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GRW-O (Generalized Regular Weakly-Open)-

Compactness, and GRW-Connectedness

in a Topological Spaces

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Abstract

GRW-closed set is introduced in [6].Here in this paper a class of compact spaces called GRWO-Compact space using GRWO-covers, GRW-connected set are introduced and some of their properties are analyzed.

Keywords: RSO-Regular semi open sets, GRW-Closed set,cl*(A)[4],GRWO-covers-GRW-open covers, GRW-connected

1. Introduction

In [3] ,Di Maio and Noiri used semi-open covers to introduce a new class of compact spaces called s-closed spaces .In[1] Balachandran, K., Sundaram, P. and Maki, J. introduce another class of compact space called GO-compact space. The notion of generalized regular weakly closed set (GRW –closed set) as a generalization of rw–closed set [2] in a Topological space is introduced in [6] and few of its behaviors are studied. Here in this paper a class of compact spaces called GRWO-Compact spaces using GRWO-covers and GRW-connected set are introduced and some of their properties are analyzed.

2. Preliminaries

Definition 2.1[6]: A GRW-Closed Sets A subset A of a space (X, τ) is called generalized regular weakly closed (GRW-closed) if cl^* (A) $\subseteq$ U whenever A $\subseteq$ U and U is regular semi open in X.

Definition 2.2[7]: A function f :X→ Y from a topological space X into a topological space Y is called GRW-continuous function if the inverse image of every closed set in Y is GRW-closed set in Y.

Definition 2.3[7]. A function f:X → Y from a topological space X into a topological space Y is called GRW- continuous –irresolute if the inverse image of every GRW-closed set in Y is GRW-closed in X.

Definition 2.4[7]: A function f: X → Y from a topological space X into a topological space Y is said to be strongly GRW-continuous if inverse image of every GRW-open set in Y is open in X.

Definition 2.5[7]: A function f: X → Y from a topological space X into a topological space Y is said to be perfectly GRW-continuous if inverse image of every GRW-open set in Y is both open and closed in X.

Definition 2.6[7] : A topological space (X, τ) is called GRWC-space if every GRW-closed set is closed.

3. GRWO-Compact space

In this section GRWO-Compact space is introduced and some of its properties are investigated.

Definition 3.1:[1] A collection $\{A_i : i \in I\}$ of g-Open sets in a topological space X is called g-Open cover of a subset B if B $\subset \cup\{A_i : i \in I\}$, where I is set all integers.

Definition 3.2:[1] : A topological spaces X is GO-Compact if every GO-cover of X has a finite sub cover .

Definition 3.3. A collection $\{A_i : i \in I\}$ of GRW-Open sets in a topological space X is called GRW-Open Cover of a subset B if B $\subset \cup\{A_i : i \in I\}$, where I is set all integers.

Definition 3.4: A topological spaces X is GRWO-Compact if every GRWO-Cover of X has a finite sub cover .

Definition 3.5: A subset B of a topological space X is said to be GRWO-Compact relative to X ,if for every collection $\{A_i : i \in I\}$ of GRW-Open sets of X such that B $\subset \cup\{A_i : i \in I\}$ there exists a finite subset I_0 of I such that B $\subset \cup\{A_i : i \in I_0\}$.

Definition 3.6: A subset B of a topological space X is GRWO-Compact if B is GRWO-Compact as the subspace of X.

Theorem 3.1: If A is a GRW-closed subset of GRWO-compact space X, then X is GRWO-Compact relative to X .

Proof:

Let A be a GRW-Closed subset of a GRWO-Compact space X.Then A^c is GRW-Open in X. Let S be a GRWO-cover of A in X. Then, S $\cup$ $\{A^c\}$ is a GRWO-cover of X. Since X is GRWO-compact, it has a finite sub cover, say $\{G_1, G_2, \ldots G_n\}$ .If this sub cover contains A^c ,discard it ,otherwise leave the sub cover as it is . Thus there is a finite GRWO-sub cover of A and so A is GRWO-Compact.

Theorem 3.2:

GRW-continuous image of a GRWO-compact set is compact.

Proof: Let f:X→ Y be a GRW-continuous function from a GRWO-compact space X onto a topological space Y. Since f is onto it will be proved that f(X)=Y is Compact .Let $\{A_i : i \in I\}$ be an open cover of Y. Then $\{f^{-1}(A_i) : i \in I\}$ is a open by the definition of GRW-continuous function and hence GRW-open cover of X from the fact that every open set is GRW-open in a topological space .Since X is GRWO-compact,it has a finite sub cover ,say $\{f^{-1}(A_1), f^{-1}(A_2), f^{-1}(A_3)......, f^{-1}(A_n)\}$ .Hence $\{A_1, A_2, \ldots A_n\}$ is an open cover of Y and so Y is compact.

Theorem 3.3 If a function f: X → Y is GRW-continuous irresolute and a subset B is GRWO-compact relative to X ,then the image f(B) is GRWO-compact relative to Y.

