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On Bitopological (1, 2)*-Generalized Homeomorphisms

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Abstract

The notions of (1,2)*-generalized homeomorphism, (1,2)*-gc-homeomorphism, (1,2)*-generalized semi-homeomorphism and (1,2)*-gsc-homeomorphism are introduced in bitopological spaces. We establish certain results relating to them.

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1. INTRODUCTION

Maki et al [19] introduced generalized homeomorphism and gc-homeomorphism which are generalizations of homeomorphism in topological spaces. Devi et al [4] defined and studied generalized semi- homeomorphism and gsc- homeomorphism in topological spaces. The concepts of (1,2)*-semi-closed set, (1,2)*-generalized closed set, (1,2)*-semi-generalized closed set, (1,2)*-generalized semi-closed set, (1,2)*-g-closed map, (1,2)*-sg-closed map and

(1,2)*-gs-closed map in bitopological spaces were initiated by Lellis Thivagar and Ravi [7,8,9,13,15]. The purpose of the present paper is to introduce a new class of mappings called (1,2)*-generalized homeomorphism, (1,2)*-gc-homeomorphism, (1,2)*-generalized semi-homeomorphism and (1,2)*-gsc- homeomorphism in bitopological spaces. We investigate some bitopological properties and the relationships between such bitopological homeomorphisms.

2. PRELIMINARIES

Throughout the present paper, (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2) (or simply X, Y and Z) denote bitopological spaces.

Definition 2.1. A subset S of X is called $\tau_{1,2}$-open [7] if $S = A \cup B$ where $A \in \tau_1$ and $B \in \tau_2$.
The complement of a $\tau_{1,2}$-open set is said to be $\tau_{1,2}$-closed.

Note 2.2 [7]. Notice that $\tau_{1,2}$-open subsets of X need not necessarily form a topology.

Definition 2.3. Let S be a subset of X. Then

(i) The $\tau_{1,2}$-closure of S defined as $\tau_{1,2}$-cl(S)= $\cap$ {F/S $\subseteq$ F and F is $\tau_{1,2}$ -closed};[10]

(ii) The $\tau_{1,2}$ -interior of S defined as $\tau_{1,2}$ -int (S)= $\cup$ {G/G$\subseteq$S and G is $\tau_{1,2}$ -open}[10].

Definition 2.4. A subset S of X is called

(i) (1,2)* -semi-open[10] if S $\subseteq \tau_{1,2}$-cl($\tau_{1,2}$-int (S));

(ii) (1,2)*- semi-closed [8] if S $\supseteq \tau_{1,2}$-int($\tau_{1,2}$-cl(S)).

The complement of a (1,2)*-semi-open set is said to be (1,2)*-semi-closed.

The family of all (1,2)*-semi-open [resp.(1,2)*-semi-closed] subsets of X is denoted by (1,2)*-SO(X) [resp.(1,2)*-SC(X)].

Let S be a subset of X. The (1,2)*-semi-closure [8] of S, denoted by (1,2)*-scl(S), is the intersection of all (1,2)*-semi-closed sets of X containing S.

The (1,2)*-semi-interior [8] of S, denoted by (1,2)*-sint(S), is the union of all (1,2)*-semi-open sets contained in S.

Definition 2.5. A subset S of X is said to be (1,2)*-generalized closed [briefly (1,2)*-g-closed] [9] if $\tau_{1,2}$-cl(S)$\subset$U whenever S$\subset$U and U is $\tau_{1,2}$-open set in X.
The complement of a (1,2)*-g-closed set is said to be (1,2)*-g-open.

Definition 2.6. A map $f : X \to Y$ is said to be (1,2)*-gc-irresolute [8] if the inverse image of each (1,2)*-g-closed set of Y is (1,2)*-g-closed in X.

Definition 2.7. A map $f : X \to Y$ is said to be (1,2)*-g-continuous [9] if the inverse image of each $\sigma_{1,2}$-closed set of Y is (1,2)*-g-closed in X.

Definition 2.8. A map f : X → Y is called
(a) pre-(1,2)*-semi-open [11] if f(U) is (1,2)*-semi-open of Y for every (1,2)*-semi-open set U in X.
(b) pre-(1,2)*-semi-closed [11] if f(U) is (1,2)*-semi-closed of Y for every (1,2)*-semi-closed set U in X.

Definition 2.9. A subset S of X said to be (1,2)*-semi-generalized closed (briefly (1,2)*-sg-closed) [8] if and only if (1,2)*-scl(S)⊂F whenever S⊂F and F is (1,2)*-semi-open set in X.
The complement of a (1,2)*-semi-generalized closed set is said to be (1,2)*-semi-generalized open (briefly (1,2)*-sg-open).

