Study of Stability of a Discrete Two-Predators and One Prey Model

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Abstract

We have studied a discrete two predator and one prey model where predators functional response is taken Halling type II and I respectively. We have find fixed point and developed the condition for which the system is stable or not at the fixed point. We have verified the result through numerical calculation.

Keywords: predator, prey, stability, unstable, Jury condition

1. Introduction

In nature it is known that the discrete-time type models described by difference equation are more appropriate than the continuous-time models. In nature there are two or more competing predators can exist with a single homogeneous prey species. When two predators and one prey are present in a system we have assumed that the mortality rate of prey depends on two predators and moreover we also assume that there is no intraspecific competition between two predators. Two predators have natural mortality rate. Two predators eat prey to survive. We also assume that prey follows logistic equation. Now a days two predator and one prey discrete model have studied extensively.

Abrams et al. [1] have studied the two-predator one prey model extensively. They studied different situation of two-predator and one-prey system. They described the chaotic dynamics of the system.
Gatto et al. [8] have considered a simple type prey-predator model to understand the dynamical behavior of fish. They analyzed the stability property of the system.

Liu and Xiao [11] have considered a discrete-time system. This system gave different type of Bifurcation namely fold bifurcation, flip bifurcation and Neimark-Saclor bifurcation and also gave a stable invariant cycle in the interior equilibrium point.

Elabbasy et al.[7] discover the global convergence result , boundedness and periodicity of solutions of recursive sequence.

Wu and Li [15] , by developing suitable Lyapunov function and using the comparison theorem of difference equation developed a sufficient condition that ensure the permanence and global attractivity of the discrete predator-prey system with Hassell-Varley type functional response.

Li and Yang [10] have developed a sufficient condition for the permanence of predator-prey systems with Beddington-DeAngelis functional response and feedback control.

Mohamad and Gopalsamy [12] have considered the single species discrete model:

\[ x(n + 1) = x(n)exp\left[ {r(n)\left( {1 - \frac{x(n)}{K(n)}} \right)} \right] \]

They derived a set of sufficient condition for which the above system gave a positive and globally asymptotically stable almost periodic solution. But using a counter example, Zhou and Zou [16] have shown that the main result of [12] was wrong. They have developed an alternative proof for existence of a positive as well as globally asymptotically stable w-periodic solution of the above system. After that Chen and Zhou [5] generalized the above system with two-species Lotka-Volterra competition system:

\[ x(n + 1) = x(n)exp\left[ {r_1(n)\left( {1 - \frac{x(n)}{K_1(n)}} \right) - \mu_2 y(n)} \right] \]
\[ y(n + 1) = y(n)exp\left[ {r_2(n)\left( {1 - \frac{y(n)}{K_2(n)}} \right) - \mu_1 x(n)} \right] \]

They obtained the sufficient condition for existence of a globally stable periodic solution of the system.

After that Wang and Lu [4] discussed the following Lotka-Volterra model:

\[ x_i(k + 1) = x_i(k)exp\left[ {r_i(k) - \sum_{j=1}^{n} a_{ij}(k)x_j(k)} \right], i = 1, 2, ..n. \]

where \( x_i(k) \) is the density of population \( i \) at \( k \)-th generation; \( r_i(k) \) is the growth rate of population \( i \) at \( k \)-th generation; \( a_{ij}(k) \) calculates the intensity of intraspecific competition or intraspecific action of species. Using Lyapunov function and covering theorem of
mathematical analysis, they derived a set of sufficient condition ascertaining the system
to be globally stable.

Chen and Chen [14] studied a discrete Volterra model with interference and Holling
type-II:

\[
\begin{align*}
x(n + 1) &= x(n)\exp \left[ r_1(n) - b_1(n)x(n) - \frac{c_1(n)}{k + x(n)} y^m(n) \right] \\
y(n + 1) &= y(n)\exp \left[ -r_2(n)b_2(n)y(n) - \frac{c_2(n)x(n)}{k + x(n)} y^{m-1}(n) \right]
\end{align*}
\]

Agiza et al. [2] have studied the chaotic and stability behavior of a discrete prey-predator
model of Holling type-II. Danca et al. [6] have also studied a nonlinear prey-predator model.
Jing and Yang [9] have considered the discrete-time predator-prey system obtained by
Euler method. They developed the condition for the existence for flip bifurcation and Hopf
bifurcation using center manifold theorem and bifurcation theory. Seno [13] have studied
a discrete prey-predator model preserving the dynamics of a structurally unstable Lotka-
Volterra model. The main aim of the paper is to investigate the stability and instability
nature of the fixed point of the discrete system modeled by two predators depending on
one prey.

