homotopy Perturbation Methods

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Abstract

In this work, series solutions are obtained for the three species Lotka – volterra, prey – predator Food web model which is governed by a system of non linear differential equations. For this purpose, the Adomian decomposition method (ADM) and Homotopy Perturbation method (HPM) are employed and it is shown that the Homotopy Perturbation method is much easier and a more efficient method.

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1. Introduction

In the study of non linear system of differential equations such as the Lotka – Volterra equations, analytical solutions are usually unknown. In order to analyze the behaviour of the system, one usually resorts to numerical integration techniques such as perturbation techniques. Perturbation technique depends on the existence of small or large parameters in the nonlinear problems. In this paper, the Adomian decomposition method (ADM) and Homotopy
Perturbation method (HPM) are employed to obtain series solutions to three species of Lotka–Volterra, prey – predator food web model which is governed by a system of non linear differential equations. The first method is the Adomian decomposition method [3], introduced by Adomian in the 1980’s. It is an iterative method that provides approximate analytical solutions in the form of an infinite series for non linear equations. This method avoids linearization, discretization and scientifically unrealistic assumptions. The second method is the Homotopy perturbation method [4], proposed by ‘He’ in 1999. In this method, the solutions are obtained as summation of an infinite series which converges to analytical solution. In this method, a homotopy is constructed with an embedding parameter \( p \in (0, 1) \), which is valid not only for small parameters but also for very large parameters. We also show that the solutions obtained by both the methods ADM and HPM are the same numerically by giving examples.

2 Adomian Decomposition Method

2.1 Analysis of the method for a three species food web model:

Let \( x_1, x_2, x_3 \) be the densities of the three species forming a food web in a closed environment. Then, their evolution is governed by the set of equations

\[
\begin{align*}
    x'_1 &= x_1 \left( a_1 - b_1 x_1 - c_1 x_2 \right) \\
    x'_2 &= x_2 \left( -a_2 + b_2 x_1 - c_2 x_2 - d_2 x_3 \right) \\
    x'_3 &= x_3 \left( -a_3 + c_3 x_2 - d_3 x_3 \right)
\end{align*}
\]

(2.1)

where,

\( a_i, b_i, c_i \) (i = 1, 2, 3), \( d_j \) (j = 2, 3) are positive constants assuming the system has a unique positive equilibrium state \((\alpha, \beta, \gamma)\) where

\[
\begin{align*}
    \alpha &= \frac{(a_1 c_2 + a_2 c_1)d_2}{(b_1 c_2 + b_2 c_1)d_3 + b_1 c_3 d_2} \\
    \beta &= \frac{(a_1 b_2 - a_2 b_1)d_3 + a_1 b_2 d_2}{(b_1 c_2 + b_2 c_1)d_3 + b_1 c_3 d_2} \\
    \gamma &= \frac{(a_1 b_2 - a_2 b_1)c_3 - (b_1 c_2 + b_2 c_1)\alpha}{(b_1 c_2 + b_2 c_1)d_3 + b_1 c_3 d_2}
\end{align*}
\]

(2.2)

Defining \( u(\tau) = \frac{x_1(\tau)}{\alpha}, \ v(\tau) = \frac{x_2(\tau)}{\beta}, \ w(\tau) = \frac{x_3(\tau)}{\gamma} \) at \( \tau = a_1 t \),

(2.3)

(2.1) are transformed to

\[
\begin{align*}
    \frac{du}{d\tau} &= u(\tau) - k_0 u^2(\tau) - k_1 u(\tau) v(\tau) \\
    \frac{dv}{d\tau} &= -k_2 v(\tau) + k_3 u(\tau) v(\tau) - k_4 v^2(\tau) - k_5 v(\tau) w(\tau) \\
    \frac{dw}{d\tau} &= -k_6 w(\tau) + k_7 v(\tau) w(\tau) - k_8 w^2(\tau)
\end{align*}
\]

