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On the Similarity Solution of Micropolar Power Law Fluid over a Vertical Plate

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Abstract

The steady 2 – D convective micropolar boundary layer driven by a pseudoplastic power law fluid on a vertical plate was studied. By using suitable similarity transformation, the governing non-dimensional boundary layer equations are reduced into ordinary differential equations containing three parameters: coupling constant, microrotation parameter and power law exponent. The numerical solution of the resulting problem is obtained using Mathcad. The effects of these parameters are presented graphically as velocity and angular velocity profiles. In addition, the result shows that the parameters have appreciable influence of the flow.

Keywords: Similarity solution, Simulation, Micropolar, Power law fluid, Boundary value

1 Introduction

In recent years, scientists have shown keen interest in the theoretical studies of the concept of micropolar fluids which deal with a class of fluids that exhibit certain microscopic effects arising from the local structure and micromotions of the fluid elements. These fluids contain dilute suspensions of rigid macromolecules with individual motions that support stress and body moments and are influence by spin inerta.

Ahmadi [1] examined the fluid flow characteristics of the boundary-layer flow of a micropolar fluid over a semi-infinite plate, using a Runge-Kutta shooting method with Newtonian iteration. Aissa and Mohammedein [2] studied the effects of the magnetic parameter; suction parameter, Eckert number and microrotation parameter on the wall jet flow of a laminar micropolar fluid past a linearly stretching, continuous sheet. Their result showed that the velocity decreases with increasing magnetic parameter, and increases with increasing microrotation parameter. Attia [3] developed a new mathematical model for steady laminar flow with heat generation of an incompressible non-Newtonian micropolar fluid impinging on a porous flat plate He examined effect of the uniform suction or blowing and the characteristics of the non-Newtonian fluid on both the flow and heat transfer. Bhargava and Rani [4] discussed the heat transfer in a micropolar fluid near a stagnation point. Gorla et al. [5] analyzed the heat transfer characteristics of a micropolar fluid over a flat plate. Hassanien and Gorla [6] studied the mixed convection in stagnation flow of micropolar fluid over a vertical surface with variable surface temperature and uniform surface heat flux. Ibrahim et. al. [7] discussed the effects of a temperature-dependent heat source on the hydromagnetic free-convective flow (set up due to temperature as well as species concentration) of an electrically conducting micropolar fluid past a steady vertical porous plate through a highly porous medium, when the free stream oscillates in magnitude. Ishak et. al. [8] presented a theoretical study of a steady boundary layer flow and heat transfer of a micropolar fluid on an isothermal continuously moving plane surface, where It is assumed that the microinertia density is variable and not constant, in the presence viscous dissipation effect. Ishak et. al [9] examind the steady stagnation flow towards a permeable vertical surface immersed in a micropolar fluid and showed that dual solutions exist in the opposing flow regime and these also continued into that of the assisting flow regime, where the buoyancy force acts in the same direction as the inertia force. Khedr et. al. [10] presented the numerical solution of a steady, laminar, MHD flow of a micropolar fluid past a stretched semi-infinite, vertical and permeable surface in the presence of temperature dependent heat generation or absorption, magnetic field and thermal radiation effects. Rahman and Sultana [11] investigated a two-dimensional steady convective flow of a micropolar fluid past a vertical porous flat plate in the presence of radiation with variable heat flux . The effects of the pertinent parameters on the local skin friction coefficient, plate couple stress and the heat transfer are calculated. And the results showed that large Darcy parameter leads to decrease the velocity while it increases the angular velocity as well as temperature of the micropolar fluids.

The rate of heat transfer in weakly concentrated micropolar fluids is higher than strongly concentrated micropolar fluids.

In practice, the theory of micropolar fluids requires the addition of an equation representing the principle of conservation of local angular momentum to the usual transport equation for the conservation of mass and momentum. Probably, due to the nonlinear nature of the stress formulation describing a non – Newtonian power law fluid, much have not been done on micropolar power law fluids.

