Int. J. Contemp. Math. Sciences, Vol. 6, 2011, no. 3, 145 - 152

On Generalized Regular Closed Sets

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Abstract

The aim of this paper is to introduce the concept of generalized regular closed sets and study some of its properties . The corresponding topological space formed by the family of these sets is also studied. It may be noted that regular closed set doesn't forms even a supra topological space. Various researchers had studied the concept of generalized closed sets and regular generalized closed sets earlier. The generalized closed set is properly placed between the generalized regular closed sets and regular generalized closed set. The connection of the topological space formed by the newly defined sets with other topological space are also discussed in this paper and some applications of this newly defined set are also shown.

Mathematics Subject Classification: 54A40

Keywords: Generalized Closed set, Regular generalized closed sets, Topological spaces etc

1 Introduction

The concept of generalized closed set and generalized open set was first introduced by N. Levin [2] in ordinary topological space. Later on N. Palaniappan

[4] studied the concept of regular generalized closed set in a topological space. In this paper the concept of generalized regular closed sets is to be introduced, which lies between regular closed set and generalized closed set. Some properties of this set are studied and the corresponding topological space is introduced.

2 Preliminaries:

Some of the required definitions and theorems are cited in this section of the paper:

2.1 A subset A of X is a regular open set if A = IntClA and a regular closed set if A = ClInt A

2.2 [2] A subset A of a space (X, τ) is called a generalized closed set (briefly g - closed) if ClA $\subseteq$ U whenever A $\subseteq$ U and U is an open set.

2.3[4] A subset A of a topological space (X, τ)is called a regular generalized closed (briefly r - g closed)set if ClA $\subseteq$ U whenever A $\subseteq$ U and U is a regular open set

2.4[1] For any subset A of (X, τ) , RCl(A) = $\bigcap\{G :$ G $\supseteq$A, G is a regular closed subset of X$\}$

2.5 Let A be a subset of X then Λ(A) = $\cap\{G :$ G $\supseteq$A. G is a open subset of X$\}$

Through out the paper (X, τ) denotes the topological space, ClA is the closure of A, IntA is the interior of A, A^C is the complement of A

3 On generalized regular closed set:

In this section, the concept of generalized regular closed set and generalized regular open sets are introduced. Some properties of these sets are cited. The corresponding topological space obtained by these sets is also introduced and some related theorems are studied.

Definition 3.1: A subset A of a topological space (X, τ) is called a generalized regular closed[resp. a generalized regular open] set (briefly g - r closed)[resp briefly g - r open] if RCl(A) $\subseteq$U [resp U $\subseteq$ R int(A)] whenever A $\subseteq$ U [resp U $\subseteq$ A] , and U is an open [resp a closed] subset of X

Example 3.2: Let X = {a, b, c} and the corresponding topological space be τ = {ϕ, X, {a}, {b,c}}. Let A = {b}. Here A is a generalized regular closed set of X. Though it is not a regular closed subset of X. Similarly it can be shown that, {b, c} is a generalized regular open subset of X.

Theorem 3.3: A subset A of (X, τ) is called generalized regular closed subset of X iff A^C is a generalized regular open subset of X

Theorem 3.4: A subset A of (X, τ) is a generalized closed set if it is a generalized regular closed set.

Proof:

It is obvious from the definition since ClA $\subseteq$ RClA

Remark 3.5: Converse of the above theorem need not be true. It follows from the following example: Let X = {a, b, c} and the corresponding topological space be τ = {X, ϕ, {a, b}, {b, c}, {b}}. Let A = {c} Obviously A is a generalized closed subset of X but not a generalized regular closed subset of X.

Theorem 3.6 : A subset A of (X, τ) is a generalized regular closed set if it is a regular closed set.

Remark 3.7:The converse of the above theorem need not be true which follows from the following example: let X = {a, b, c}, and the corresponding topological space be denoted as τ = {ϕ, X, {a, b}} Let A = {a, c}. A is a generalized regular closed set but not a regular closed set.

Remark 3.8: RClA is a generalized regular closed set since RclA is a regular closed set and from theorem 3.6 it is a generalized regular closed subset of X for any subset A of X.

Therefore we a have a relational structure between the sets as

Regular closed set $\Longrightarrow\delta$- closed set $\Longrightarrow$ δgeneralized closed set $\Longrightarrow$ Generalized Regular closed set $\Longrightarrow$ Generalized closed set $\Longrightarrow$Regular Generalized closed set

$\Downarrow$

g* closed set

Theorem 3.9: If A be an open subset and a generalized regular closed subset of (X, τ) then it is a regular closed subset of (X, τ).