We have organized this paper as follows. We have described the model in section
2. In section 3, we find the nature of fixed point. In section 4, a brief discussion with
numerical example is given. A brief discussion is given in section 5.

2. The Model

We have first considered a model which is used by Armstrong and McGehee (1980)
[3] to describe the nonequilibrial coexistence of two species on a single biotic resource.
Firstly we modify the model studied in [1]. The modified model is represented by:

\[
\begin{align*}
\frac{dp}{dt} &= p \left[ r \left( 1 - \frac{p}{k} \right) - \frac{C_1 x}{1 + hC_1 p} - C_2 y \right] \\
\frac{dx}{dt} &= x \left( \frac{B_1 C_1 p}{1 + hC_1 p} - D_1 - m_1 x \right) \\
\frac{dy}{dt} &= y \left( B_2 C_2 p - D_2 \right)
\end{align*}
\]

Here \( p \) denotes prey density and \( x \) and \( y \) that of predators. The parameter \( B_i \) is the
conversion efficiency of food into offspring for predator \( i \), \( h \) is the handling time per prey
item for predator \( x \), \( D_1 \) is a density independent death rate, \( C_1 \) is a searching predator’s
attack rate, and \( r \) and \( K \) are logistic growth parameters. \( m_1 \) is intra specific competition
between predator.
The discrete version of the above model is given by

\[ p(n + 1) = p(n) + p(n) \left[ r \left( 1 - \frac{p(n)}{k} \right) - \frac{C_1 x(n)}{1 + hC_1 p(n)} - C_2 y(n) \right] \]  

(4)

\[ x(n + 1) = x(n) + x(n) \left( \frac{B_1 C_1 p(n)}{1 + hC_1 p(n)} - D_1 - m_1 x(n) \right) \]  

(5)

\[ y(n + 1) = y(n) + y(n) \left( B_2 C_2 p(n) - D_2 \right) \]  

(6)

where meaning of the parameters are as system (1-3). We have now studied system (4-6).

3. Fixed point and Stabiliy Analysis of the fixed Points of systems (4-6)

For finding the fixed point of system (4-6) we first write \( p(n + 1) = p(n) = p^* \), \( x(n + 1) = x(n) = x^* \) and \( y(n + 1) = y(n) = y^* \) respectively.

Therefore the fixed points of the system (4-6) are \( E_0(0,0,0), E_1(k,0,0) \) and \( E\left( \frac{D_2}{B_2 C_2}, \frac{1}{m_2} \left[ \frac{B_1 C_1 D_2}{B_2 C_2 + hC_1 D_2} - D_1 \right], y^* \right) \) where \( y^* = \frac{1}{C_2} \left[ r \left( 1 - \frac{D_2}{B_2 C_2} \right) - \frac{C_1 C_2 D_2}{B_2 C_2 + hC_1 D_2} \frac{1}{m_1} \left( \frac{B_1 C_1 D_2}{B_2 C_2 + hC_1 D_2} - D_1 \right) \right] \) are arbitrary constants that are determined by the initial conditions. Note that \( \lambda = \frac{B_1 C_1 - D_1}{B_2 C_2 + hC_1 D_2} > D_1 \) when \( \frac{B_1 C_1 D_2}{B_2 C_2 + hC_1 D_2} \). Now the Jacobian matrix is \( J \) at any fixed point \((x, y, p)\) is given by

\[
J = \begin{pmatrix} 1 + r \left( 1 - \frac{p}{k} \right) - \frac{C_1 x}{1 + hC_1 p} - C_2 y & -\frac{C_1 p}{1 + hC_1 p} & -C_2 p \\ \frac{B_1 C_1 x}{(1 + hC_1 p)^2} & 1 + \frac{B_1 C_1 p}{1 + hC_1 p} - D_1 - 2m_1 x & 0 \\ B_2 C_2 y & 0 & 1 + B_2 C_2 p - D_2 \end{pmatrix}
\]  

(7)

To discuss the nature of the fixed point of given system we first state the result known as Jury Condition [1].

