On the Solution of a Special Type of Large Scale
Integer Linear Vector Optimization Problems
with Uncertain Data through TOPSIS Approach

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Abstract
This paper focuses on the solution of a Large Scale Integer Linear Vector Optimization Problems with chance constraints (CHLSILVOP) of a special type through the Technique for Order Preference by Similarity Ideal Solution (TOPSIS) approach, where such problems have block angular structure of the constraints. Chance constraints involve random parameters in the right hand sides. TOPSIS is extended to solve CHLSILVOP with constraints of block angular structure. Compromise (TOPSIS) control minimizes the measure of distance, provided that the closest solution should have the shortest distance from the positive ideal solution (PIS) as well as the longest distance from the negative ideal solution (NIS). As the measure of “closeness” $d_P$-metric is used. Thus, I reduce a $k$-dimensional objective space to a two-dimensional space by a first-order compromise procedure. The concept of a membership function of fuzzy set theory is used to represent the satisfaction level for both criteria. Moreover, I derive a single objective largescale Integer programming problem using the max–min operator for the second-order compromise operation. Also, An interactive decision making algorithm for generating integer Pareto optimal (compromise) solution for CHLSILVOP through TOPSIS approach is provided where the decision maker (DM) is asked to specify the relative importance of objectives. Finally, a numerical illustrative example is given to clarify the main results developed in this paper.

Mathematics Subject Classification: 03E72, 90C06, 90C10, 90C15, 90C29
Keywords: TOPSIS; Interactive decision making; Large Scale Systems; Vector Optimization Problems; Integer programming; Fuzzy set theory; Stochastic programming; Block angular structure; Compromise (satisfactory solution); Positive ideal solution; Negative ideal solution

(1) Introduction

Stochastic programming deals with a class of optimization models and algorithms in which some or all of the data may be subject to significant uncertainty. In real world decision situations, when formulating a Large Scale Vector Optimization problem (LSVOP), some or all of the parameters of the optimization problem are described by stochastic (or random or probabilistic) variables rather than by deterministic quantities [20, 22, 23, 24]. Also, various variables of the real system may be constrained to take only integer values [13, 25].

Most CHLSILVOP arising in applications have special structures that can be exploited. One familiar structure is the block angular structure to the constraints, and several kinds of decomposition methods for linear and nonlinear programming problems with block angular structure have been proposed [13, 16, 22]. An algorithm for parametric large scale integer multiple objective decision- making problems with block angular structure has been presented in [13]. Recently, a significant number of studies have indeed been reported on single and multiple objective linear and nonlinear programming problems [2, 3, 4, 5, 7, 8, 9, 13, 14, 15, 16, 17, 18, 20, 21, 24, 26, 28].

In the last decade, a significant number of papers have been published on the TOPSIS approach. Abo-Sinna [1] extends TOPSIS approach to solve multi-objective dynamics programming (MODP) problems. He shows that using the fuzzy max-min operator with nonlinear membership functions, the obtained solutions are always nondominated solutions of the original MODP problems. Deng et al. [12] formulate the inter-company comparison process as a multi-criteria analysis model, and presents an effective approach by modifying TOPSIS for solving such problem. Chen [7] extends the concept of TOPSIS to develop a methodology for solving multi-person multi-criteria decision-making problems in fuzzy environment. Chen and Tzeng [8] combine the grey relation model with the ideas of TOPSIS for selecting an expatriate host country. Yang and Chou [27] use the TOPSIS approach to solve the multiresponse simulation-optimization problem. Chenget al [9] proposea hierarchy multiple criteria decision-making (MCDM) model based on fuzzy-sets theory to deal with the supplier selection problems in the supply chain system. According to the concept of the TOPSIS, a closeness coefficient is defined to determine the ranking order of all suppliers by calculating the distances to the both fuzzy positive-ideal solution (FPIS) and fuzzy negative-ideal solution (FNIS) simultaneously.


