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A New Approach to Solve the Classical Symmetric Traveling Salesman Problem by Zero Suffix Method

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Abstract

This paper presents zero suffix method for solving the classical symmetric traveling salesman problem (TSP). It is conjectured that the chance of improving a good solution by moving a node to a position far away from its original one is small, it is possible to further improve a TSP tour that cannot be improved by other local search methods. To test the performance of the proposed method an example is solved. Thus this paper shows that algorithm proposed is efficient for solving the TSPs.

Keywords: Local search, Symmetric traveling salesman problem, Zero suffix method

1 Introduction

Traveling salesman problem is one of the classical challenging combinatorial optimization problems. The objective of the TSP is to minimize the total distance traveled by visiting all the nodes once and only once and then returning to the depot node. The classical formulation of the TSP is stated as follows. Let a network $G = (N, A, C)$ be defined with N denoting the set of nodes on the network, A denoting the set of arcs and $C = [C_{ij}]$ denoting the matrix of costs.

A common application of the TSP is the movement of people, equipment and vehicles around tours of duty to minimize the total traveling cost. For example, in a school bus routing problem, it is required to schedule a school bus to pick up waiting students from the pre-specified locations. Post routing is another application of the TSP. The postman problem is modeled as traversing a given set of streets in a city, rather than visiting a set of specified locations. Moreover, the TSP plays an important role in general post problem, where the houses or streets are far away from each other. Besides the above mentioned applications, some other seemly unrelated problems are solved by formulating them as the TSP. The genome sequencing problem occurs in the field of bio-engineering. The aim of this problem is to find the genome sequence based on the markers that serve as landmarks for the genome maps. The TSP plays an important role in genome sequencing by providing a tool for building sequences from experimental data on the proximity of individual pairs of markers. By considering markers as cities, a genome sequence can be viewed as a TSP path traveling through each marker once and only once. The drilling problem is another application of the TSP with the objective of minimizing the total travel time of the drill. In the electronic industry, printed circuit boards are widely found in common electronic devices. The printed circuit board normally has a very large number of holes used for mounting components or integrated chips. These holes are typically drilled by automated drilling machines that move between specified locations to drill a hole one after another. Therefore, the locations in the drilling problem correspond to the cities in the TSP. The applications of the TSP are not limited to the examples described above. A detailed review of the applications of the TSP can be found in [1]-[2]. It has been proved that TSP is NP-hard in [3] which imply that a polynomial bounded exact algorithm for TSP is unlikely to exist. In this paper, we presented a zero suffix method for solving the TSP and its performance is illustrated based on the benchmark TSP instances. By doing local search using the Generalized Crossing (GC) method, which is developed in [5] for the vehicle routing problem (VRP), each block is explored intensively in order to improve the existing solution. This paper is organized as follows. In 2, a literature review is given on the TSP exact and approximate algorithms. The detail of the proposed method is presented in 3 and the numerical given 4.

2 Literature Review

The TSP has been studied intensively during the last 50 years and many exact and heuristic algorithms have been developed. These algorithms include construction algorithms, iterative improvement algorithms, branch-and-bound and branch-and-cut exact algorithms, and many metaheuristic algorithms, such as simulated annealing (SA), tabu search (TS), ant colony (AC) and genetic algorithm (GA). Some of the well known tour construction procedures are the nearest neighbor procedure by Rosenkrantz et al. [6], the Clarke and Wright savings' algorithm [7], the insertion procedures [6], the partitioning approach by Karp [8] and the minimal spanning tree approach by Christofides [9] etc. The branch exchange is perhaps the best known iterative improvement algorithm for the TSP. The 2-opt and 3-optheuristics were described in Lin [10]. Lin and Kernighan [11] made a great improvement in quality of tours that can be obtained by heuristic methods. Even today, their algorithm remains the key ingredient in the most successful approaches for finding high-quality tours and is widely used to generate initial solutions for other algorithms. Or [12] developed a simplified edge exchange procedure requiring only $O(n_2)$ operations at each step, but producing tour nearly as good as the average performance of 3-opt algorithm. One of the earliest exact algorithms is due to Dantzig et al. [13], in which linear programming (LP) relaxation is used to solve the integer formulation by adding suitably chosen linear inequality to the list of constraints continuously. Branch and bound (B&B) algorithms are widely used to solve the TSPs. Several authors have proposed (B&B) algorithm based on assignment problem (AP) relaxation of the original TSP formulation. These authors include Eastman [14], Held and Karp [15], Smith et al. [16], Carpaneto and Toth [17], Balas and Crhistofides [18]. Some branch and cut (B&C) based exact algorithms were developed by Crowder and Padberg [19], Padberg and Hong [20], Grotschel and Holland [21]. Besides the above mentioned exact and heuristic algorithms, metaheuristic algorithms have been applied successfully to the TSP by a number of researchers.

SA algorithms for the TSP were developed by Bonomi and Lutton [22], Golden and Skiscim [23] and Nahar et al. [24], Lo and Hus [25] etc. Tabu search metaheuristic algorithms for the TSP have been proposed by Knox [26], and Fiechter [27] etc. The AC is a relative new metaheuristic algorithm which is applied successfully to solve the TSP. Some work based on AC technology was reported by Dorigo et al. [28], Gomez and Banan [29], Bullnheimer et al. [30] and Tsai et al. [31].Genetic algorithms for the TSP were reported by Grefenstette et al. [32], Whitley et al. [33], and Nguyen et al. [34]. Comprehensive review of the techniques developed for the TSP can be found in [1]-[2], [35]-[37].Here we had introduced a new method known as zero suffix method to solve TSPs.

