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On Generalized Minimal Closed Sets

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Abstract

The aim of this paper is to introduce a new concept of generalized minimal closed set. It can be shown that, a subset A of X is a generalized minimal closed set iff the minimal open sets containing A in that topological space are also closed sets. The generalized maximal continuous functions on this type of topological space are also studied. It is to be noted that this set is an independent concept of open sets and closed sets but is a stronger form of generalized closed set.

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1. Introduction

The concept of minimal open set was introduced first by Nakaoka F. and Oda Nobuyuki [4] .The concept of generalized closed set was first introduced by N.Levine [5]. In this paper the concept of generalized minimal closed set is introduced. It can be shown that generalized minimal closed set is a generalized closed set but not conversely. Also it can be shown that, a subset A of X is a generalized minimal closed set iff the minimal open sets containing A in that topological space are also closed sets. The generalized maximal continuous functions on this type of topological space are also studied.

2. Preliminaries

Some important preliminaries required to go further through this paper are cited below.

2.1 [4] Let (X, T) be a topological space. A nonempty set U is said to be a minimal open set iff any open set, which is contained in U, is U or ϕ.

2.2 [5] A subset A of X is a generalized closed set iff ClA $\subseteq$ U whenever A $\subseteq$ U, where U is any open subset of X.

2.3 A function f : (X, T) $\longrightarrow$ (Y, σ) is said to be a continuous function iff the inverse image of a closed subset V of Y is a closed set in X

2.4 [2]A function f : (X, T)$\longrightarrow$ (Y, σ) is said to be a generalized continuous function iff the inverse image of a closed subset V of Y is a generalized closed set in X

2.5 [6] A partition space is a topological space where every open set is also a closed set.

3. On generalized minimal closed set

In this section the concept of generalized minimal closed set is introduced and the corresponding topological structure obtained by the collection of these sets is studied

Definition 3.1:A subset A of X is a minimal closed set if it doesn't contain any other non-null closed set except itself.

Remark 3.2: The complement of maximal open set is a minimal closed set and the complement of a minimal open set is a maximal closed set

Definition 3.3: A subset A of X is said to be a generalized minimal closed [resp generalized maximal open] set if A is contained in [resp A contains] at least one minimal open [resp at least one maximal closed] subset U of X such that ClA $\subseteq$ U [resp Int A $\supseteq$ U]

Example 3.4: Let X = {a, b, c} and the corresponding topological space be T = {ϕ, X, {a, b}, {c}}. Let A= {a} be any subset of X. Clearly A $\subseteq$ {a, b} be the minimal open set. Here ClA = {a, b} $\subseteq$ {a, b}. So, A is a generalized minimal closed set.

Theorem 3.5: A is a generalized minimal closed set iff A^C is a generalized maximal open set.

Proof is obvious

Theorem 3.6: A subset A of X is a generalized minimal closed set iff the minimal open sets containing A in that topological space are also closed sets.

Proof: First let us consider that the minimal open sets U containing A are also closed sets, then ClA $\subseteq$ ClU = U [Since U is a closed set]. Hence from the definition A is a generalized minimal closed set.

Conversely, let if possible A be a generalized minimal closed set. Then A $\subseteq$ U where U is a minimal open set of X, ClA $\subseteq$ U. Since complement of every open subset is a closed set. The set ClA is a closed set and it contains A, so its complement open set should not contain A i.e. IntA^C, must contain elements without A. Now, if IntA^C is not disjoint from U then their intersection will be the minimal element which is not possible since U is the minimal element. Again if IntA^C and U are disjoint then their union will be either X or some open subset of X. If it is X then the minimal open set is also closed which is the theorem. If not then the complement of the union open set may contain A and which is also not possible. So, U is a minimal clopen set. Hence the theorem.

Theorem 3.7: If a subset A of X is a generalized minimal closed set then it is a generalized closed set.

Proof: Let A $\subseteq$ U, U being a minimal open subset of X. Now A being a generalized minimal closed set ClA $\subseteq$U. Since U is a minimal open subset of X, U is a subset of any other open set. So, ClA $\subseteq$ U and any other open set containing A. Hence from definition A is a generalized closed set.