Definition 2.10. A subset S of X is said to be (1,2)*-generalized-semi closed (briefly (1,2)*-gs-closed)[8] if (1,2)*-scl(S)⊂F whenever S⊂F and F is $\tau_{1,2}$-open.
The complement of a (1,2)*-gs-closed set is said to be (1,2)*-gs-open.

Definition 2.11. A map f : X → Y is called
(i) (1,2)*-g-closed [9] if f(U) is (1,2)*-g-closed set in Y for every $\tau_{1,2}$-closed set U in X;
(ii) (1,2)*-g-open [9] if f(U) is (1,2)*-g-open set in Y for every $\tau_{1,2}$-open set U in X;
(iii) (1,2)*-gs-closed [8] if f(F) is (1,2)*-gs-closed set in Y for every $\tau_{1,2}$-closed set F in X;
(iv) (1,2)*-gs-open [8] if f(F) is (1,2)*-gs-open set in Y for every $\tau_{1,2}$-open set F in X;
(v) (1,2)*-gs-continuous [8] if $f^{-1}(V)$ is (1,2)*-gs-closed in X for every $\sigma_{1,2}$-closed set V in Y;
(vi) (1,2)*-gs-irresolute [8] if $f^{-1}(V)$ is (1,2)*-gs-closed in X for every (1,2)*-gs-closed set V in Y;
(vii) (1,2)*-continuous [14] if $f^{-1}(V)$ is $\tau_{1,2}$-closed in X for every $\sigma_{1,2}$-closed set V in Y;
(viii) (1,2)*-semi-continuous [10] if if $f^{-1}(V)$ is (1,2)*-semi-closed in X for every $\sigma_{1,2}$-closed set V in Y.

We introduce a class of maps as follows.

Definition 2.12. A bijection f : X → Y is called
(i) (1,2)*-homeomorphism [15] if f is bijection, (1,2)*-continuous and (1,2)*-open;
(ii) (1,2)*-generalized homeomorphism (briefly (1,2)*-g-homeomorphism) if f is both (1,2)*-g-continuous and (1,2)*-g-open.
(iii) (1,2)*-gc-homeomorphism if f is (1,2)*-gc-irresolute and its inverse f^{-1} is also (1,2)*-gc-irresolute.
(iv) (1,2)*-generalized semi-homeomorphism (briefly (1,2)*-gs-homeomorphism) if f is both (1,2)*-gs-continuous and (1,2)*-gs-open.

(v) (1,2)*-gsc-homeomorphism if f is (1,2)*-gs-irresolute and its inverse f^{-1} is (1,2)*-gs-irresolute.

Example 2.13 Let X = {a, b, c}, τ_1 = {φ, X, {a}} and τ_2 = {φ, X}. Let Y = {p, q, r}, σ_1 = {φ, Y, {p}} and σ_2 = {φ, Y}. The sets in {φ, X, {b, c}} are $\tau_{1,2}$ -closed in X. The sets in {φ, X, {b}, {c}, {a, b}, {a, c}, {b, c}} are (1,2)*-g-closed in X. The sets in {φ, Y, {q, r}} are $\sigma_{1,2}$-closed in Y. The sets in {φ, Y, {q}, {r}, {p, q}, {p, r},{q, r}} are (1,2)*-g-closed in Y. Define f: X → Y by f(a)=q, f(b)=r, f(c)=p. Then f is bijective, (1,2)*-g-continuous and (1,2)*-g-open. Thus f is (1,2)*-g-homeomorphism.

Example 2.14 Let X = {a, b, c}, τ_1 = {φ, X, {b}} and τ_2 = {φ, X, {a, b}}. Let Y = {p, q, r}, σ_1={φ, Y, {p}} and σ_2 = {φ, Y}. Define f: X → Y by f(a)=q, f(b)=p, f(c)=r. Then f is bijective, (1,2)*-gs-continuous and (1,2)*-gs-open. Hence f is (1,2)*-gs-homeomorphism.

Example 2.15 Let X = {a, b, c}, τ_1 = {φ, X, {a}} and τ_2 = {φ, X, {a, b}}. Let Y = {p, q, r}, σ_1 = {φ, Y, {p}, {p, q}} and σ_2 = {φ, Y, {q}, {q, r}}. Define f: X → Y by f(a)=p, f(b)=q, f(c)=r. Then f is bijective, (1,2)*-gs-irresolute. Also f^{-1} is (1,2)*-gs-irresolute. Hence f is (1,2)*-gsc-homeomorphism.

Result 2.16[8] Every $\tau_{1,2}$-closed set is (1,2)*-semi-closed but not conversely.