**Result Jury Condition:**

Let us consider

\[ z_{n+1} = M z_n \]  

(8)

where \( z_n \) is an m-vector and \( M \) an \( m \times m \) -matrix, with \( z_0 \) given. Let the solution be in the form \( z_n = \lambda^n c \), where \( c \) is an m-vector. The general solution of the equation (8) is \( z_n = \sum_{i=1}^{m} \lambda_i^{n} c_i \) where \( c_i \) is the eigenvector corresponding to eigenvalues \( \lambda_i \). The \( \lambda_i \) are arbitrary constants that are determined by the initial conditions. Note that – if each \( |\lambda_i| < 1 \) then \( |z_n| \to 0 \) as \( n \to \infty \), and
If there exists $i$ such that $|\lambda_i| > 1$, and if $A_i \neq 0$, then $|z_n| \longrightarrow n \longrightarrow \infty$.

For $m = 3$, with eigenvalues equation $\lambda^3 + \lambda^2 a_1 + \lambda a_2 + a_3 = 0$, the necessary and sufficient condition for asymptotic stability, $|\lambda_i| < 1$ for $i = 1, 2, 3$ are the Jury Conditions $|a_1 + a_3| < a_2 + 1$, $|a_3| < 1$ and $|a_2 - a_3a_1| < |1 - a_3^2|$.

**Theorem 1.** The fixed point $E_0$ of system (4-6) is always unstable.

**Proof.** At the point $E_0(0,0,0)$ the Jacobian matrix is given by

$$J(E_0) = \begin{bmatrix} 1 + r & 0 & 0 \\ 0 & 1 - D_1 & 0 \\ 0 & 0 & 1 - D_2 \end{bmatrix}$$

The eigenvalues of $J(E_0)$ are $\lambda_1 = 1 - D_1$, $\lambda_2 = 1 - D_2$ and $\lambda_3 = 1 + r$. If $0 < D_1 < 2$ then $|\lambda_1| < 1$, $0 < D_2 < 2$ then $|\lambda_2| < 1$, and $-2 < r < 0$ then $|\lambda_3| < 1$. Thus we say that the system (4-6) is stable when $0 < D_1 < 2$, $0 < D_2 < 2$ and $-2 < r < 0$ hold. But the parameter $r$ can not be negative and hence the system (4-6) is unstable. This completes the proof.

**Theorem 2.** The fixed point $E_1$ of system (4-6) is stable if $\frac{B_1C_1k}{1 + hC_1k} < D_1 < 2 + \frac{B_1C_1k}{1 + hC_1k}, B_2C_2k < D_2 < 2 + B_2C_2k$ and $0 < r < 2$ hold and unstable otherwise.

**Proof.** The Jacobian matrix at the point $E_1(k,0,0)$ is given by

$$J(E_1) = \begin{bmatrix} 1 - r & -\frac{C_1k}{1 + hC_1k} & -C_2k \\ 0 & 1 + \frac{B_1C_1k}{1 + hC_1k} - D_1 & 0 \\ 0 & 0 & 1 + B_2C_2k - D_2 \end{bmatrix}$$

The eigenvalues of $J(E_1)$ are $\lambda_1 = 1 + \frac{B_1C_1k}{1 + hC_1k} - D_1$, $\lambda_2 = 1 + B_2C_2k - D_2$ and $\lambda_3 = 1 - r$.

When $\frac{B_1C_1k}{1 + hC_1k} < D_1 < 2 + \frac{B_1C_1k}{1 + hC_1k}$ then $|\lambda_1| < 1$, when $B_2C_2k < D_2 < 2 + B_2C_2k$ then $|\lambda_2| < 1$ and when $0 < r < 2$ then $|\lambda_3| < 1$. Hence system (4-6) is stable when $\frac{B_1C_1k}{1 + hC_1k} < D_1 < 2 + \frac{B_1C_1k}{1 + hC_1k}, B_2C_2k < D_2 < 2 + B_2C_2k$ and $0 < r < 2$ hold. Otherwise the system (4-6) is unstable. This completes the proof.