Remark 3.8: Converse of the above theorem need not be true which follows from the following example: let X = $\{$a, b, c, d$\}$ and the corresponding topological space be T = $\{\phi$, X, $\{$a, b$\}, \{$a, b, c$\}, \{$c, d$\}, \{$c$\}\}$. Let A = $\{$a$\}$. Obviously A is a generalized minimal closed set. Let B = $\{$c, d$\}$ $\subseteq$ $\{$c, d$\}$, X .ClB = B $\subseteq$ $\{$c, d$\}$, X . So, B is a generalized closed set. But B is not a generalized minimal closed set. Since B is not contained in any minimal open set .

So, we get the relational structure as

Generalized closed set $\Leftarrow$ Generalized minimal closed set

Generalized open set $\Leftarrow$ Generalized maximal open set

Theorem 3.9: Let if possible A be closed subset of X, then A is a generalized minimal closed subset of X iff A is a minimal clopen subset of X.

Proof: Let if possible A be a closed set i.e. ClA = A. Let A $\subseteq$ U, a minimal open set. Then ClA = A $\subseteq$ U. But U is a minimal clopen set from theorem 3.6. Now if there exist any other closed set contained in U then its complement, which is an open set, has an element common with U and their intersection element is also an open set less than the minimal open set, which is not possible. So, A is a minimal clopen set. Converse of the above theorem is obvious.

Remark 3.10: A generalized closed set is a generalized minimal closed set that is the converse of the theorem 3.7 is true if every open set containing A is also a minimal open set in that topological space. i.e. the topological space contains only X and the null set and no other element.

Theorem 3.11: Any non - null open set is a generalized minimal closed set iff it is a minimal clopen set.

Proof: Let if possible, A is a non-null open set. A is not contained in any minimal open set unless it is itself a minimal open set which follows from the definition of the minimal open set. Let A be a minimal open set. Now if it is a generalized minimal open set then ClA $\subseteq$ A. But we know that A $\subseteq$ ClA. I.e. A = ClA i.e. A is a clopen set. Hence the set is a minimal clopen set. Converse part is obviously true.

Remark 3.12: From theorem 3.9 and theorem 3.11 it can be written that non-null open set and a non-null closed set is a non-null generalized minimal closed set iff the set is a minimal open set and a closed set.

Theorem 3.13:

(i) ϕ is a generalized minimal closed set but X is not

(ii) Finite union of generalized minimal closed set is a generalized minimal closed set if it is contained in any minimal open set

(iii) Arbitrary intersection of generalized minimal closed set is a generalized minimal closed set

Proof: (i) Since Cl(ϕ) = ϕ subset of any minimal open set containing ϕ. So, ϕ is a generalized minimal closed set. But X is not contained in any minimal open set, so X is not a generalized minimal closed set.

(ii) Let if possible {A_i : i = 1, 2, ., n } be a finite collection of generalized minimal closed sets. Let $A_i \subseteq \cup\{A_i$: i = 1, 2, ., n } $\subseteq$ U, a minimal open subset of X. Since {A_i : i = 1, 2, ., n }is a finite collection of generalized minimal closed set , so, {Cl(A_i): i = 1, 2, ., n } $\subseteq$ U . i.e. $\cup${Cl(A_i): i =

1, 2, ., n} = {Cl(∪A_i : i = 1, 2, ., n)} ⊆ U [1] i.e. finite union of generalized minimal closed set is a generalized minimal closed set if it is contained in any minimal open set

(iii) Let if possible {A_i : i∈I} be an arbitrary collection of generalized minimal closed sets. Let A = ∩{A_i : i∈I} ⊆ U a minimal open set. If all {A_i : i∈I} ⊆ U_i , some independent minimal open set then ∩{A_i : i∈I}= ϕ , a generalized minimal closed set. Now let if possible ∩{A_i : i∈I} ≠ ϕ, then ∩{A_i : i∈I} ⊆ U implies {A_i : i∈I} ⊆ U. Since {A_i : i∈I} is an arbitrary collection of generalized minimal closed sets, Cl{A_i : i∈I} ⊆ U. Now ∩{A_i : i∈I} ⊆ {A_i : i∈I}. So, Cl{∩{A_i : i∈I}} ⊆ ∩ Cl{A_i : i∈I} ⊆ U. So, arbitrary intersection of generalized minimal closed set is a generalized minimal closed set.

Remark 3.14: Since arbitrary union of closed set need not be a closed set, so arbitrary union of generalized minimal closed set is not a generalized minimal closed set.