(2.4)
where
\[ k_0 = \frac{b_1}{a_1} \alpha, \quad k_1 = \frac{c_1}{a_1} \beta, \quad k_2 = \frac{a_2}{a_1}, \quad k_3 = \frac{b_2}{a_1} \alpha, \quad k_4 = \frac{c_2}{a_1} \beta, \quad k_5 = \frac{d_2}{a_1} \beta, \quad k_6 = \frac{a_3}{a_1}, \quad k_7 = \frac{c_3}{a_1} \beta, \quad k_8 = \frac{d_3}{a_1} \gamma \]

Consider the system (2.4) of the form
\[
\frac{du(\tau)}{d\tau} = u(\tau) - k_0 f(u(\tau)) - k_1 g(u(\tau), v(\tau))
\]
\[
\frac{dv(\tau)}{d\tau} = -k_2 v(\tau) + k_3 g(u(\tau), v(\tau)) - k_4 h(v(\tau)) - k_5 \phi_2(v(\tau), w(\tau))
\]
\[
\frac{dw(\tau)}{d\tau} = -k_6 w(\tau) + k_7 \phi_2(v(\tau), w(\tau)) + k_8 \phi_1(w(\tau))
\]
(2.5)

with initial conditions \( u(0) = u_0, \quad v(0) = v_0, \quad w(0) = w_0 \). Here \( f, g, h, \phi_1 \) and \( \phi_2 \) are non linear functions of \( u, (u, v), v, w \) and \( (v, w) \) respectively. The decomposing method consists of approximating the solutions of (2.5) as an infinite series.

\[
u = \sum_{n=0}^{\infty} u_n
\]
\[
v = \sum_{n=0}^{\infty} v_n
\]
\[
w = \sum_{n=0}^{\infty} w_n
\]
(2.6)

de decomposing \( f, g, h, \phi_1 \) and \( \phi_2 \) as
\[
f(u) = \sum_{n=0}^{\infty} A_n
\]
\[
h(v) = \sum_{n=0}^{\infty} C_n
\]
\[
\phi_1(w) = \sum_{n=0}^{\infty} D_n
\]
\[
g(u(\tau), v(\tau)) = \sum_{n=0}^{\infty} B_n
\]
\[
\phi_2(v(\tau), w(\tau)) = \sum_{n=0}^{\infty} E_n
\]
(2.7)

Here \( A_n, B_n, C_n, D_n \) and \( E_n \) are the Adomian Polynomials, applying the decomposition method, the system (2.5) can be written as
\[
Lu = u(\tau) - k_0 f(u(\tau)) - k_1 g(u(\tau), v(\tau))
\]
\[
Lv = -k_2 v(\tau) + k_3 g(u(\tau), v(\tau)) - k_4 h(v(\tau)) - k_5 \phi_2(v(\tau), w(\tau))
\]
\[ Lw = -k_6 w(\tau) + k_7 \phi_2 (v(\tau), w(\tau)) + k_8 \phi_1 (w(\tau)) \]  
\[(2.9)\]

Where \( L \) represents \( \frac{\partial}{\partial \tau} \), the linear differential operator, assuming integration inverse operator \( L^{-1} \) exists and is defined as 
\[ L^{-1} = \int_0^\tau d\tau \]

Applying inverse operator to (2.9), we obtain
\[ u = u_0 + L^{-1} u(\tau) - k_0 L^{-1} f(u(\tau)) - k_1 L^{-1} g(u(\tau), v(\tau)) \]
\[ v = v_0 - k_2 L^{-1} v(\tau) + k_3 L^{-1} g(u(\tau), v(\tau)) - k_4 L^{-1} h(v(\tau)) - k_5 L^{-1} \phi_2 (v(\tau), w(\tau)) \]
\[ w = w_0 - k_6 L^{-1} w(\tau) + k_7 L^{-1} \phi_2 (v(\tau), w(\tau)) + k_8 L^{-1} \phi_1 (w(\tau)) \]  
\[(2.10)\]