Proof: Let $\{A_i : i \in I\}$ be any collection of GRW-open subsets of Y such that f(B) $\subset \cup\{A_i : i \in I\}$.Then B $\subset \cup\{f^{-1}(A_i) : i \in I\}$ and $f^{-1}(A_i)$ is GRW-open for

$i \in I$ .By using assumption there exists a finite subset I_0 of I such that B $\subset \cup\left\{ f^{-1}(A_i) : i \in I_0 \right\}$.Therefore, f(B) $\subset \cup\left\{ A_i : i \in I_0 \right\}$ which shows that f(B) is GRWO-compact relative toY.

Theorem 3.4: If f :X→Y is a strongly GRW-continuous function from a compact space X onto a topological space Y, then Y is GRWO-compact.

Proof:Let $\left\{ A_i : i \in I \right\}$ be a GRW-open cover of Y .Then $\left\{ f^{-1}(A_i) : i \in I \right\}$ is an open cover of X since f is strongly GRW-continuous .Since X is compact ,it has a finite sub cover say $\left\{ f^{-1}(A_1), f^{-1}(A_2), f^{-1}(A_3)......, f^{-1}(A_n) \right\}$ is a finite GRW-open cover of Y .Therefore Y is GRWO-compact.

Theorem 3.5: If X is GRWO-compact the it is GO-compact.

Proof: Proof follows from the fact that g-open sets are GRW-open[6].

The following observation is made

GRWO-compact →GO-compact→compact

Theorem 3.6:If A is GRW-closed in a topological space X then A $\times$Y is GRW-closed .

Proof:Let U$\times$Y be a RSO-set in X$\times$Y such that A$\times$Y$\subset$ U$\times$Y,since A is GRW-closed ,cl^*(A) $\subset$U since U is RSO-set in X ,hence cl^*(A$\times$Y) $\subset$ cl^*(A)$\times$Y $\subset$ U$\times$Y that is A$\times$Y is GRW-Closed .

Theorem 3.5: If the product space of two nonempty spaces is GRWO-compact then each factor space is GRWO-compact.

Proof: Let X$\times$Y be the product of nonempty spaces X and Y.To show that the projection

f: X$\times$Y →X from X$\times$Y onto X is GRW-irresolute function. Let F be any GRW-closed set in X .Then, it follows from the theorem 3.4 F$\times$Y is GRW-closed in X$\times$Y .But f^{-1}(F) = F $\times$Y and hence f is GRW-irresolute .Now, suppose X $\times$Y is GRWO-Compact .By using the theorem 3.3, the GRW-continuous –irresolute image of f, that is f(X$\times$Y) is GRWO-Compact. f(X$\times$Y) =X and so X is GRWO-Compact. The proof is similar for Y.

4. GRW-connectedness

In this section a new class of space called GRW-connected space is introduced and some of its properties are studied.

Definition 4.1[1] A topological space X is said to be GO-connected if X cannot be written as a disjoint union of two non-empty g-open sets. A sub set of X is GO-connected if it is GO-connected as a subspace.

Definition^{new} 4.2: A topological space X is said to be GRW-connected if X cannot be written as a disjoint union of two non-empty GRW-open sets. A sub set of X is GRW-connected if it is GRW-connected as a subspace.

Theorem 4.1: In a topological X, the following are equivalent:

i) X is GRW-Connected

ii) The only subset of X which is both GRW-open and GRW-closed are the empty set ϕ and X.

iii) Each GRW-continuous function of X into a discrete space Y with atleast two points is a constant function.

Proof:i) implies ii) Let U be a GRW-open set and GRW-closed set in X.Then X – U is both GRW-open and GRW-closed .Since X is the disjoint union of the GRW-open sets U and X-U,one of these must be empty, that is U $=\phi$ or U- X.

ii) Implies i). Suppose that X = AU B where A and B are disjoint non empty GRW-open subsets of X. Then A is both GRW-open set and GRW-closed set in X. By assumption, A $=\phi$ or X. Therefore X is GRW-connected.

ii) Implies iii) Let f: X→Y be a GRW-continuous function .Then X is covered by GRW-open and GRW-closed covering $\{f^{-1}(y):y\in Y\}$.By assumption, $f^{-1}(y) =\phi$ or X for each y $\in$ Y.If $f^{-1}(y) =\phi$ for all $y\in Y$ then f fails to be a function .Therefore there exists only one point $y\in Y$ such that $f^{-1}(y) \neq \phi$ and hence $f^{-1}(y)=X$ which shows that f is a constant function.

iii)Implies ii) Let U be both GRW-open and GRW-closed in X. Suppose $U\neq\phi$.Let f:X→ Y be a GRW-continuous function defined by f(U) ={y} and f(X-U) ={w} for some distinct points y and w in Y .By assumption, f is constant .Therefore ,y=w and hence U= X.