Remark 3.15: The collection of all generalized minimal closed set forms a pseudo special dual topological space. If X is included then it forms a special dual topological space and is denoted as (X, gM). Also the collection of all generalized maximal open set forms a pseudo special topological space and if f is included then it forms a special topological space denoted as (X, gMT) and is named as generalized maximal topological space

Theorem 3.16: For a topological space (X, T) the following conditions are equivalent

1. X is a partition space with only one element
2. Every subset of X is a generalized minimal open set.

Proof is obvious

Definition 3.17: A space is said to be a minimal $T_{1/2}$ space if every generalized minimal closed set is also a closed set.

Theorem 3.18: If a space is a T $_{1/2}$ space then it is a minimal $T_{1/2}$ space.

Proof: Let if possible a space X be a T $_{1/2}$ space i.e. every generalized closed set is also a closed set. Since from theorem 3.7, every generalized minimal closed set is a generalized closed set so, the space is a minimal $T_{1/2}$ space

4. On generalized maximal continuous function

In section 3, we have shown that the collection of all maximal open set with the null set forms a topological space. In this section the concept of generalized maximal continuous function is introduced and some related theorems are studied.

Definition 4.1: A function f : (X, T) $\longrightarrow$ (Y, σ) is said to be generalized maximal continuous function if the inverse image of every open set in Y is a generalized maximal open set in X

Theorem 4.2: A function f : (X, T) $\longrightarrow$ (Y, σ) is said to be generalized maximal continuous function iff the inverse image of every closed set in Y is a generalized minimal closed set in X

Proof is obvious

Example 4.3: Let f : (X, T) $\longrightarrow$ (Y, σ) be a mapping such that f(x) = x. Here X= {a, b, c}, Y = {a, b} and the corresponding topological space be T= {X, ϕ, {a, b}, {c}}, σ = {X, ϕ, {b}}. Here let A = {a} then f^{-1}(A) = A is a generalized minimal closed set in X. A is a closed set in Y. So, f is a generalized maximal continuous function

Theorem 4.4: Let X be a minimal $T_{1/2}$ space. Let f : (X, T) $\longrightarrow$ (Y,σ) is a generalized maximal continuous function then it is a continuous function

Proof: Let if possible f : (X, T) $\longrightarrow$ (Y,σ) is a generalized maximal continuous function i.e. inverse image of a closed set in Y is a generalized minimal closed set in X. Since X is a minimal $T_{1/2}$ space, so every generalized minimal closed set in X is also a closed set in X i.e. f is a continuous function.

Theorem 4.5: A function f : (X, T) $\longrightarrow$ (Y, σ) is a generalized maximal continuous function then it is a generalized continuous function

Proof: Since from the structural relation after remark 3.6, generalized minimal closed set implies generalized closed set.

Remark 4.6: Converse of the above theorem need not be true which follows from the following example: Let f : (X, T) $\longrightarrow$ (X, σ) be a mapping such that f(x) = x. Here X= {a, b, c, d} and the corresponding topological space be T={X, ϕ, {a, b}, {c, d}, {a, b, c}, {c}}, σ = {X, ϕ, {a, b}, {c,d}, {d}, {a, b, c}, {c}}. Here the inverse images of every closed set are a generalized minimal closed set but the inverse image of {d} a generalized minimal closed set in σ is not a generalized minimal closed set in T. So, f is generalized continuous but not generalized maximal continuous.

Remark 4.7: If X is a generalized minimal $T_{1/2}$ space then generalized maximal continuous function, continuous function and generalized continuous function are the similar concepts.

Theorem 4.8: Let f : (X, T) $\longrightarrow$(Y, σ) and g : (Y, σ) $\longrightarrow$ (Z, α) be two functions between two topological spaces. Then

If g is continuous function and f is a generalized maximal continuous function then gof is a generalized maximal continuous function

Proof is obvious

Remark 4.9: A function f : (X, T) $\longrightarrow$ (Y, σ) is a generalized maximal continuous function iff f : (X, gM_X) $\longrightarrow$ (Y, gM_Y) is a continuous function . Since every generalized minimal closed set in (Y, σ) is a closed set in (Y, gM_Y).

Conclusion: In this paper the concept of generalized minimal closed set is introduced which is a weaker form of generalized closed set. Also the concept of generalized maximal continuous function is introduced which implies a continuous function. Also various connections with other sets are studied. This set may be very helpful in applied field and many other new types of sets can be formed from this set which may be very helpful to deal in applied field of research work.

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