Result 2.17[11] A map f: X → Y is called (1,2)*-semi-irresolute if f^{-1} (V) is (1,2)*-semi-open in X for every (1,2)*-semi-open set V in Y.

Result 2.18[10] Every (1,2)*-continuous function is (1,2)*-semi-continuous but not conversely.

3. CHARACTERIZATIONS AND PROPERTIES

Proposition 3.1 For any bijection f: X → Y the following statements are equivalent.

(a) f^{-1}: Y → X is (1,2)*-gs-continuous.

(b) f is (1,2)*-gs-open.

(c) f is (1,2)*-gs-closed.

Proof. To Prove (a) ⇒ (b). Let F be any $\tau_{1,2}$-open set of X. Then X–F is $\tau_{1,2}$-closed in X. Since f^{-1} is (1,2)*-gs-continuous, $(f^{-1})^{-1}$(X–F)=f(X–F)=Y–f(F) is (1,2)*-gs-closed in Y. Then f(F) is (1,2)*-gs-open in Y. Hence f is (1,2)*-gs-open.

To Prove (b) ⇒ (c). Let F be any $\tau_{1,2}$-closed set in X. Then X–F is $\tau_{1,2}$-open in X. Since f is (1,2)*-gs-open, f(X–F)=Y–f(F) is (1,2)*-gs-open in Y. Then f(F) is (1,2)*-gs-closed in Y. Hence f is (1,2)*-gs-closed.

To Prove (c) $\Rightarrow$ (a). Let V be any $\tau_{1,2}$-closed set in X. Since f : X $\to$ Y is (1,2)*-gs-closed, f(V) is (1,2)*-gs-closed in Y. (i.e) $(f^{-1})^{-1}(V)$ is (1,2)*-gs-closed in Y. Hence f^{-1} is (1,2)*-gs-continuous.

Proposition 3.2 Let f: X $\to$ Y be a bijective and (1,2)*-gs-continuous map. Then the following statements are equivalent.

(a) f is (1,2)*-gs-open.
(b) f is (1,2)*-gs-homeomorphism.
(c) f is (1,2)*-gs-closed.

Proof. (a) $\Rightarrow$ (b). Given f is bijective, (1,2)*-gs-continuous and (1,2)*-gs-open. Hence f is (1,2)*-gs-homeomorphism.
(b) $\Rightarrow$ (c) Let f be (1,2)*-gs-homeomorphism. Hence f is (1,2)*-gs-open. By Proposition 3.1 f is (1,2)*-gs-closed.
(c) $\Rightarrow$ (a) Follows from Proposition 3.1

Remark 3.3 The composition of two (1,2)*-gs-homeomorphism need not be a (1,2)*-gs-homeomorphism as the following example shows.

Example 3.4 Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be as in Example 2.14. Z = {x, y, z}, η_1 = {φ, Z, {x, y}}, η_2 = {φ, Z, {y}}. The sets in {φ, X, {b}, {a, b}} are $\tau_{1,2}$-open in X. Then (1,2)*-SO(X) ={ φ, X, {b}, {a, b}, {b, c}}. The sets in {φ, X, {a}, { b}, {a, b}, {b, c}} are (1,2)*-gs-open in X; The sets in {φ, Y, {p}} are $\sigma_{1,2}$-open in Y; (1,2)*-SO(Y) = {φ, Y, {p}, {p, q}, {p, r}}; The sets in {φ, Y, {p}, {q}, {r}, {p, q}, {p, r}} are (1,2)*-gs-open in Y; The sets in {φ, Z, {x, y}, {y}} are $\eta_{1,2}$-open in Z; (1,2)*-SO(Z)={φ, Z, {y}, {x, y}, {y, z}}; The sets in {φ, Z, {x}, {y}, {x, y}, {y, z}} are (1,2)*-gs-open in Z. Define f: X $\to$ Y by f(a)=q, f(b)=p, f(c)=r and g: Y $\to$ Z by g(p)=x, g(q)=z, g(r)=y. Clearly f and g are (1,2)*-gs-homeomorphism. Let gof: X $\to$ Z be defined as (gof) (a) =z, (gof) (b) =x, (gof) (c)=y. Clearly gof is bijective. gof: X $\to$ Z is not (1,2)*-gs-continuous because $(gof)^{-1}$ ({x, z})={a, b} is not (1,2)*-gs-closed in X where {x, z} is $\eta_{1,2}$-closed in Z. Hence gof is not (1,2)*-gs-homeomorphism.

Proposition 3.5 Every (1,2)*-sg-closed set is (1,2)*-gs-closed set.

Proof. Let F be $\tau_{1,2}$-open set of X such that S $\subset$ F. Then F is (1,2)*-semi-open such that S $\subset$ F. Since S is (1,2)*-sg-closed, (1,2)*-scl(S) $\subset$ F. Thus S is (1,2)*-gs-closed.