Using (2.7) and (2.8), it follows that
\[ \sum_{n=0}^{\infty} u_n = u_0 + L^{-1} \sum_{n=0}^{\infty} (u_n - k_0 L^{-1} A_n - k_1 L^{-1} B_n) \]
\[ \sum_{n=0}^{\infty} v_n = v_0 - k_2 L^{-1} \sum_{n=0}^{\infty} (v_n + k_3 L^{-1} B_n - k_4 L^{-1} C_n - k_5 L^{-1} E_n) \]
\[ \sum_{n=0}^{\infty} w_n = w_0 - k_6 L^{-1} \sum_{n=0}^{\infty} (w_n + k_7 L^{-1} E_n + k_8 L^{-1} D_n) \]  
\[(2.11)\]

Now determine the iterates using recurrence relations as
\[ u(0) = u_0 \]
\[ u_{n+1} = L^{-1} u_n - k_0 L^{-1} A_n - k_1 L^{-1} B_n, n = 0, 1, 2,... \]
\[ v(0) = v_0 \]
\[ v_{n+1} = -k_2 L^{-1} v_n + k_3 L^{-1} B_n - k_4 L^{-1} C_n - k_5 L^{-1} E_n, n = 0, 1, 2,... \]
\[ w(0) = w_0 \]
\[ w_{n+1} = -k_6 L^{-1} w_n + k_7 L^{-1} E_n - k_8 L^{-1} D_n, n = 0, 1, 2,... \]  
\[(2.12)\]

Writing the solution of the initial value problem (2.5) as
\[ (u, v, w) = \left( \lim_{n \to \infty} \sum_{k=0}^{n} u_k , \lim_{n \to \infty} \sum_{k=0}^{n} v_k , \lim_{n \to \infty} \sum_{k=0}^{n} w_k \right) \]  
\[(2.13)\]

3. Application of Adomian decomposition method

3.1 An example for the food web model

We consider (2.5) with initial values \( u(0) = u_0, v(0) = v_0, w(0) = w_0 \) and proceeding as in section 2.1, we take
\[ f(u) = u^2, h(v) = v^2, \phi_1 (w) = w^2 \]
\[ g(u,v) = u v, \phi_2 (v, w) = vw \]  
\[(3.1)\]

The Adomian polynomials for \( f(u) = u^2, g(u,v) = u v, h(v) = v^2 \) can be derived as follows
\[ A_n = \frac{1}{n!} \frac{d^n}{dx^n} \left( f \left( \sum_{n=0}^{\infty} x^n u_n \right) \right) \quad n \geq 0 \]  

\[ f(u) = u^2 = \sum_{n=0}^{\infty} A_n = (u_0 + u_1 + u_2 + \ldots)^2 \]

\[ = (u_0)^2 + (2u_0u_1) + (2u_0u_2 + u_1^2) + (2u_0u_3 + 2u_1u_2) + (2u_0u_4 + 2u_1u_3 + u_2^2) + (2u_0u_5) + 2u_1u_4 + 2u_2u_3) + \ldots \]  

Therefore, we get the following Adomian polynomials
\[ A_0 = u_0^2 \]
\[ A_1 = 2u_0u_1 \]
\[ A_2 = 2u_0u_2 + u_1^2 \]
\[ A_3 = 2u_0u_3 + 2u_1u_2 \]
\[ A_4 = 2u_0u_4 + 2u_1u_3 + u_2^2 \]
\[ A_5 = 2u_0u_5 + 2u_1u_4 + 2u_2u_3 \text{ and so on.} \]  

\[ g(u, v) = uv = \sum_{n=0}^{\infty} B_n = \left( \sum_{n=0}^{\infty} u_n \right) \left( \sum_{n=0}^{\infty} v_n \right) \]

\[ = \sum_{n=0}^{\infty} \left( \sum_{k=0}^{n} u_k v_{n-k} \right) \]

Therefore, \( B_n = \sum_{k=0}^{n} u_k v_{n-k}, \quad n = 0, 1, 2, \ldots \)