**Theorem 3.** The fixed point $E$ of system (4-6) is stable if (i) $\frac{B-1}{A} < r < \frac{B+1}{A}$

(ii) $rC + D > 0$

(iii) $|M + Nr + Gr^2| < |1 - (rA - B)^2|$ hold simultaneously and otherwise system (4-6) is unstable; where

$$A = \left[1 + D_1 - \frac{B_1C_1D_2}{B_2C_2 + hC_1D_2}\right] \left[1 + D_2 - kB_2C_2\right] \frac{D_2}{kB_2C_2}$$
Therefore the eigenvalues of the above Jacobian matrix are the roots of the equation
\[\lambda^3 - a_1 \lambda^2 + a_2 \lambda - a_3 = 0\]
where
\[a_1 = -3 + m_1 x^* - p^* \left( \frac{r}{k} + \frac{hC_2^2 x^*}{(1+hC_1 p^*)^2} \right)\]
\[a_2 = 3 - 2m_1 x^* + (2 - m_1 x^*)p^* \left( \frac{r}{k} + \frac{hC_2^2 x^*}{(1+hC_1 p^*)^2} \right) + \frac{B_1 C_2^2 p^* x^*}{(1+hC_1 p^*)^2} + B_2 C_2^2 p^* y^*\]
\[a_3 = - \left[ (1 - m_1 x^*) (1 + B_2 C_2^2 y^* p^*) + (1 - m_1 x^*) p^* \left( \frac{r}{k} + \frac{hC_2^2 x^*}{(1+hC_1 p^*)^2} \right) + B_1 C_2^2 p^* x^* \right]\]
The eigenvalues of the Jacobian matrix \(J(E)\) will be negative if the Jury condition holds i.e. \(|a_1 + a_3| < a_2 + 1\), \(|a_3| < 1\) and \(|a_2 - a_3 a_1| < |1 - a_3^2|\) are satisfied.

Now \(|a_3| < 1\) implies that
\[\frac{B - 1}{A} < r < \frac{B + 1}{A}\]  
where
\[A = \left[ 1 + D_1 - \frac{B_1 C_2^2}{B_2 C_2 + hC_1 D_2} \right] \left( 1 + D_2 - k B_2 C_2 \right) \frac{D_2}{k B_2 C_2}\]
\[ B = (1 - m_1 x^*) \left[ 1 - \frac{B_2 C_1 C_2 p^* x^*}{1 + h C_1 p^*} + \frac{h C_1 p^* x^*}{(1 + h C_1 p^*)^2} \right] + \frac{B_3 C_2^2 x^* p^*}{(1 + h C_1 p^*)^3} \]

Since \( r > 0 \) then \( \frac{B_2 C_2^2}{A} > 0 \) and \( \frac{B_3 C_2^2}{A} > 0 \). Here two cases arises.

Case 1: \( A > 0, B > 1 \) and \( B > -1 \). \( 1 - m_1 x^* > 0 \) and \( 1 + D_2 > k B_2 C_2 > D_2 \) and \( B > \max[-1, 1] \).

\[ 1 - m_1 x^* < 0 \text{ and } 1 + D_2 < k B_2 C_2 > D_2 \]

i.e. \[ B = \left( 1 - \left[ \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} - D_1 \right] \right) \left[ 1 - \frac{C_1 D_2 \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} - D_1}{B_2 C_2 + h C_1 D_2} + \frac{h C_1 D_2 B_2 C_2 \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} - D_1}{(B_2 C_2 + h C_1 D_2)^2} \right] + \]

\[ \frac{B_1 C_2^2 D_2 B_2 C_2^2}{1 - m_1} \left[ \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} - D_1 \right] \]

\[ B = \left( 1 - \left[ \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} - D_1 \right] \right) \left[ 1 - \frac{C_1 D_2 \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} - D_1}{B_2 C_2 + h C_1 D_2} + \frac{h C_1 D_2 B_2 C_2 \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} - D_1}{(B_2 C_2 + h C_1 D_2)^2} \right] + \]

\[ \frac{B_1 C_2^2 D_2 B_2 C_2^2}{1 - m_1} \left[ \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} - D_1 \right] \]