Remark 3.6[11] A bijection f: X $\to$ Y is pre-(1,2)*-semi-open if and only if f is pre-(1,2)*-semi-closed.

Theorem 3.7 If a map f: X $\to$ Y is (1,2)*-semi-irresolute and pre-(1,2)*-semi-closed, then (a). For every (1,2)*-sg-closed set A of Y, $f^{-1}(A)$ is (1,2)*-sg-closed set in X and (b). For every (1,2)*-sg-closed set B of X, f(B) is (1,2)*-sg-closed set in Y.

Proof. (a)Let A be a (1,2)*-sg-closed set of Y. Suppose that $f^{-1}(A) \subset O$ where O is (1,2)*-semi-open in X. Since f is (1,2)*-semi-irresolute, we have $f((1,2)^*\text{-scl}(f^{-1}(A)) \cap (X \backslash O)) \subset (1,2)^*\text{-scl}(f(f^{-1}(A))) \cap f(f^{-1}(Y \backslash A)) \subset (1,2)^*\text{-scl}(A) \backslash A$. This means that (1,2)*-scl(A)\A contains a (1,2)*-semi-closed subset $f((1,2)^*\text{-scl}(f^{-1}(A) \cap (X \backslash O))$, Since f is pre-(1,2)*-semi-closed. We have $f((1,2)^*\text{-scl}(f^{-1}(A) \cap (X \backslash O)) = \phi$ and hence $(1,2)^*\text{-scl}(f^{-1}(A)) \subset O$. This implies that $f^{-1}(A)$ is (1,2)*-sg-closed in X.

(b). Let B be a (1,2)*-sg-closed set in X. Let $f(B) \subset O$ where O is any (1,2)*-semi-open set of Y. Then, $B \subset f^{-1}(O)$ holds, and $f^{-1}(O)$ is (1,2)*-semi-open in X because f is (1,2)*-semi-irresolute. Since B is (1,2)*-sg-closed, $(1,2)^*\text{-scl}(B) \subset f^{-1}(O)$, and hence $f((1,2)^*\text{-scl}(B)) \subset O$. Since (1,2)*-scl(B) is (1,2)*-semi-closed set in X and f is pre-(1,2)*-semi-closed, f((1,2)*-scl(B)) is (1,2)*-semi-closed in Y. Then $(1,2)^*\text{-scl}(f((1,2)^*\text{-scl}(B))) = f((1,2)^*\text{-scl}(B))$. Therefore, we have $(1,2)^*\text{-scl}(f(B)) \subset (1,2)^*\text{-scl}(f((1,2)^*\text{-scl}(B))) = f((1,2)^*\text{-scl}(B)) \subset O$. Hence f(B) is (1,2)*-sg-closed in Y.

Theorem 3.8

(a) If $f: X \rightarrow Y$ is (1,2)*-semi-irresolute and pre-(1,2)*-semi-closed, then for every (1,2)*-sg-closed set A of Y, $f^{-1}(A)$ is (1,2)*-gs-closed.

(b) If $f: X \rightarrow Y$ is (1,2)*-continuous and pre-(1,2)*-semi-closed, then for every (1,2)*-gs-closed set A of X, f(A) is (1,2)*-gs-closed.

Proof (a) Let A be (1,2)*-sg-closed set in Y. By Theorem 3.7, $f^{-1}(A)$ is (1,2)*-sg-closed set in X. By Proposition 3.5, $f^{-1}(A)$ is (1,2)*-gs-closed set in X.

(b). Let O be a $\sigma_{1,2}$-open set of Y such that $f(A) \subset O$. Then $A \subset f^{-1}(O)$ implies $(1,2)^*\text{-scl}(A) \subset f^{-1}(O)$ since A is (1,2)*-gs-closed and $f^{-1}(O)$ is $\tau_{1,2}$-open in X. Since f is pre-(1,2)*-semi-closed, $(1,2)^*\text{-scl}[f((1,2)^*\text{-scl}(A))] = f[(1,2)^*\text{-scl}(A)] \subset O$ and hence $(1,2)^*\text{-scl}(f(A)) \subset O$. Therefore f(A) is (1,2)*-gs-closed set.

Remark 3.9 The union of two disjoint (1,2)*-gs-open sets is not, in general, (1,2)*-gs-open as the following example shows.

Example 3.10 Refer Example 3.4 {q} and {r} are two disjoint (1,2)*-gs-open sets in Y. But their union {q,r} is not (1,2)*-gs-open in Y.

Theorem 3.11[13] Let S be a subset of X. Then $(1,2)^*\text{-scl}(S) = S \cup \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(S))$.