In this paper, TOPSIS is extended for solving CHLSILVOP. TOPSIS was first developed by Hwang and Yoon [16] for solving a multiple attribute decision making problem. It is based upon the principle that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). The single criterion of the shortest distance from the given goal or the PIS may be not enough to decision makers. In practice, we might like to have a decision which not only makes as much profit as possible, but also avoids as much risk as possible. A similar concept has also been pointed out by Zeleny [30].

The formulation of CHLSILVOP with block angular structure is introduced in section 2. The family of \( \text{d}_p \)-distance and its normalization is discussed in section 3. The TOPSIS approach for CHLSILVOP is presented in section 4. In section 5, an interactive decision making algorithm for generating a Pareto optimal (compromise) integer solution for CHLSILVOP through TOPSIS approach is provided where the decision maker (DM) is asked to specify the relative importance of objectives. For the sake of illustration, a numerical example is given in section 6. Finally, conclusions will be given in section 7.

### (2) Formulation of the Problem

Consider the following CHLSILVOP with a block angular structure of the constraints:

\[
\text{Maximize } \left(z_1(X), z_2(X), \ldots, z_k(X)\right) \quad (1 - a)
\]

subject to

\[
X \in M = \{X \in \mathbb{R}^n : P\{\sum_{j=1}^q \sum_{i=1}^n \alpha_{ijh_0} x_{ijh_0} \leq v_{h_0} \} \geq \alpha_{h_0}, \quad h_0 = 1, 2, \ldots, m_0, \}
\]

\[
P\{\sum_{i=1}^n b_{ijh_j} x_{ijh_0} \leq v_{h_j} \} \geq \alpha_{h_j}, \quad h_j = m_{j-1} + 1, m_{j-1} + 2, \ldots, m_j,
\]

\[
x_{ij} \geq 0 \text{ and integer, } i \in \mathbb{N}, j = 1, 2, \ldots, q, q > 1 \quad (1 - b)
\]

where

- \( k \) : the number of objective functions,
- \( q \) : the number of subproblems,
- \( m \) : the number of constraints,
- \( n \) : the number of variables,
- \( n_j \) : the number of variables of the \( j \)th subproblem, \( j = 1, 2, \ldots, q \),
- \( m_0 \) : the number of the common constraints represented by

\[
P\{\sum_{j=1}^q \sum_{i=1}^n \alpha_{ijh_0} x_{ijh_0} \leq v_{h_0} \} \geq \alpha_{h_0}, \quad h_0 = 1, 2, \ldots, m_0,
\]
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\( m_j \): the number of independent constraints of the \( j^{th} \) subproblem represented by

\[
P\{ \sum_{i=1}^{n} b_{ijh} x_{ijh_0} \leq v_{h_j} \} \geq \alpha_{h_j}, \quad h_j = m_{j-1} + 1, m_{j-1} + 2, \ldots, m_j.
\]

\( \mathbb{R} \) : the set of all real numbers,

\( X \) : an \( n \)-dimensional column vector of variables,

\( K = \{1,2,\ldots,k\} \),

\( \mathbb{N} = \{1,2,\ldots,n\} \),

\( \mathbb{R}^n = \{X = (x_1, x_2, \ldots, x_n)^T : x_i \in \mathbb{R}, i \in \mathbb{N} \} \).

In addition, \( P \) means probability, \( \alpha_{h_0} \) and \( \alpha_{h_j} \) are a specified probability levels. For the sake of simplicity, consider that the random parameters, \( v_{h_0} \) and \( v_{h_j} \) are distributed normally and independently of each other with known means \( E\{v_{h_0}\} \) and \( E\{v_{h_j}\} \) and variances \( \text{Var}\{v_{h_0}\} \) and \( \text{Var}\{v_{h_j}\} \).