We have Adomian Polynomials
\[ B_0 = u_0v_0 \]
\[ B_1 = u_0v_1 + u_1v_0 \]
\[ B_2 = u_0v_2 + u_1v_1 + u_2v_0 \]
\[ B_3 = u_0v_3 + u_1v_2 + u_2v_1 + u_3v_0 \]
\[ B_4 = u_0v_4 + u_1v_3 + u_2v_2 + u_3v_1 + u_4v_0 \text{ and so on.} \]

\[ h(v) = v^2 = \sum_{n=0}^{\infty} C_n = (v_0 + v_1 + v_2 + \ldots)^2 \]

\[ = v_0^2 + (2v_0v_1) + (2v_0v_2 + v_1^2) + (2v_0v_3 + 2v_1v_2) + (2v_0v_4 + 2v_1v_3 + v_2^2) + (2v_0v_5 + 2v_1v_4 + 2v_2v_3) + \ldots \]

Therefore, we get the following Adomian polynomials
\[ C_0 = v_0^2 \]
\[ C_1 = 2v_0v_1 \]
\[ C_2 = 2v_0v_2 + v_1^2 \]
\[ C_3 = 2v_0v_3 + 2v_1v_2 \]
\[ C_4 = 2v_0v_4 + 2v_1v_3 + v_2^2 \]
C_5 = 2v_0 v_5 + 2v_1 v_4 + 2v_2 v_3 and so on \hspace{1cm} (3.9)

\phi_i (w) = w^2 = \sum_{n=0}^{\infty} D_n = (w_0 + w_1 + w_2 + \ldots)^2

= (w_0^2) + (2w_0w_1) + (2w_0w_2 + w_1^2) + (2w_0w_3 + 2w_1w_2) + (2w_0w_4 + 2w_1w_3 + w_2^2)

+ (2w_0w_5 + 2w_1w_4 + 2w_2w_3) + \ldots \hspace{1cm} (3.10)

We get the following Adomian polynomials

D_0 = w_0^2
D_1 = 2w_0w_1
D_2 = 2w_0w_2 + w_1^2
D_3 = 2w_0w_3 + 2w_1w_2
D_4 = 2w_0w_4 + 2w_1w_3 + w_2^2
D_5 = 2w_0w_5 + 2w_1w_4 + 2w_2w_3 + \ldots \hspace{1cm} (3.11)

\phi_j (v, w) = \sum_{n=0}^{\infty} E_n = \left( \sum_{n=0}^{\infty} v_n \right) \left( \sum_{n=0}^{\infty} w_n \right) = \sum_{n=0}^{\infty} \left( \sum_{k=0}^{n} v_k w_{n-k} \right) \hspace{1cm} (3.12)

E_n = \sum_{k=0}^{n} v_k w_{n-k}, n=0, 1, 2, \ldots \hspace{1cm} (3.13)

It follows that

E_0 = v_0 w_0
E_1 = v_0 w_1 + v_1 w_0
E_2 = v_0 w_2 + v_1 w_1 + v_2 w_0
E_3 = v_0 w_3 + v_1 w_2 + v_2 w_1 + v_3 w_0
E_4 = v_0 w_4 + v_1 w_3 + v_2 w_2 + v_3 w_1 + v_4 w_0 and so on \hspace{1cm} (3.14)

For numerical purpose, we take

a_1 = 1, a_2 = 1, a_3 = 1, b_1 = 1, b_2 = 2, c_1 = 1/4, c_2 = 1/2, c_3 = -2,

b_2 = 1, b_3 = 1 and initial values u(0) = 1, v(0) = 2, w(0) = 3.