Case 2: \( A < 0, B < 1 \) and \( B < -1 \). \( 1 - m_1 x^* > 0 \) and \( 1 + D_2 < k B_2 C_2 > D_2 \) holds or.

\[ 1 - m_1 x^* < 0 \text{ and } 1 + D_2 > k B_2 C_2 > D_2 \]

i.e.\[ B = \left( 1 - \left[ \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} - D_1 \right] \right) \left[ 1 - \frac{C_1 D_2 \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} - D_1}{B_2 C_2 + h C_1 D_2} + \frac{h C_1 D_2 B_2 C_2 \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} - D_1}{(B_2 C_2 + h C_1 D_2)^2} \right] + \]

\[ \frac{B_1 C_2^2 D_2 B_2 C_2^2}{1 - m_1} \left[ \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} - D_1 \right] \]

\[ B = \left( 1 - \left[ \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} - D_1 \right] \right) \left[ 1 - \frac{C_1 D_2 \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} - D_1}{B_2 C_2 + h C_1 D_2} + \frac{h C_1 D_2 B_2 C_2 \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} - D_1}{(B_2 C_2 + h C_1 D_2)^2} \right] + \]

\[ \frac{B_1 C_2^2 D_2 B_2 C_2^2}{1 - m_1} \left[ \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} - D_1 \right] \]

Here we see that \( 1 + D_1 > \frac{B_2 C_2}{B_2 C_2 + h C_1 D_2} > D_1 \) and \( 1 + D_2 > k B_2 C_2 > D_2 \) and \( B > 1 \).

Hence \( 1 - m_1 x^* > 0 \) and \( 1 + D_1 > \frac{B_2 C_2}{B_2 C_2 + h C_1 D_2} > D_1 \) and \( 1 + D_2 > k B_2 C_2 > D_2 \) and \( B > 1 \) holds.

Again \( |a_1 + a_3| < 1 + a_2 \) implies that \( m_1 x^* B_2 C_2^2 y^* p^* > 0 \) and

\[ r C + D > 0. \] (13)

where

\[ C = \left[ 2 + D_1 - \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} \right] [k B_2 C_2 - D_2 - 2] \]

\[ D = \left[ 2 - \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} - D_1 \right] \left[ 4 - \frac{C_1 D_2 \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} - D_1}{B_2 C_2 + h C_1 D_2} + \frac{h C_1 D_2 B_2 C_2 \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} - D_1}{(B_2 C_2 + h C_1 D_2)^2} \right] + \]

\[ \frac{B_1 C_2^2 D_2 B_2 C_2^2}{1 - m_1} \left[ \frac{B_3 C_2 D_2}{B_2 C_2 + h C_1 D_2} - D_1 \right] \]

and

moreover \( |a_2 - a_3| < |1 - a_3^2| \) implies that

\[ |M + N r + G r^2| < |1 - (r A - B)^2| \] (14)
holds where \(1 - a_3^2 = 1 - (rA - B)^2\) and \(a_2 - a_3a_1 = M + Nr + Gr^2\)
\[
M = \left[1 + D_1 - \frac{B_2C_2D_2}{B_2C_2 + hC_1D_2}\right] \left[1 - B + (1 - B) \left[1 + \frac{1}{m_1} \frac{B_2C_2D_2}{B_2C_2 + hC_1D_2} - D_1 \frac{hC_2B_3C_2}{(B_2C_2 + hC_1D_2)^2}\right]\right.
- \frac{1}{m_1} \left[\frac{B_1C_1D_2}{B_2C_2 + hC_1D_2} - D_1\right]^2 \frac{B_2C_2C_1D_2}{B_2C_2 + hC_1D_2} - D_1\right] + \frac{B_1C_1D_2}{B_2C_2 + hC_1D_2}\right]\right]
\[
N = \frac{A D_3}{k B_2 C_2} \left[3 - \frac{B_2C_2D_2}{B_2C_2 + hC_1D_2} + D_1 + \frac{hC_2B_2C_2}{(B_2C_2 + hC_1D_2)^2} \frac{1}{m_1} \frac{B_2C_2D_2}{B_2C_2 + hC_1D_2} - D_1\right]\right]
\[
G = \frac{-A D_3}{k B_2 C_2}
\]

If system (4-6) satisfies the conditions (12),(13) and (14) at the interior fixed point \(E\) then system (4-6) is stable and otherwise if any one of the conditions of (12),(13) and (14) are violated then system (4-6) is unstable.