If the objective functions are linear, then the objective function can be written as follows:

\[
z(X) = \sum_{j=1}^{q} z_{ij} = \sum_{j=1}^{q} c_{ij} x_j, \quad i = 1,2,\ldots,k
\]

Using the chance constrained programming technique \[22, 23\], the deterministic version of the CHLSILVOP problem (1) can be written as follows:

\[
\text{Maximize} \quad (z_1(X), z_2(X), \ldots, z_k(X))
\]

subject to

\[
X \in M^f = \{X \in \mathbb{R}^n : \sum_{j=1}^{q} \sum_{i=1}^{\infty} a_{ijh_0} x_{ijh_0} \leq E\{v_{h_0}\} + k_{\alpha_{h_0}} \sqrt{\text{Var}\{v_{h_0}\}}, h_0 = 1,2,\ldots,m_0,
\]

\[
\sum_{i=1}^{n} b_{ijh} x_{ijh} \leq E\{v_{h_j}\} + k_{\alpha_{h_j}} \sqrt{\text{Var}\{v_{h_j}\}}, h_j = m_{j-1} + 1, m_{j-1} + 2,\ldots,m_j
\]

\( x_{ij} \geq 0, \text{and integer}, i \in \mathbb{N}, j = 1,2,\ldots,q, q > 1 \)

where \( k_{\alpha_{h_j}}, j = 0,1,2,\ldots,q \), is the standard normal value such that \( \Phi(k_{\alpha_{h_j}}) = 1 - \alpha_{h_j} \), \( j = 0,1,\ldots,q \) and \( \Phi \) represents the cumulative distribution function of the standard normal distribution.

(3) Some Basic Concepts of distance Measures

The compromise programming approach \[29\] has been developed to perform multiple objective decision making problems, reducing the set of nondominated solutions. The compromise solutions are those which are the closest by some distance measure to the ideal one.

The point \( z_i(X^*) = \sum_{j=1}^{q} z_{ij}(X^*) \) in the criteria space is called the ideal point (reference point). As the measure of “closeness”, \( d_p \)-metric is used. The \( d_p \)-metric
defines the distance between two points, \( z_i(X) = \sum_{j=1}^{q} z_{ij}(X) \) and \( z_i(X^\ast) = \sum_{j=1}^{q} z_{ij}(X^\ast) \) (the reference point) in k-dimensional space [3] as:

\[
d_p = \left( \sum_{i=1}^{k} (z_i^\ast - z_i)^p \right)^{1/p} = \left( \sum_{i=1}^{k} \left( \sum_{j=1}^{q} z_{ij}^\ast - \sum_{j=1}^{q} z_{ij} \right)^p \right)^{1/p}
\]

where \( p \geq 1 \).

Unfortunately, because of the incommensurability among objectives, it is impossible to directly use the above distance family. To remove the effects of the incommensurability, we need to normalize the distance family of equation (4) by using the reference point [3,29,30] as:

\[
d_p = \left( \sum_{i=1}^{k} \left( \frac{\sum_{j=1}^{q} z_{ij}^\ast - \sum_{j=1}^{q} z_{ij}}{\sum_{j=1}^{q} z_{ij}^\ast} \right)^p \right)^{1/p}
\]

where \( p \geq 1 \).

To obtain a compromise solution for problem (1), the global criteria method[3,15,21] for large scale problems uses the distance family of equation (3) by the ideal solution being the reference point. The problem becomes how to solve the following auxiliary problem :

\[
\text{Minimize}_{x \in M} \quad d_p = \left( \sum_{i=1}^{k} \left( \frac{\sum_{j=1}^{q} z_{ij}(X^\ast) - \sum_{j=1}^{q} z_{ij}(X)}{\sum_{j=1}^{q} z_{ij}(X^\ast)} \right)^p \right)^{1/p}
\]

where \( X^\ast \) is the PIS and \( p = 1, 2, \ldots, \infty \).

Usually, the solutions based on PIS are different from the solutions based on NIS. Thus, both \( PIS(z^\ast) \) and \( NIS(z^\ast) \) can be used to normalize the distance family and obtain [3,16,19] :

\[
d_p = \left( \sum_{i=1}^{k} \left( \frac{\sum_{j=1}^{q} z_{ij}^\ast - \sum_{j=1}^{q} z_{ij}}{\sum_{j=1}^{q} z_{ij}^\ast} \right)^p \right)^{1/p}
\]

where \( p \geq 1 \).