Therefore,

u_0 = 1
v_0 = 2
w_0 = 3

\begin{align*}
u_1 &= L^{-1}u_0 - k_3 L^{-1}A_0 - k_1 L^{-1}B_0 = 0.4167 \tau \\
v_1 &= -k_3 L^{-1}v_0 + k_4 L^{-1}B_0 - k_4 L^{-1}C_0 - k_4 L^{-1}E_0 = -4.333 \tau \\
w_1 &= -k_4 L^{-1}w_0 + k_5 L^{-1}E_0 + k_5 L^{-1}D_0 = 8 \tau \\
u_2 &= L^{-1}u_1 - k_3 L^{-1}A_1 - k_1 L^{-1}B_1 = 0.3958 \tau^2 \\
v_2 &= -k_3 L^{-1}v_1 + k_4 L^{-1}B_1 - k_4 L^{-1}C_1 - k_4 L^{-1}E_1 = 3.6806 \tau^2 \\
w_2 &= -k_4 L^{-1}w_1 + k_5 L^{-1}E_1 + k_5 L^{-1}D_1 = 6 \tau^2 \\
u_3 &= L^{-1}u_2 - k_3 L^{-1}A_2 - k_1 L^{-1}B_2 = -0.0966 \tau^3 \\
v_3 &= -k_3 L^{-1}v_2 + k_4 L^{-1}B_2 - k_4 L^{-1}C_2 - k_4 L^{-1}E_2 = -3.2130 \tau^3
\end{align*}
The rest of the terms of the decomposing series have been calculated using mathcad7. Substituting these terms in (2.6)

\( u(\tau) = u_0(\tau) + u_1(\tau) + u_2(\tau) + u_3(\tau) + u_4(\tau) + \ldots \)

\[ = 1 + 0.4167\tau + 0.3958\tau^2 - 0.0966\tau^3 + 0.1168\tau^4 - 0.1492\tau^5 \ldots \] \hspace{1cm} (3.15)

\( v(\tau) = v_0(\tau) + v_1(\tau) + v_2(\tau) + v_3(\tau) + v_4(\tau) + v_5(\tau) + \ldots \)

\[ = 2 - 4.333\tau + 3.6806\tau^2 - 3.2130\tau^3 + 3.9427\tau^4 - 4.8092\tau^5 + \ldots \] \hspace{1cm} (3.16)

\( w(\tau) = w_0(\tau) + w_1(\tau) + w_2(\tau) + w_3(\tau) + w_4(\tau) + w_5(\tau) + \ldots \)

\[ = 3 + 8\tau + 6\tau^2 + 3.9444\tau^3 + 9.5509\tau^4 + 11.2421\tau^5 + \ldots \] \hspace{1cm} (3.17)

4. Homotopy perturbation method

The combination of the perturbation method and homotopy method is called the homotopy perturbation method which has eliminated the limitations of the traditional perturbation method. Consider the nonlinear differential equation.

\[ M(u) - g(s) = 0, \quad s \in S \] \hspace{1cm} (4.1)

with the boundary conditions

\[ N\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad s \in \Gamma \]

where,

\( M \) is a differential operator, \( N \) is a boundary operator, \( g(s) \) is an known analytic function. \( \Gamma \) is the boundary of the domains. Divide the operator \( M \) into a linear operator \( R \) and a nonlinear operator \( Q \). Therefore (4.1) can be expressed as

\[ R(u) + Q(u) = g(s) \] \hspace{1cm} (4.2)

By the homotopy technique, constructing a homotopy

\( \nu(s,p): S \times [0,1] \rightarrow R \) which satisfies

\[ L(\nu, p) = (1-p) \left[ R(u) - R(u_0) + p \left( M(u) - g(s) \right) \right] = 0, \quad p \in (0,1), \quad s \in S. \]

\[ L(\nu, p) = R(u) - R(u_0) + pR(u_0) + p \left[ M(u) - g(s) \right] = 0 \] \hspace{1cm} (4.3)

where,
p ∈ [0,1] is an embedding parameter, \( u_0 \) is an initial approximation of (4.1) which satisfies the boundary conditions.

\[
L(\nu,0) = R(u) - R(u_0) = 0 \quad (4.4)
\]

\[
H(\nu,1) = M(\nu) - g(s) = 0 \quad (4.5)
\]

Changing of \( p \) from zero to unity is just that of \( \nu[s,p] \) from \( u_0(s) \) to \( u(s) \). By the method of HPM, we use the embedding parameter \( p \) as a small parameter and assume that the solutions of (4.3) and (4.4) can be written as a power series of \( p \).

\[
\nu = v_0 + p v_1 + p^2 v_2 + p^3 v_3 + .... \quad (4.6)
\]

Setting \( p \to 1 \) results on the approximating solutions of (4.1) as

\[
u = \lim_{{p \to 1}} v = v_0 + v_1 + v_2 + .... \quad (4.7)
\]

The series (4.7) is convergent for most cases. However, the convergent rate depends on the nonlinear operator \( M(\nu) \).