Proposition 3.12 If A is $\tau_{1,2}$-open and (1,2)*-gs-closed set in X, then A is (1,2)*-semi-closed.

Proof. Since A is $\tau_{1,2}$-open and (1,2)*-gs-closed, By definition of (1,2)*-gs-closedness, (1,2)*-scl(A)$\subset$A. By Theorem 3.11, (1,2)*-scl(A)= A$\cup\tau_{1,2}$-int($\tau_{1,2}$-cl(A)). Thus $\tau_{1,2}$-int($\tau_{1,2}$-cl(A))$\subset$A. Hence A is (1,2)*-semi-closed.

Proposition 3.13 For any bijection f: X $\to$ Y the following statements are equivalent.

(a) f^{-1}: Y $\to$ X is (1,2)*-g-continuous.
(b) f is (1,2)*-g-open.
(c) f is (1,2)*-g-closed.

Proof. (a) $\Rightarrow$ (b). Let U be any $\tau_{1,2}$-open set of X. Then X–U is $\tau_{1,2}$-closed in X. Since f^{-1} is (1,2)*-g-continuous, $(f^{-1})^{-1}$ (X–U) = f(X–U)=Y–f(U) is (1,2)*-g-closed in Y. Then f(U) is (1,2)*-g-open in Y. Hence f is (1,2)*-g-open.
(b) $\Rightarrow$ (c). Let U be any $\tau_{1,2}$-closed set in X. Then X–U is $\tau_{1,2}$-open set in X. Since f is (1,2)*-g-open, f(X–U)=Y–f(U) is (1,2)*-g-open in Y. Then f(U) is (1,2)*-g-closed in Y. Hence f is (1,2)*-g-closed.
(c) $\Rightarrow$ (a). Let U be $\tau_{1,2}$-closed set of X. Since f is (1,2)*-g-closed, f(U) is (1,2)*-g-closed in Y. Then $(f^{-1})^{-1}$(U) is (1,2)*-g-closed in Y. Hence f^{-1} is (1,2)*-g-continuous.

Proposition 3.14 Let f: X $\to$ Y be a bijective and (1,2)*-g-continuous map. Then the following statements are equivalent.

(a) f is (1,2)*-g-open.
(b) f is (1,2)*-g-homeomorphism.
(c) f is (1,2)*-g-closed.

Proof. (a) $\Rightarrow$ (b). Since f is bijective, (1,2)*-g-continuous and (1,2)*-g-open, f is (1,2)*-g-homeomorphism.
(b) $\Rightarrow$ (c). Since f is (1,2)*-g-homeomorphism, f is (1,2)*-g-open. By Proposition 3.13 f is (1,2)*-g-closed.
(c) $\Rightarrow$ (a). Follows from Proposition 3.13.

Remark 3.15 The composition of two (1,2)*-g-homeomorphisms need not be a (1,2)*-g-homeomorphism as the following example shows.

Example 3.16 Refer Example 2.13 Clearly f is (1,2)*- g-homeomorphism. Let Z = {x, y, z}, η_1 ={φ, Z, {y}} and η_2 ={φ, Z, {x, y}}. The sets in {φ, Z, {y}, {x, y}} are $\eta_{1,2}$-open. The sets in {φ, Z, {x}, {y}, {x, y}} are (1,2)*-g-open in Z. Define g: Y□Z by g(p)=x, g(q)=z, g(r)=y. Clearly g is bijective, (1,2)*-g-continuous and (1,2)*-g-open. Hence g is also (1,2)*-g-homeomorphism. Let gof: X □ Z be defined as (gof) (a)=z, (gof)(b)=y, (gof) (c)=x. Clearly gof is bijective. But gof is not (1,2)*-g-continuous because $(gof)^{-1}$ ({z})={a} is not (1,2)*-g-closed in X where {z} is $\eta_{1,2}$-closed in Z. Hence gof is not (1,2)*-g-homeomorphism.

Proposition 3.17 If A is (1,2)*-gs-closed set in X and $A \subseteq B \subseteq$ (1,2)*-scl(A), then B is (1,2)*-gs-closed in X.

Proof Let $B \subset U$ where U is $\tau_{1,2}$-open in X. Since A is (1,2)*-gs-closed set and $A \subseteq U$, (1,2)*-scl(A) $\subseteq U$. Since $B \subseteq$(1,2)*-scl(A), (1,2)*-scl(B) $\subseteq$(1,2)*-scl(A) $\subseteq U$. Hence (1,2)*-scl(B) $\subseteq U$ and so B is (1,2)*-gs-closed in X.