A non – Newtonian power law fluid obeys the Ostwald – de Waele rheological model,

$$\tau_{yx} = -m\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}$$

This two parameter rheological equation is also known as the power law model. When $n = 1$, the equation represents a Newtonian fluid with a dynamic coefficient of viscosity m. Therefore, deviation of n from a unity indicates the degree of deviation from Newtonian behaviour. For $n < 1$, the fluid is pseudoplastic and for $n > 1$, the fluid is dilatant. n, is power law exponent and m is the consistency coefficient. Using micropolar fluids one is able to describe the behaviour of colloidal solutions, suspension solutions, liquid crystals, blood e.t.c, that exhibit non – Newtonian characteristics. Hence, the objective of this work is to present the numerical solution of the steady 2 – D convective micropolar boundary layer driven by a pseudoplastic power law fluid on a vertical plate.

2 Problem Formulations

We consider the steady two-dimensional micropolar flow in a pseudoplastic power law fluid past a vertical plate. The x-axis is chosen along the plate to the direction of the flow with the origin at the leading edge of the plate and the y-axis is taken normal to it.

The governing equations of continuity, momentum and angular momentum are given by

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

Momentum Equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\upsilon\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^{n}+k_1\frac{\partial N}{\partial y} \qquad (2)$$

Angular momentum equation

$$G_1\frac{\partial^2 N}{\partial y^2}-(2N+q(x)\frac{\partial u}{\partial y})=0 \qquad (3)$$

Boundary conditions

$$\left.\begin{array}{ll} u=U,\ v=0,\ \ N=0,\text{at}\ \ y=0 & \\ u=0, N=0 & \text{as}\ \ y\to\infty \end{array}\right\} \qquad (4)$$

Here *u*, *v* are the velocity components along *x*, *y* coordinates, respectively, *v* is the kinematic viscosity, *ρ*, *N* the microrotation component, *K*1, *(K*1 > 0*)* the coupling constant, G_1 the microrotation constant.

2.1 Similarity Solutions

We define the following similarity Transformations:

$$u=\frac{\partial\psi}{\partial y}\qquad v=-\frac{\partial\psi}{\partial x}$$

$$\eta=\frac{Ay}{x^{\frac{1}{2n-1}}}$$

$$\psi=Uf(\eta)$$

$$N=\frac{A}{x^{\frac{2n-2}{2n-1}}}g(\eta)$$

$$q(x)=\frac{A}{x^{\frac{2n-2}{2n-1}}} \qquad (5)$$

The momentum and angular momentum equations (2)-(3), after some simplifications, reduce to the following forms (Using (5)):

$$\upsilon n U^{n-2} A^{2n-1} \left(-f''\right)^{n-1} f''' - \frac{1}{2n-1} f'^{2} + k\ g' = 0 \qquad (6)$$

$$G\left(g'' - f''\right) - 2g = 0 \qquad (7)$$

The transformed boundary conditions are given by

$$\begin{aligned} &f'(0) = 1, f(0) = 0, g(0) = 0 \\ &f'(\infty) = 0, g(\infty) = 0 \end{aligned} \qquad (8)$$

Where $k = \dfrac{k_1 x^{\frac{2}{2n-1}}}{U^2}$ is the coupling constant parameter

$G = \dfrac{(G_1 - U)A^2}{x^{\frac{2}{2n-1}}}$ is the microrotation parameter.

3 Results and Discussion

In this paper a version of the Mathcad-14 solver have been used to solve ordinary differential equations (6)–(7) along with the boundary conditions (8).

The The parameters involved in the present problem are G, k and n. here we have considered power law exponent as $n < 1, n = 1, n > 1$ indicating pseudoplastic fluids, Newtonian fluid and dilatant fluids respectively. The values of G and k are chosen arbitrary.

Figs. 1–2 show the dimensionless velocity and angular velocity for various fluids like as pseudoplastic, Newtonian and dilatant. We can see that both the flow profiles decreases with power exponent indicating that flow at the same edge velocity a power law fluid generally smaller boundary layer thickness than a Newtonian even dilatant fluids with the same density. It is also observe that dilatant fluid decreases very quickly at the far from boundary.