3 Zero suffix method

We, now introduce a new method called the zero suffix method for finding an optimal solution to the traveling salesman problem.

The zero suffix method proceeds as follows.

Step 1: Construct the TSP table.

Step 2: Subtract each row entries of the TSP table from the corresponding row minimum after that subtract each column entries of the TSP table from the corresponding column minimum.

Step 3: In the reduced cost matrix there will be at least one zero in each row and column, then find the suffix value of all the zeros in the reduced cost matrix by following simplification, the suffix value is denoted by S,

Therefore $S = \left\{ \frac{\text{Add the costs of nearest adjacent sides of zero which are greater than zero}}{\text{No. of costs added}} \right\}$

Step 4: Choose the maximum of S, if it has one maximum value encircle the corresponding zero first then, if it has more equal values then choose arbitrarily.

Step 5: After encircling omit the corresponding row and column, the resultant matrix must possess at least one zero is each row and column, else repeat 3.

Step 6: Repeat 3 to 3 until each row possess one encircled zero.

Step 7: Check the TSP condition now, if the condition is satisfied then optimal solution is obtained else goto next step.

Step 8: Now choose the next minimum (non-zero) cost element in cost matrix, if the next minimum value occurs at two places then consider all the cases separately until the optimal solution is reached.

Example 3.1.

		To			
		A	*B*	*C*	*D*
FROM	*A*	–	46	16	40
	B	41	–	50	40
	C	82	32	–	60
	D	40	40	36	–

By applying Zero suffix Method, we get

$$\begin{pmatrix} \infty & 27 & [0] & 21 \\ \otimes & \infty & 13 & [0] \\ 49 & [0] & \infty & 28 \\ [0] & 1 & \otimes & \infty \end{pmatrix}$$

Since each row and each column contains exactly one encircled zero, the current assignment is optimal for the assignment problem.

Therefore the optimum assignment schedule is given by

$A \to C,\ B \to D,\ C \to B,\ D \to A,$

i.e., $A \to C,\ C \to B,\ B \to D,\ D \to A,$

i.e., $A \to C \to B \to D \to A$

Check whether the route conditions are satisfied.

$A \to C \to B \to D \to A$ are satisfies the route condition.

Therefore the required minimum costs.

= (16+32+40+40) units of cost.

= 128/- units of cost.

Example 3.2.

		To				
		A	*B*	*C*	*D*	*E*
	A	–	3	6	2	3
FROM	*B*	3	–	5	2	3
	C	6	5	–	6	4
	D	2	2	6	–	6
	E	3	3	4	6	–

By applying Zero suffix Method, we get

$$\begin{pmatrix} \infty & \otimes & 2 & [0] & \otimes \\ [0] & \infty & 1 & \otimes & \otimes \\ 2 & 1 & \infty & 3 & [0] \\ \otimes & [0] & 3 & \infty & 4 \\ \otimes & \otimes & [0] & 4 & \infty \end{pmatrix}$$

Since each row and each column contains exactly one encircled zero, the current assignment is optimal.

Therefore the optimum assignment schedule is given by

$$A \to D,\ B \to A,\ C \to E,\ D \to B,\ E \to C$$

i.e., $A \to D \to B \to A,\ C \to E \to C$

and the corresponding optimum (minimum) assignment cost

= (2+3+4+2+4) units of cost.

= 15/- units of costs.

But the assignment schedule does not provide the solution of the problem, because it does not satisfy the 'route condition'.

We try to find the next best solution which satisfies the route condition also. The next minimum (non-zero) cost element in the cost matrix is 1. So we try to begin 1 in the solution. But 1 occurs at two places. We shall consider all the case separately until the acceptable solution is reached.

By using Zero suffix method, making an assignment at (2,3) instead of zero assignment at (2,1). The resulting feasible solution will then be

$$\begin{pmatrix} \infty & \otimes & 2 & [0] & \otimes \\ \otimes & \infty & [1] & \otimes & \otimes \\ 2 & 1 & \infty & 3 & [0] \\ \otimes & [0] & 3 & \infty & 4 \\ [0] & \otimes & \otimes & 4 & \infty \end{pmatrix}$$

Therefore the optimum assignment is given by

$$A \to D,\ B \to C,\ C \to E,\ D \to B,\ E \to A$$

i.e., $A \to D \to B \to C \to E \to A$

Also when an assignment is made at (3,2) instead of zero assignment at (3,5) the resulting feasible solution will be

$$\begin{pmatrix} \infty & \otimes & 2 & \otimes & [0] \\ \otimes & \infty & 1 & [0] & \otimes \\ 2 & [1] & \infty & 3 & \otimes \\ [0] & \otimes & 3 & \infty & 4 \\ \otimes & \otimes & [0] & 4 & \infty \end{pmatrix}$$

Therefore the optimum assignment is given by

$$A \to E,\ B \to D,\ C \to B,\ D \to A,\ E \to C$$

i.e., $A \to E \to C \to B \to D \to A$

For the given problem, the optimum assignment schedule is given by

$A \to D \to B \to C \to E \to A$ *(or)*

$A \to E \to C \to B \to D \to A$

In both cases, the optimum (minimum) assignment cost is 16/- units of cost.

4 Conclusions and Remarks

This paper presents an zero suffix method to solve TSPs., the proposed method is able to obtain better solutions for TSPs when compared with a coadaptive neural network method proposed in the literature. We believe that the performance of our method can be further improved by hybridizing with metaheuristic algorithms, such as tabu search and ant colony optimization, and so this is an area of further research. In addition, the method to the TSP as described in this paper can be extended to other similar or related combinational optimization problems as well, such as the vehicle routing problems and machine scheduling problems.

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