5 Application of Homotopy Perturbation method

5.1 An example for the food web model

We now solve (2.4) using HPM with initial conditions \( u(0) = 1, \nu(0) = 2, w(0) = 3 \) as taken in example 3.1. We rewrite (2.4) as

\[
\frac{du}{d\tau} = p \left[ u(\tau) - k_0 u^2(\tau) - k_1 u(\tau)\nu(\tau) \right]
\]

\[
\frac{dv}{d\tau} = p \left[ -k_2 \nu(\tau) + k_3 u(\tau)\nu(\tau) - k_4 \nu^2(\tau) - k_5 \nu(\tau)w(\tau) \right]
\]

\[
\frac{dw}{d\tau} = p \left[ -k_6 w(\tau) + k_7 u(\tau)w(\tau) - k_8 w^2(\tau) \right] \quad (5.1)
\]

where,

\( p \) is an embedding parameter belongs to [0,1] as a ‘small’ parameter, the constants \( k_i \) (i = 0,1,2, ... 8) have the same meaning as in section 2.1.

Assuming the solutions of (5.1), \((u, \nu, w)\) are expressed as power series

\[
\begin{align*}
\nu &= v_0 + p v_1 + p^2 v_2 + p^3 v_3 + .... \\

w &= w_0 + p w_1 + p^2 w_2 + p^3 w_3 + .... \quad (5.2)
\end{align*}
\]

Substituting (5.2) in (5.1) and equating the like coefficients of powers of \( p \), we obtain the following differential equations.

\[
p^0 : \begin{cases}
u_0 = 0, \quad v_0 = 0, \quad w_0 = 0 \\
          u(0) = 1, \quad \nu(0) = 2, \quad w(0) = 3.
\end{cases}
\]
Study of series solutions of three species

Thus solving the above system of equations yields (taking the same values for \(k_i\) \((i = 0, 1, 2, ..., 8)\) in sec. 3, example 3.1)