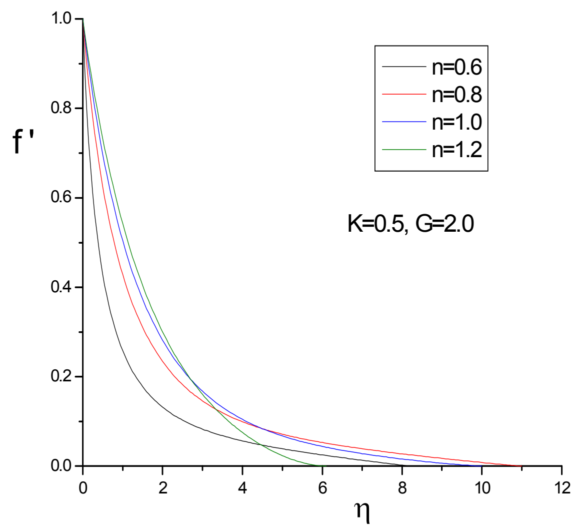

Fig.1: Velocity profile as a function of similarity variable for different values of power law exponent

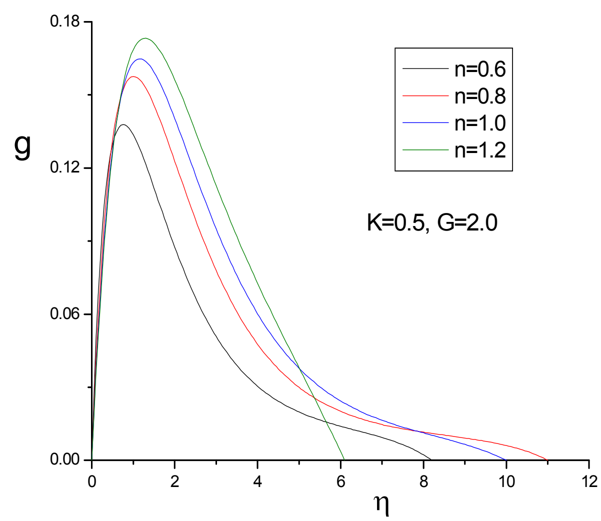

Fig.2: Angular velocity profile as a function of similarity variable for different values of power law exponent

Fig.3 shows the variations of the dimensionless velocity,against the similarity variable η , for pseudoplastic fluid and for different values of the coupling constant , whereas the corresponding variations for the dimensionless angular velocity are shown in Fig.4. From fig.3, it is seen that with increasing k, the flow is almost no influence around $\eta = 1$, however, the effect of k decreases the velocity profile which occurs at $\eta > 1$.

It is remarkable from fig.4, which angular velocity increases rapidly with the similarity variable from the leading edge to the vertical plate taking their maximum value in the region $\eta \approx 1.3$. A little far downstream, a corresponding decrement take place up to the point where they take their minimum value $\eta \approx 3$.

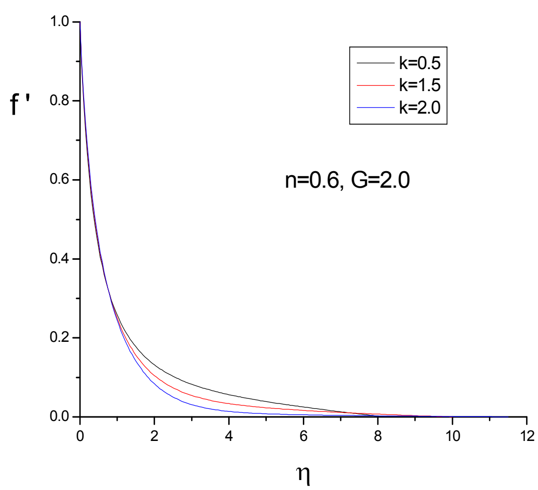

Fig.3: Velocity profile as a function of similarity variable for different values of coupling constant