Theorem 3.18 If f: $X \to Y$ is (1,2)*-continuous (1,2)*-gs-closed and A is (1,2)*-g-closed set of X, then f(A) is (1,2)*-gs-closed in Y.

Proof. Let $f(A) \subset O$ where O is $\sigma_{1,2}$-open set in Y. Then $A \subset f^{-1}(O)$. Since f is (1,2)*-continuous, $f^{-1}(O)$ is $\tau_{1,2}$-open set in X. Hence $\tau_{1,2}$-cl(A) $\subset f^{-1}(O)$ as A is (1,2)*-g-closed set. Therefore f($\tau_{1,2}$-cl(A))$\subset$O. Since f is (1,2)*-gs-closed and $\tau_{1,2}$-cl(A) is $\tau_{1,2}$-closed in X, f($\tau_{1,2}$-cl(A)) is (1,2)*-gs-closed in Y. Thus (1,2)*-scl[f($\tau_{1,2}$-cl(A))]$\subset$O. Since f(A) $\subset$ f($\tau_{1,2}$-cl(A)), (1,2)*-scl(f(A))$\subset$(1,2)*-scl[f($\tau_{1,2}$-cl(A))]$\subset$O. Therefore f(A) is (1,2)*-gs-closed in Y.

Proposition 3.19 Every (1,2)*-g-closed set is (1,2)*-gs-closed.

Proof. Let F be any $\tau_{1,2}$-open set of X such that $S \subseteq F$. Since S is (1,2)*-g-closed, $\tau_{1,2}$-cl (S) $\subseteq$ F. But (1,2)*-scl(S) $\subseteq \tau_{1,2}$-cl (S) . Hence (1,2)*-scl(S) $\subseteq$ F. Hence S is (1,2)*-gs-closed.

Theorem 3.20 If f: $X \to Y$ is (1,2)*-g-closed and g: $Y \to Z$ is (1,2)*-continuous and (1,2)*-gs-closed, then gof :$X \to Z$ is (1,2)*-gs-closed.

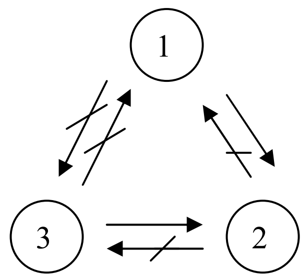
Proof. Let F be any $\tau_{1,2}$-closed set of X. Since f is (1,2)*-g-closed, f(F) is (1,2)*-g-closed set in Y. Since g is (1,2)*-continuous and (1,2)*-gs-closed and f(F) is (1,2)*-g-closed set of Y, By Theorem 3.18, g(f(F)) is (1,2)*-gs-closed in Z. Hence gof is (1,2)*-gs-closed.

Theorem 3.21 If f: $X \to Y$ is (1,2)*-closed and g: $Y \to Z$ is (1,2)*-gs-closed, then gof :$X \to Z$ is (1,2)*-gs-closed.

Proof. Let F be any $\tau_{1,2}$-closed set of X. Since f is (1,2)*-closed, f(F) is $\sigma_{1,2}$-closed in Y. Since g is (1,2)*-gs-closed, g(f(F))=(gof)(F) is (1,2)*-gs-closed in Z. Hence gof is (1,2)*-gs-closed.

4. COMPARISONS

Remark 4.1 For the sets we considered above we have the following diagram of implications where A $\not\to$ B means A does not necessarily imply B.



Where (1)=(1,2)*-g-closed, (2)=(1,2)*-gs-closed, (3)=(1,2)*-sg-closed.

Remark 4.2[8]. (1,2)*-g-closed set and (1,2)*-sg-closed sets are, in general, independent.

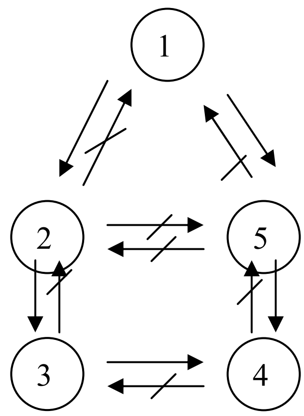
Remark 4.3 A (1,2)*-gs-closed set need not be (1,2)*-g-closed as the following example shows.

Example 4.4 Let Z={x,y,z), η_1={φ,Z,{y}} and η_2={φ,Z,{x,y}}. Then {x} is (1,2)*-gs-closed set but it is not (1,2)*-g-closed.

Remark 4.5 A (1,2)*-gs-closed set need not be (1,2)*-sg-closed as the following example shows.

Example 4.6 Let X={a, b, c), τ_1={φ,X} and τ_2={φ,X,{a}}. Then {a, b} is (1,2)*-gs-closed set but it is not (1,2)*-sg-closed.