In this study, the concept of TOPSIS is extended to obtain a Pareto Optimal (compromise) integersolution for CHLSILVOP.

(4) **TOPSIS for CHLSILVOP**

Consider the following CHLSILVOP:

Maximize/Minimize \( (z_1(X), z_2(X), \ldots, z_k(X)) \)

subject to

\( X \in M \)

where

\( \sum_{j=1}^{q} z_{ij}(X) : \) Objective Function for Maximization, \( t \in K_1 \subset K \),
\[ \sum_{j=1}^{q} z_{vj}(X) : \text{Objective Function for Minimization}, v \in K_2 \subseteq K. \]

In order to use the distance family of equation (7) to resolve problem (8), we must first find \( PIS(z^*) \) and \( NIS(z^-) \) which are[3]:

\[
z^* = \operatorname{Maximize} (or \ \operatorname{Minimize}) \sum_{j=1}^{q} z_{tj}(X) \left( \frac{\sum_{j=1}^{q} z_{vj}(X)}{\sum_{j=1}^{q} z_{vj}} \right), \forall t (and \ \nu) \quad (9 - a)
\]

\[
z^- = \operatorname{Minimize} (or \ \operatorname{Maximize}) \sum_{j=1}^{q} z_{tj}(X) \left( \frac{\sum_{j=1}^{q} z_{vj}(X)}{\sum_{j=1}^{q} z_{vj}} \right), \forall t (and \ \nu) \quad (9 - b)
\]

where \( K = K_1 \cup K_2. \quad z^* = (z^*_1, z^*_2, \ldots, z^*_k) \) and \( z^- = (z^-_1, z^-_2, \ldots, z^-_k) \) are the individual positive(negative) ideal solutions.

Using the PIS and the NIS, we obtain the following distance functions from them, respectively:

\[
d_p^{PIS} = \left( \sum_{t \in K_1} w_t^p \left( \frac{\sum_{j=1}^{q} z_{tj} - \sum_{j=1}^{q} z^*_t}{\sum_{j=1}^{q} z^*_t} \right)^p \right) + \sum_{v \in K_2} w_v^p \left( \frac{\sum_{j=1}^{q} z_{vj} - \sum_{j=1}^{q} z^*_v}{\sum_{j=1}^{q} z^*_v} \right)^p \quad (10 - a)
\]

and

\[
d_p^{NIS} = \left( \sum_{t \in K_1} w_t^p \left( \frac{\sum_{j=1}^{q} z_{tj} - \sum_{j=1}^{q} z^-_t}{\sum_{j=1}^{q} z^-_t} \right)^p \right) + \sum_{v \in K_2} w_v^p \left( \frac{\sum_{j=1}^{q} z_{vj} - \sum_{j=1}^{q} z^-_v}{\sum_{j=1}^{q} z^-_v} \right)^p \quad (10 - b)
\]

where \( w_t = 1, 2, \ldots, k, \) are the relative importance (weights) of objectives, and \( p = 1, 2, \ldots, \infty. \)

In order to obtain a compromise solution, transfer problem (8) into the following bi-objective problem with two commensurable (but often conflicting) objectives[3,19]:

Minimize \( d_p^{PIS}(X) \)

Maximize \( d_p^{NIS}(X) \)

subject to (11)

\[ X \in M'/ \]

where \( p = 1, 2, \ldots, \infty. \)