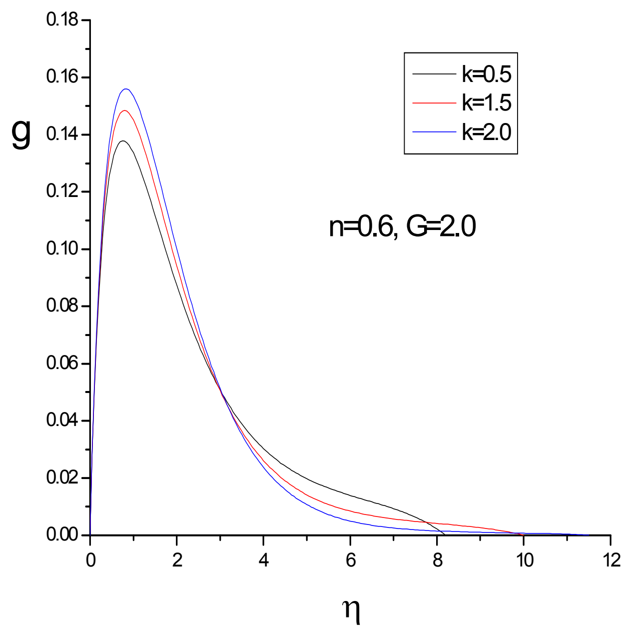

Fig.4: Angular velocity profile as a function of similarity variable for different values of coupling constant

Figs. 5- 6 show the variations of the dimensionless velocity and angular velocity profiles, respectively, against the similarity variable η , for pseudoplastic fluid and for different values of the microrotation parameter.

It is seen from fig.5 that velocity profiles han an impact far from the boundary around $\eta > 4$. On the other hand, as G increases, the thickness of the hydrodynamic boundary layer increases.

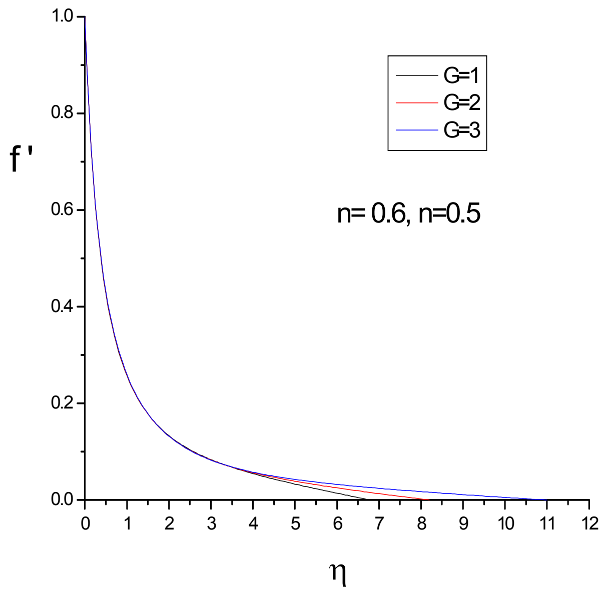

Fig.5: Velocity profile as a function of similarity variable for different values of microrotation

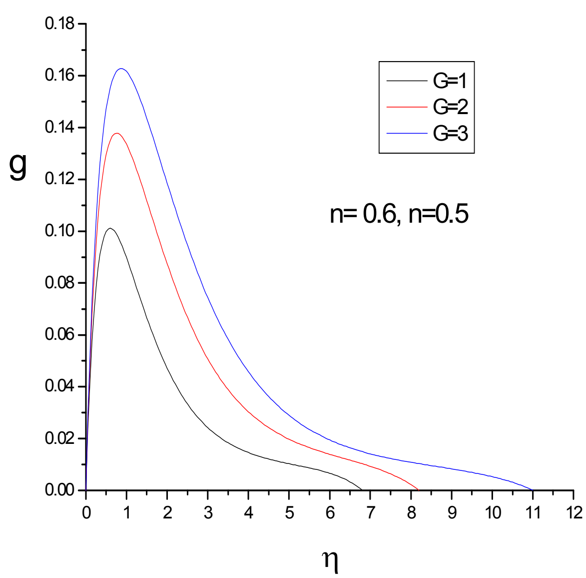

Fig.6: Angular velocity profile as a function of similarity variable for different values of microrotation

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