\[
\begin{align*}
\text{for } & p^1: \\
& u_1' = u_0 - k_0 u_0^2 - k_1 u_0 v_0 \\
& v_1 = -k_2 v_0 + k_3 u_0 v_0 - k_4 v_0^2 - k_5 v_0 w_0 \\
& w_1 = -k_6 w_0 + k_7 v_0 w_0 + k_8 w_0^2 \\
& u_1(0) = 0, \ v_1(0) = 0, \ w(0) = 0 \\
\text{for } & p^2: \\
& u_2' = u_1 - 2k_0 u_0 u_1 - k_1 (u_0 v_1 + u_1 v_0) \\
& v_2 = -k_2 v_1 + k_3 (u_0 v_1 + u_1 v_0) - 2k_4 v_0 v_1 - k_5 (v_0 w_1 + v_1 w_0) \\
& w_2 = -k_6 w_1 + k_7 (v_0 w_1 + v_1 w_0) + 2k_8 w_0 w_1 \\
& u_2(0) = 0, \ v_2(0) = 0, \ w(0) = 0 \\
\text{for } & p^3: \\
& u_3' = u_2 - 2k_0 u_0 u_2 - u_1^2 - k_1 (u_0 v_2 + u_1 v_1 + u_2 v_0) \\
& v_3 = -k_2 v_2 + k_3 (u_0 v_2 + u_1 v_1 + u_2 v_0) - k_4(2v_0 v_2 + v_1^2) - k_5(v_0 w_2 + v_1 w_1 + v_2 w_0) \\
& w_3 = -k_6 w_2 + k_7 (v_0 w_2 + v_1 w_1 + v_2 w_0) + k_8 (2w_0 w_2 + w_1^2) \\
& u_3(0) = 0, \ v_3(0) = 0, \ w(0) = 0 \\
\text{for } & p^4: \\
& u_4' = u_3 - k_0 (2u_0 u_3 + 2u_1 u_2) - k_1 (u_0 v_3 + u_1 v_2 + u_2 v_1 + u_3 v_0) \\
& v_4 = -k_2 v_3 + k_3 (u_0 v_3 + u_1 v_2 + u_2 v_1 + u_3 v_0) - k_4(2v_0 v_3 + 2v_1 v_2) - k_5(v_0 w_3 + v_1 w_2 + v_2 w_1 + v_3 w_0) \\
& w_4 = -k_6 w_3 + k_7 (v_0 w_3 + v_1 w_2 + v_2 w_1 + v_3 w_0) + k_8 (2w_0 w_3 + 2w_1 w_2) \\
& u_4(0) = 0, \ v_4(0) = 0, \ w(0) = 0 \\
\text{for } & p^5: \\
& u_5' = u_4 - k_0 (2u_0 u_4 + 2u_1 u_3 + u_2^2) - k_1 (u_0 v_4 + u_1 v_3 + u_2 v_2 + u_3 v_1 + u_4 v_0) \\
& v_5 = -k_2 v_4 + k_3 (u_0 v_4 + u_1 v_3 + u_2 v_2 + u_3 v_1 + u_4 v_0) - k_4(2v_0 v_4 + 2v_1 v_3 + v_2^2) \\
& w_5 = -k_6 w_4 + k_7 (v_0 w_4 + v_1 w_3 + v_2 w_2 + v_3 w_1 + v_4 w_0) + k_8 (2w_0 w_4 + 2w_1 w_3 + w_2^2) \\
& u_5(0) = 0, \ v_5(0) = 0, \ w(0) = 0 \\
\end{align*}
\]
The rest of the terms of the decomposing series have been calculated using MathCad7. Substituting $u_i$, $v_i$, $w_i$ ($i = 1, 2, \ldots 5$) in to (5.2), we have

\[ u(\tau) = 1 + 0.4167\tau + 0.3958\tau^2 - 0.0966\tau^3 + 0.1168\tau^4 - 0.1492\tau^5 + \ldots \]
\[ v(\tau) = 2 - 4.333\tau + 3.6806\tau^2 - 3.2130\tau^3 + 3.9427\tau^4 - 4.8092\tau^5 + \ldots \]
\[ w(\tau) = 3 + 8\tau + 6p\tau^2 + 3.9444p^3\tau^3 + 9.5509p^4\tau^4 + 11.2421p^5\tau^5 + \ldots \]

Letting $p \rightarrow 1$, we obtain

\[ u(\tau) = 1 + 0.4167\tau + 0.3958\tau^2 - 0.0966\tau^3 + 0.1168\tau^4 - 0.0492\tau^5 + \ldots \]
\[ v(\tau) = 2 - 4.333\tau + 3.6806\tau^2 - 3.2130\tau^3 + 3.9427\tau^4 - 4.8092\tau^5 + \ldots \]
\[ w(\tau) = 3 + 8\tau + 6\tau^2 + 3.9444\tau^3 + 9.5509\tau^4 + 11.2421\tau^5 + \ldots \]

which are exactly the same solutions as those obtained in (3.15), (3.16) and (3.17) in sec. 3, example 3.1.

**Conclusion**

In each of the examples (3.1 and 5.1) described in the paper for the purpose of validation of the work, we have solved the same problem for two different methods i.e. Adomian Decomposition method & Homotopy Perturbation method. Though we obtain the same solutions for both methods, the advantage of the Homotopy Perturbation method is its straightforwardness and calculation procedure is simple when compared to the Adomian Decomposition method, where the calculation of Adomian polynomials is tedious. Hence, we conclude that the Homotopy Perturbation method is much easier and a more efficient method than the Adomian Decomposition method.

**REFERENCES**

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