Remark 4.1: Every GO-Connected space is connected but not conversely [1].

Theorem 4.2:Every GRWO-Connected space is GO-connected but not conversely.

Proof: Let X be GRWO –Connected space. If possible let X be not GO-connected. Then X can be written as X=AUB where A and B are disjoint non empty g-open sets in X .Since every open set is GRW-open ,X,=AUB where A and B are disjoint non empty and GRW-open sets in X ,but X is GRW-connected so X cannot be written as union of two disjoint non-empty GRW-open sets ,hence the assumption is wrong that is X is GO-connected .Therefore X is GO-connected.

The converse need not be true as seen from the following example.

Example 4.1: Let X={a,b,c} and $\tau = \{\phi, \{a\}, \{a,c\}, \{a,b\}, X\}$. Then the topological space (X,τ) is GO-connected .However ,since {a,b} is both GRW-open and GRW-closed , X is not GRW-connected by theorem 4.1.

From the above results, the following observation is made

GRW-connected → GO-Connected → Connected

Theorem 4.3: In a topological space (X,τ) with at least two points, if $\tau = \beta$ where β is the family of all closed sets, then X is not-GRW-connected.

Proof: Using the hypothesis and theorem 2.10 to Levine [5] there is a proper non empty subset of X which is both g-open and g-closed hence GRW-open and GRW-closed set in X .Hence by the theorem 4.1 X is not GRW-connected.

Theorem 4.4: Suppose that X is a GRWC-space, and then X is connected if and only if X is GRWO-connected.

Proof: Assume that X is GRWC-space and Connected .If possible ,let X be not GRW-connected .then X can be written in the for X = AUB where A and B are non empty ,disjoint and GRW-open sets in X .Since X is GRWC-space ,every GRW-open set is open and so X =A U B where A and B are disjoint ,nonempty and open sets in X .This contradicts the fact that X is Connected .Therefore X is GRW-connected.

Converse follows from the theorem 4.2.

Theorem 4.5:

i) If f:X→ Y is a GRW-continuous surjection and X is GRWO-connected then Y is Connected .

ii) If f:X→ Y is a GRW-continuous irresolute surjection and X is GRW-connected then Y is GRW-Connected .

Proof: i)Suppose that Y is not connected .Let Y =A U B where A and B are disjoint non empty open sets in Y.Since f is GRW-continuous and onto, X = f^{-1}(A) Uf^{-1}(B) where f^{-1}(A) and f^{-1}(B) are disjoint nonempty and GRW-open sets in X. This contradicts the fact that X is GRWO-connected. Hence Y is connected.

ii) If possible ,assume that Y is not GRWO-connected ,then Y = A U B where A and B are non empty ,disjoint and GRW-open sets in Y.Since f is GRW-continuous –irresolute , $f^{-1}(A)$ and $f^{-1}(B)$ are GRW-open in X.Since f is onto , $f^{-1}(A)$ and $f^{-1}(B)$ are non empty .

Now X= $f^{-1}(Y)$ =$f^{-1}(A)$U $f^{-1(}B)$ .This contradicts the fact that X is GRWO-connected .Therefore ,Y is GRWO-connected .

Theorem 4.6: Let f be a GRW-continuous function from a topological space (X,τ) into a topological space (Y,σ).Then f(H) is a connected subset of Y for every closed and GRWO-connected subset H of X.

Proof: The restriction $f_{/H}$ of f to H is GRW-continuous by theorem 3.5[second paper].The image of the GRWO-connected space (H, $\tau_{/H}$) under $f_{/H}$: (H, $\tau_{/H}$) → (f(H), $\sigma_{/f(H)}$) is connected , using the theorem 4.5.Thus (f(H), $\sigma_{/f(H)}$) is connected .Therefore f(H) is connected subset of Y.

Theorem 4.7: If f:X→ Y is a strongly GRW-continuous function from a connected space X onto a topological space Y then Y is GRWO-connected .

Proof: If possible, let Y be not GRWO-connected .Then Y can be written in the form Y =AUB where A and B are disjoint nonempty GRW-open sets in Y. Since f is strongly GRW-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are open sets in X. Also X= $f^{-1}(Y)$= $f^{-1}(AUB)$= $f^{-1}(A)$U $f^{-1}(B)$.This contradicts the fact that X is connected .Therefore Y is GRWO-Connected .

Theorem 4.8:If product space of two nonempty spaces is GRWO-connected ,then each factor space is GRWO-connected .

Proof: Let X×Y be the product space of nonempty spaces X and Y. It will be proved that the projection function f: X×Y → X is GRW-continuous –irresolute .Let A be any GRW-closed set in X .Then by theorem 3.4 ,A×Y is GRW-closed set in X ×Y. But $f^{-1}(A)$ =A×Y and so f is GRW-continuous –irresolute .Now assume that X×Y is GRWO-connected .Further f(X×Y) =X and so X is GRWO-connected .The proof of Y is GRWO-connected is similar.

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