Since these two objectives are usually conflicting to each other, we can simultaneously obtain their individual optima. Thus, we can use membership
functions to represent these individual optima. Assume that the membership functions ($\mu_1(X)$ and $\mu_2(X)$) of two objective functions are linear. Then, based on the preference concept, we assign a larger degree to the one with shorter distance from the PIS for $\mu_1(X)$ and assign a larger degree to the one with farther distance from NIS for $\mu_2(X)$. Therefore, as shown in figure 1, $\mu_1(X)$ and $\mu_2(X)$ can be obtained as the following [3, 22, 31]:

$$
\begin{align*}
\mu_1(X) &= \begin{cases} 
1, & \text{if } d_p^{PIS}(X) < (d_p^{PIS})^*, \\
1 - \frac{d_p^{PIS}(X) - (d_p^{PIS})^*}{(d_p^{PIS})^* - (d_p^{PIS})^-}, & \text{if } (d_p^{PIS})^- \geq d_p^{PIS} \geq (d_p^{PIS})^*, \\
0, & \text{if } d_p^{PIS}(X) > (d_p^{PIS})^-,
\end{cases} \\
\mu_2(X) &= \begin{cases} 
1, & \text{if } d_p^{NIS}(X) > (d_p^{NIS})^*, \\
1 - \frac{d_p^{NIS}(X) - (d_p^{NIS})^-}{(d_p^{NIS})^- - (d_p^{NIS})^*}, & \text{if } (d_p^{NIS})^- \leq d_p^{NIS} \leq (d_p^{NIS})^*, \\
0, & \text{if } d_p^{NIS}(X) < (d_p^{NIS})^-,
\end{cases}
\end{align*}
$$

(12 - a)

and

$$
\begin{align*}
\mu_1(X) &= \begin{cases} 
1, & \text{if } d_p^{PIS}(X) < (d_p^{PIS})^*, \\
1 - \frac{d_p^{PIS}(X) - (d_p^{PIS})^*}{(d_p^{PIS})^* - (d_p^{PIS})^-}, & \text{if } (d_p^{PIS})^- \geq d_p^{PIS} \geq (d_p^{PIS})^*, \\
0, & \text{if } d_p^{PIS}(X) > (d_p^{PIS})^-,
\end{cases} \\
\mu_2(X) &= \begin{cases} 
1, & \text{if } d_p^{NIS}(X) > (d_p^{NIS})^*, \\
1 - \frac{d_p^{NIS}(X) - (d_p^{NIS})^-}{(d_p^{NIS})^- - (d_p^{NIS})^*}, & \text{if } (d_p^{NIS})^- \leq d_p^{NIS} \leq (d_p^{NIS})^*, \\
0, & \text{if } d_p^{NIS}(X) < (d_p^{NIS})^-,
\end{cases}
\end{align*}
$$

(12 - b)

where

$$
(d_p^{PIS})^* = \min_{x \in M} d_p^{PIS}(X) \text{ and the solution is } X^{PIS},
$$

$$
(d_p^{NIS})^* = \max_{x \in M} d_p^{NIS}(X) \text{ and the solution is } X^{NIS},
$$

$$
(d_p^{PIS})^- = d_p^{PIS}(X^{NIS}) \text{ and } (d_p^{NIS})^- = d_p^{NIS}(X^{PIS})
$$

Figure 1. The membership functions of $\mu_1(X)$ and $\mu_2(X)$
Now, by applying the max-min decision model which is proposed by Bellman and Zadeh [6] and extended by Zimmermann [31], we can resolve problem (11). The satisfying decision, $X^*$, may be obtained by solving the following model:

$$
\mu_D(X^*) = \max_{X \in \mathcal{X}} \{ \min_{1 \leq i \leq n} (\mu_1(X) - \mu_2(X)) \}
$$

(13)

Finally, if $\delta = \min(\mu_1(X), \mu_2(X))$, the model (13) is equivalent to the form of Tchebycheff model [11], which is equivalent to the following model:

Maximize $\delta$,

subject to

$$
\begin{align*}
\mu_1(X) &\geq \delta, \\
\mu_2(X) &\geq \delta, \\
X &\in \mathcal{M}', \quad \delta \in [0,1]
\end{align*}
$$

(14 - a)

where $\delta$ is the satisfactory level for both criteria of the shortest distance from the PIS and the farthest distance from the NIS.