4. A Numerical example

We explain our result by several examples.
Example 1. Let us consider \(B_1 = 1, C_1 = 1, h = 1, D_1 = 0.1, m_1 = 1, B_2 = 1, C_2 = 1, D_2 = 0.5, r = 0.7, k = 2\). The fixed points are \(E_0(0, 0, 0), E_1(2, 0, 0)\) and \(E(0.5, 0.233, 0.370)\). The eigenvalues at \(E_0(0, 0, 0)\) are 1.7, 0.9, 0.5 which can not satisfy the condition of Theorem 1 and so system (4-6) is unstable at \(E_0\). On the other hand the eigenvalues at \(E_1(2, 0, 0)\) are 0.3, 2.5, 1.5667 which can not satisfy the stability condition of Theorem 2 and hence system (4-6) is unstable at \(E_1\). At the fixed point \(E(0.5, 0.233, 0.370)\) the eigenvalues are 0.802, 0.921+i0.460, 0.921-i0.460 and hence system (4-6) is stable at \(E\).

Example 2. If we consider \(B_1 = 1, C_1 = 1, h = 1, D_1 = 1, m_1 = 1, B_2 = 1, C_2 = 1, D_2 = 1, r = 1, k = 1\). The fixed points are \(E_0(0, 0, 0), E_1(1, 0, 0)\) and \(E(1, -0.5, 0.250)\). The eigenvalues at \(E_0(0, 0, 0)\) are 2, 0, 0 which can not satisfy the condition of Theorem 1 and so system (4-6) is unstable at \(E_0\). On the other hand the eigenvalues at \(E_1(1, 0, 0)\) are 0, 0.5, 1 which satisfy the stability condition of Theorem 2 and hence system (4-6) is stable at \(E_1\). But the fixed point \(E(1, -0.5, 0.250)\) can not lie in the first quadrant and the initial population can not be negative and so system (4-6) is unstable at \(E\).

Example 3. If we consider \(B_1 = 1, C_1 = 1, h = 1, D_1 = 1, m_1 = 1, B_2 = 1, C_2 = 1, D_2 = 1, r = 2, k = 1\). The fixed points are \(E_0(0, 0, 0), E_1(2, 0, 0)\) and \(E(1, 1, 0.5)\). The eigenvalues at \(E_0(0, 0, 0)\) are 3, 0, 0 which can not satisfy the condition of Theorem 1 and so system (4-6) is unstable at \(E_0\). On the other hand the eigenvalues at \(E_1(2, 0, 0)\) are -1, 0.667, 2 which satisfy the unstability condition of Theorem 2 and hence system (4-6) is unstable at \(E_1\). But at the fixed point \(E(1, 1, 0.5)\) system (4-6) is stable as for this choice of parameter system (4-6) have satisfied all the Jury condition and hence system (4-6) is stable.

Thus from above discussion we have seen that given system is unstable at the origin in every respect. On the other hand we say that system is stable at the interior fixed point and boundary fixed point after satisfying some criterion that state in Theorem 2 and 3 earlier. But we can not say the global stability of given system.
5. Discussion
We have considered a discrete system containing two predators and one prey. Moreover we see that the predator-I of system (4-6) have a functional response of Holling type-II and predator-II of system (4-6) follows the functional response of Holling type-I and prey population of same system (4-6) also follows the functional response of Holling type-II. Here we see that the fixed point (0, 0, 0) of system (4-6) is always unstable without any restriction of parameters of system (4-6). On the other hand the boundary fixed point of system (4-6) is stable only when the parameters are satisfied the condition of Theorem 2. But the interior fixed point of system (4-6) is stable if the parameters of system (4-6) have satisfied the conditions of Theorem 3. Thus our system (4-6) is stable at the boundary point and at interior fixed point if the parameters satisfy the conditions of Theorem 2 and 3.

References


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