Proof: Let if possible, A is a generalized regular closed subset and an open subset of X. Therefore A = IntA (an open subset of X) Hence from definition RclA $\subseteq$ IntA = A. But we know that A $\subseteq$RclA. So, A = RclA i.e., A is a regular closed subset of (X, τ).

Theorem 3.10: ϕ and X are generalized regular closed subset of X

Remark 3.11: The finite union or intersection of generalized regular closed set needs not be a generalized regular closed set. It follows from the following two examples Let X = {a, b, c d, e} and the corresponding topological space be τ = {ϕ, X, {a, b}, {c}, {a, b, c}, } . Let A = {a, c, d}. Obviously A is a generalized regular closed subset of X. Again let B = { b, c, e} . B is also a generalized regular closed subset of X. But A $\cap$ B = c is not a generalized regular closed subset of X . Since A $\cap$ B = c$\subseteq$The open sets c, {a, b, c}, X. But Rcl A $\cap$ B $\subsetneq$ c. So finite intersection of generalized regular closed sets need not be a generalized regular closed subset of X. Again let X = {a, b, c, d, e} and the corresponding topological space be τ = { ϕ, X, {a, b}, {c}, {a, b, c}, {d, e}}Let A = {d} and B = { e} . Here A and B are generalized regular closed subset of X . But A $\cup$ B = {d, e} is not a generalized regular closed subset of X since the RCl(A $\cup$ B)= {d, e, f} $\subseteq${d, e}, an open subset of X. Hence finite union of generalized regular open set need not be generalized regular open subset of X

Remark 3.12: The collection of all generalized regular open subset of X forms an (X,m_X) space.

Theorem3.13:Let A and B be two regular closed subset of (X, τ) then A$\bigcup$B is a generalized regular closed subset of (X, τ)

Theorem 3.14: The intersection of a generalized regular closed set and a closed set is a generalized closed set

Proof: Let A be a generalized regular closed subset of X. If U is an open subset of X with A $\bigcap$ F $\subseteq$U then A $\subseteq$ U $\cup$ (X\F) . So, RclA $\subseteq$ U $\cup$ (X\F) . Cl(A $\bigcap$ F) $\subseteq$ ClA $\bigcap$ ClF $\subseteq$ RclA $\bigcap$ ClF = Rcl A $\bigcap$ F $\subseteq$U. So A $\bigcap$ F is a generalized closed set.

Remark 3.15: The intersection of a generalized regular closed set and a regular closed set is a generalized regular closed set i.e. the intersection of two regular closed sets is a generalized regular closed set.

Theorem 3.16: ClA is a generalized regular closed subset of X in a space where every closed subset of X is also a regular closed subset of X.

Proof: From remark 3.8, RclA is a generalized regular closed subset of X and from theorem 3.15; RclA $\bigcap$ Cl A is a generalized regular closed subset of X. Since ClA is a closed subset of X. Since every closed subset of x are also regular closed subset of X , so RClA = ClA i.e. RclA $\bigcap$Cl A= ClA, a generalized regular closed subset of X.

Theorem 3.17: Let A $\subseteq$ B $\subseteq$ RclA and A is a generalized regular closed subset of (X, τ) then B is also a generalized regular closed subset of (X, τ).

Proof: Since A is a generalized regular closed subset of (X, τ). So, RclA $\subseteq$ U, whenever A $\subseteq$ U, U being an open subset of X. Let A $\subseteq$ B $\subseteq$ RclA i.e. Rcl A = Rcl B. Let if possible, there exist an open subset V of X such that B $\subseteq$ V. So, A $\subseteq$ V and A being generalized regular closed subset of X, Rcl A $\subseteq$ V i.e. Rcl B $\subseteq$V. Hence B is also a generalized regular closed subset of X

Remark 3.18: Let A $\subseteq$ B $\subseteq$ ClA and A is a generalized regular closed subset of (X, τ) then B is a generalized closed subset of (X, τ).

Remark 3.19: If A is a generalized regular closed subset of X and since A $\subseteq$ ClA $\subseteq$ RClA .So from theorem 3.17, ClA is a generalized regular closed subset of X. Converse need not be true which follows from the following example: Let X = {a, b, c} and the corresponding topological space be $\tau = \{\phi$, X, {a}, {b}, {a, b}}. Let A = {A, B}, ClA = X obviously a generalized regular closed set but A is not so.

Theorem 3.20: Every subset A of a topological space (X, τ) is a generalized regular closed set iff X is a partition space.

Proof: Let if possible we assume that X is a partition space. Let A $\subseteq$ U, where U is an open subset of X and A is an arbitrary subset of X. Since X is a partition space then is clopen. Thus RClA $\subseteq$ RCl U = U. i.e. every subset of X is a generalized regular closed set. Conversely, let U $\subseteq$ X is open then since every subset of X is a generalized regular closed set, so, RCl U $\subseteq$ U i.e. U is a regular closed set and thus a closed subset of X i.e. X is a partition space.