### (5) The algorithm of TOPSIS for solving CHLSILVOP

Thus, I can introduce the following algorithm to generate a set of Pareto Optimal (compromise) integer solutions for the CHLSILVOP:

**The algorithm (Alg-I):**

**Step 1.** Formulate CHLSILVOP (1) which have linear objective functions as in Eq. (2).

**Step 2.** Use the standard normal distribution table to find $k_{a_{hj}}$, $j = 0, 1, 2, \ldots, q$.

**Step 3.** Transform problem (1) to the form of problem (3).

**Step 4.** Transform problem (3) to the form of problem (8).

**Step 5.** Ignore the integer requirement.

**Step 6.** Construct the PIS payoff table of problem (8) by using the decomposition algorithm [10, 20, 24] and problem(9-a). Thus, $z^* = (z_1^*, z_2^*, \ldots, z_k^*)$, the individual positive ideal solutions are obtained.

**Step 7.** Construct the NIS payoff table of problem (8) by using the decomposition algorithm [10, 20, 24] and problem(9-b). Thus, $z^- = (z_1^-, z_2^-, \ldots, z_k^-)$, the individual negative ideal solutions are obtained.

**Step 8.** Use equation (10) and the above step to construct $d^P_{PIS}$ and $d^N_{NIS}$.

**Step 9.** Transform problem (8) to problem (11).

**Step 10.** Ask the DM to select $p = p^* \in \{1, 2, \ldots, \infty\}$.

**Step 11.** Ask the DM to select $w_i = w_i^*$, $i = 1, 2, \ldots, k$, where $\sum_{i=1}^{k} w_i = 1$.

**Step 13.** Construct the payoff table of problem (11):

- At $p=1$, use the decomposition algorithm [10, 20, 24].
- At $p \geq 2$, use the generalized reduced gradient method [20, 21] and obtain $d^*_p = ((d^P_{PIS})^*, (d^N_{NIS})^*)$, $d^*_p = ((d^P_{PIS})^*, (d^N_{NIS})^*)$. 

Step 14. Construct problem (14) by the use of the membership functions (12).
Step 15. Solve problem (14).
Step 16. If the solution of problem (14) yields optimal integer solution, then go to step 18. Otherwise go to step 17.
Step 18. If the DM is satisfied with the current solution, go to step 19. Otherwise, go to step 10.
Step 19. Stop.

(6) An illustrative numerical example

Consider the following CHLSILVOP which has the angular structure:

\[
\begin{align*}
\text{Maximize } z_1(X) &= 2x_1 + 4x_2 \\
\text{Minimize } z_2(X) &= x_1 - 2x_2
\end{align*}
\]

subject to

\[
\begin{align*}
P\{x_1 + x_2 \leq b_0\} &\geq 0.7257, \\
P\{x_1 \leq b_1\} &\geq 0.5, \\
P\{2x_2 \leq b_2\} \geq 0.4013, \\
x_1, &\quad x_2 \geq 0 \text{ and integer}
\end{align*}
\]

where

\[
\begin{align*}
E(b_0) &= 8, E(b_1) = 2, E(b_2) = 7, Var(b_0) = 25, Var(b_1) = 4 \text{ and } Var(b_2) = 16.
\end{align*}
\]

Use (Alg-I) to solve the above problem.

By using problem (3), problem (15) can be written as follows:

\[
\begin{align*}
\text{Maximize } z_1(X) &= 2x_1 + 4x_2 \\
\text{Minimize } z_2(X) &= x_1 - 2x_2
\end{align*}
\]

subject to

\[
\begin{align*}
x_1 + x_2 &\leq 5, \\
x_1 &\leq 2, \\
x_2 &\leq 4, \\
x_1, &\quad x_2 \geq 0
\end{align*}
\]

Obtain PIS and NIS for problem (16), (Tables 1&2).