Remark 4.7 For the maps we considered above, we have the following diagram where A $\not\rightarrow$ B means A does not necessarily imply B.



Where
(1)=(1,2)*-homeomorphism, (2)=(1,2)*-gc-homeomorphism,
(3)=(1,2)*-g-homeomorphism, (4)=(1,2)*-gs-homeomorphism,
(5)=(1,2)*-gsc-homeomorphism.

Theorem 4.8 If f: X → Y is (1,2)*-semi-continuous and (1,2)*-open map, then f is (1,2)*-semi-irresolute.

Proof. Obvious

Proposition 4.9 If f: X → Y is (1,2)*-homeomorphism, then f and its inverse f^{-1} are pre-(1,2)*-semi-closed and also (1,2)*-semi-irresolute.

Proof. Since f is (1,2)*-open and (1,2)*-continuous, by Theorem 4.8 f is (1,2)*-semi-irresolute. The bijectivity of f implies that its inverse f^{-1} is pre-(1,2)*-semi-closed. Similarly since f^{-1} is (1,2)*-open and (1,2)*-continuous we have that f^{-1} is (1,2)*-semi-irresolute and f is pre-(1,2)*-semi-closed.

Result 4.10[8] Every (1,2)*-continuous map is (1,2)*-g-continuous but not conversely.

Result 4.11[8] If f: X → Y is bijective, (1,2)*-open and (1,2)*-g-continuous map, then f is (1,2)*-gc-irresolute.

Theorem 4.12 Every (1,2)*-homeomorphism is (1,2)*-gc-homeomorphism.

Proof. Let f be (1,2)*-homeomorphism. Then f and f^{-1} are continuous. By Result 4.10, f and f^{-1} are (1,2)*-g-continuous. By Result 4.11, f and f^{-1} are (1,2)*-gc-irresolute, Hence f is (1,2)*-gc-homeomorphism.

Remark 4.13 The converse of the above Theorem need not be true as the following example shows.

Example 4.14 Let X={a, b, c}, τ_1 ={φ, X, {a, b}} and τ_2 ={φ, X}. Let Y ={p, q, r}, σ_1={φ, Y, {q}} and σ_2={φ, Y, {p, q}}. Define f: X → Y by f(a)=p, f(b)=q, f(c)=r. Then f is bijective, both f and f^{-1} are (1,2)*-gc-irresolute. Hence f is (1,2)*-gc-homeomorphism. But f is not (1,2)*-homeomorphism, since f is not (1,2)*-continuous.

Proposition 4.15 Every (1,2)*-gc-homeomorphism is (1,2)*-g-homeomorphism.

Proof. Let f be (1,2)*-gc-homeomorphism. Then f is (1,2)*-gc-irresolute and its inverse f^{-1} is also (1,2)*-gc-irresolute. Every (1,2)*-gc-irresolute map is (1,2)*-g-continuous [8]. Hence f and f^{-1} are (1,2)*-g-continuous. By Proposition 3.13, f is (1,2)*-g-open. By Proposition 3.14, f is (1,2)*- g-homeomorphism.

Remark 4.16 The following example shows that the converse of the above Proposition need not be true.

Example 4.17 Refer Example 2.13. Then f is (1,2)*-g-homeomorphism. But f is not (1,2)*-gc-homeomorphism because f is not (1,2)*-gc-irresolute.

Proposition 4.18 Every (1,2)*-g-continuous map is (1,2)*-gs-continuous.

Proof. Let V be $\sigma_{1,2}$-closed set of Y. Let f: X → Y be (1, 2)*-g-continuous. Then f^{-1} (V) is (1, 2)*-g-closed in X. By Proposition 3.19, f^{-1}(V) is (1,2)*-gs-closed in X. Hence f is (1, 2)*-gs-continuous.

Proposition 4.19 Every (1,2)*-g-open map is (1,2)*-gs-open.

Proof. Let f: X → Y be (1,2)*-g-open and F be any $\tau_{1,2}$-open set of X. Then f (F) is (1,2)*-g-open in Y and Y–f(F) is (1,2)*-g-closed in Y. By Proposition 3.19, Y–f(F) is (1,2)*-gs-closed in Y. Therefore f(F) is (1,2)*-gs-open in Y. Hence f is (1,2)*-gs-open map.

Proposition 4.20 Every (1,2)*- g-homeomorphism is (1,2)*-gs-homeomorphism.

Proof. Follows from Proposition 4.18 and Proposition 4.19.

Remark 4.21 The converse of the above Proposition need not be true as the following example shows.