Theorem 3.21: A subset A of (X, σ), [an Alexandroff space] is a generalized regular closed subset of X iff RclA $\subseteq \Lambda$(A)

Proof: In an Alexandroff space, arbitrary intersection of open sets is open sets. Therefore if A is a generalized regular closed sets in an Alexandroff space then there exist an open set U of X containing A such that RClA $\subseteq$ U, since A $\subseteq \Lambda$(A), an open set in an Alexandroff space, therefore RClA $\subseteq \Lambda$(A) The other part is as in the previous theorem.

Theorem 3.22: A Λ set A of (X, σ), [an Alexandroff space] is a generalized regular closed subset of X iff it is a regular closed set.

Proof: Let if possible A be a generalized regular closed subset of X then, RClA $\subseteq \Lambda$(A). Since A is a Λ set, RClA $\subseteq \Lambda$(A) = A. But we know that A $\subseteq$RclA. Therefore A = RclA i.e. A is a regular closed subset of an Alexandroff space (X, σ) Converse part follows from theorem, 3.6

Remark 3.23: In an Alexandroff space if Λ(A) is a generalized regular closed subset of X then Λ(A) $\supseteq$ RClΛ(A). But we know that Λ(A) $\subseteq$ RClΛ(A). Therefore, Λ(A) = RClΛ(A)

Theorem 3.24: A subset A of X is generalized regular closed sets iff RClA $\cap$ A^C contains no nonzero closed set in X.

Proof: Let if possible A be a generalized regular closed subset of X. Also if possible let, F be a closed subset of X such that F $\subseteq$ RClA $\bigcap$ A^C i.e. F $\subseteq$ RClA and F $\subseteq$ A^C . Since F is a closed subset of X, F^C is an open subset of X containing A. A being generalized regular open subset of X, RClA $\subseteq$ F^C. But F $\subseteq$ RClA. So, we get a contradiction, which leads to the conclusion that F = ϕ. So the condition. Conversely, let A $\subseteq$ G, G being an open subset of X. Then $G^C \subseteq A^C$, G^C is a closed subset of X. Let if possible RClA $\subseteq$ G. Then RClA $\bigcap$ G^C is a nonzero closed subset of RClA $\bigcap$ A^C , which is a contradiction. Hence A is a generalized regular closed subset of X.

Remark 3.25: Let A be an open and a generalized regular closed subset of X. From theorem 3.9, A is a regular closed subset of X. Hence, RClA $\bigcap$ A^C = A $\bigcap$ $A^C = \phi$ i.e. if A be an open and a generalized regular closed subset of X then RClA $\bigcap$ $A^C = \phi$. On the other hand RClA $\bigcap$ $A^C = \phi$ implies that A is a generalized regular closed set but need not be an open subset of X.

Theorem 3.26: If A×B is a generalized regular open subset of (X×Y, $\tau \times \sigma$), iff A is a generalized regular open subset in (X, τ) and B is a generalized regular open subset in (Y, σ).

Proof: Let if possible A×B is a generalized regular open subset of (X×Y, τ×s). Let H be a closed subset of (X, τ) and G be a closed subset of (Y, σ) such that H $\subseteq$ A, G $\subseteq$ B. Then H×G is closed in (X×Y, τ×s) such that H×G $\subseteq$ A × B. By assumption A×B is a generalized regular open subset of (X×Y, $\tau \times \sigma$) and so H×G$\subseteq$ Rint(A × B) $\subseteq$ Rint(A) × Rint(B) i.e. H $\subseteq$ Rint(A), G $\subseteq$ Rint(B) and hence A is a generalized regular open subset in (X, τ) and B is a generalized regular open subset in (Y, σ). Conversely, Let F be a closed subset of (X×Y, $\tau \times \sigma$) such that F $\subseteq$ A × B. For each (x, y) $\subseteq$ F, Cl(x) × Cl(y) $\subseteq$ Cl(F) = F $\subseteq$ A × B . Then the two closed sets Cl(x) and Cl(y) are contained in A and B respectively . By assumption Cl(x) $\subseteq$ RintA and Cl(y) $\subseteq$ RintB hold. This implies that for each (x, y) $\subseteq$ F, (x,y) $\subseteq$ RintA × RintB $\subseteq$ Rint(A×B) and hence F $\subseteq$ Rint(A × B) . A ×B is a generalized regular open subset of (X×Y, $\tau \times \sigma$). Hence the theorem.

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Received: March, 2010