Table 1: PIS payoff table of problem (17)

<table>
<thead>
<tr>
<th></th>
<th>$z_1$</th>
<th>$z_2$</th>
<th>$x_1$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Maximize } z_1(X)$</td>
<td>18*</td>
<td>-7</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>$\text{Minimize } z_2(X)$</td>
<td>16</td>
<td>-8*</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

PIS: $z^* = (18, -8)$
Table 2: NIS payoff table of problem (17)

<table>
<thead>
<tr>
<th></th>
<th>$z_1$</th>
<th>$z_2$</th>
<th>$x_1$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Minimize</strong> $z_1(X)$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Maximize</strong> $z_2(X)$</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

NIS: $z^- = (0, 2)$

Next, compute equation (10) and obtain the following equations:

$$d_p^{PIS} = w_1^p \left( \frac{18 - z_1(X)}{18 - 0} \right)^p + w_2^p \left( \frac{z_2(x) - (-8)}{2 - (-8)} \right)^p$$

$$d_p^{NIS} = w_1^p \left( \frac{z_1(X) - 0}{18 - 0} \right)^p + w_2^p \left( \frac{2 - z_2(x)}{2 - (-8)} \right)^p$$

Thus, problem (11) is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p = 0.5$ and $p = 2$ (Table 3),

Table 3: PIS payoff table of equation (11), when $p = 2$

<table>
<thead>
<tr>
<th></th>
<th>$d_2^{PIS}$</th>
<th>$d_2^{NIS}$</th>
<th>$z_1$</th>
<th>$z_2$</th>
<th>$x_1$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Min.</strong></td>
<td>0.0365*</td>
<td>0.5677*</td>
<td>11.3888</td>
<td>-2.3056</td>
<td>0.5339</td>
<td>4</td>
</tr>
<tr>
<td><strong>Max.</strong></td>
<td>0.05-</td>
<td>0.6592*</td>
<td>10</td>
<td>-7</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

$d_2^* = (0.0365, 0.6592), d_2 = (0.05, 0.6577)$.

Now, it is easy to compute (14):

Maximize $\delta$

subject to

$x_1 + x_2 \leq 5, x_1 \leq 2, x_2 \leq 4,$

$x_1, x_2 \geq 0$ and integer,

$$\left( \frac{d_2^{PIS}(X) - 0.0365}{0.0135} \right) \geq \delta, \left( \frac{0.6592 - d_2^{NIS}(X)}{0.0015} \right) \geq \delta, \delta \in [0, 1].$$

The maximum “satisfactory level” ($\delta = 1$) is achieved for the solution $x_1 = 1, x_2 = 1$.

(7) Conclusions

In this paper, a TOPSIS approach has been extended to solve CHLSILVOP. The CHLSILVOP using TOPSIS approach provides an effective way to find the compromise (satisfactory) solution of such problems. Generally TOPSIS,
Vector optimization problems

provides a broader principle of compromise for solving Vector Optimization Problems. It transfers k-objectives (criteria), which are conflicting and non-commensurable, into two objectives (the shortest distance from the PIS and the longest distance from the NIS), which are commensurable and most of time conflicting. Then, the bi-objective problem can be solved by using membership functions of fuzzy set theory to represent the satisfaction level for both criteria and obtain TOPSIS, compromise solution by a second-order compromise. The max-min operator is then considered as a suitable one to resolve the conflict between the new criteria (the shortest distance from the PIS and the longest distance from the NIS).

Also, in this paper, an algorithm of generating Pareto Optimal (compromise) integer solutions of CHLSILVOP problem has been presented. It is based on the decomposition algorithm of CHLSILVOP with block angular structure via TOPSIS approach for \( p=1 \) and Generalized reduced gradient method for \( p \geq 2 \). This algorithm combines CHLSILVOP and TOPSIS approach to obtain TOPSIS's compromise solution of the problem. Finally, a numerical illustrative example clarified the various aspects of both the solution concept and the proposed algorithm.

References


Received: September, 2010