Example 4.22 Let X= {a, b, c}, τ_1= {φ, X, {a}, {a, b}} and τ_2 = {φ, X, {a, c}}. Let Y={p, q, r}, σ_1= {φ, Y, {q}} and σ_2 ={φ, Y, {p, q}}. Define f: X → Y by f(a)=q, f(b)=p, f(c)=r. Then f is (1,2)*-gs-homeomorphism. But f is not (1,2)*-g-open because f({a, c}) ={q, r} is not (1,2)*-g-open in Y where {a, c} is $\tau_{1,2}$-open in X. Hence f is not (1,2)*-g-homeomorphism.

Proposition 4.23 If f is (1,2)*-open and (1,2)*-gc-irresolute, then f is (1,2)*-gs-irresolute.

Proof. Follows from Proposition 3.19.

Proposition 4.24 Every (1,2)*- homeomorphism is (1,2)*-gsc-homeomorphism.

Proof. Let f: X → Y be (1,2)*- homeomorphism. By Theorem 4.12, f is (1,2)*-gc-homeomorphism. Hence f and f^{-1} are both (1,2)*-gc-irresolute. By Proposition 4.23, f and f^{-1} are both (1,2)*-gs-irresolute. Hence f is (1,2)*-gsc-homeomorphism.

Remark 4.25 The converse of the above Proposition need not be true as the following example shows.

Example 4.26 Refer Example 2.15 f is (1,2)*-gsc-homeomorphism. But it is not (1,2)*-homeomorphism because f is not (1,2)*-continuous.

Proposition 4.27 Every (1,2)*- gsc-homeomorphism is (1,2)*-gs-homeomorphism.

Proof. Let f: X → Y be (1,2)*-gsc- homeomorphism. Hence f and f^{-1} are (1,2)*-gs-irresolute and hence (1,2)*-gs-continuous. Since f^{-1} is (1,2)*-gs-continuous, by Proposition 3.1 f is (1,2)*-gs-open. Hence f is (1,2)*-gs-homeomorphism.

Remark 4.28 The converse of the above Proposition 4.27 need not be true as shown by the following example.

Example 4.29 Let X={a, b, c}, τ_1 ={φ, X, {b}} and τ_2={φ, X, {a, b}}. Let Y={p, q, r}, σ_1={φ, Y} and σ_2 ={φ, Y, {p}}. Define f: X → Y by f(a)=q, f(b)=p, f(c)=r. Then f is (1,2)*-gs-homeomorphism. But f is not (1,2)*-gsc- homeomorphism because f^{-1}({p, q})={a, b} is not (1,2)*-gs-closed in X where {p, q} is (1,2)*-gs-closed in Y.

Remark 4.30 The following two examples show that the concepts of (1,2)*-gsc-homeomorphism and (1,2)*-gc- homeomorphism are independent of each other.

Example 4.31 Let X= {a, b, c}, τ_1 ={φ, X} and τ_2= {φ, X, {a, b}}. Let Y= {p, q, r}, σ_1= {φ, Y, {q}} and σ_2= {φ, Y, {p, q}}. Define f: X → Y by f(a)=p, f(b)=q, f(c)=r. Then f is (1,2)*-gc-homeomorphism but f is not (1,2)*-gsc-homeomorphism because f^{-1}({p})={a} is not (1,2)*-gs-closed in X where {p} is (1,2)*-gs-closed in Y.

Example 4.32 Let X={a, b, c}, τ_1= {φ, X, {a}} and τ_2= {φ, X, {a, b}}. Let Y={p, q, r}, σ_1= {φ, Y, {p}, {p, q}} and σ_2 ={φ, Y, {q}, {q, r}}. Define f: X → Y by f(a)=p, f(b)=q, f(c)=r. Then f is (1,2)*-gsc-homeomorphism but f is not (1,2)*-gc- homeomorphism because f^{-1}({q})={b} is not (1,2)*-g-closed in X where {q} is (1,2)*-g-closed in Y.

5. (1,2)*-SGO-compact spaces and (1,2)*-GSO-compact spaces.

Definition 5.1 (i) A space (X, τ_1, τ_2) is called a (1,2)*-T_d space if every (1,2)*-gs-closed subset of X is (1,2)*-g-closed in X.

(ii) A space (X, τ_1, τ_2) is called a (1,2)*-T_b space if every (1,2)*-gs-closed subset of X is $\tau_{1,2}$-closed in X.

(iii) A space (X, τ_1, τ_2) is called a (1,2)*-$T_{1/2}$ space [9] if every (1,2)*-g-closed subset of X is $\tau_{1,2}$-closed in X.

Proposition 5.2 (a) Every (1,2)*-gsc- homeomorphism from a (1,2)*- T_d space onto itself is a (1,2)*-gc- homeomorphism.
(b) Every (1,2)*-gc-homeomorphism from a (1,2)*- $T_{1/2}$ space onto itself is a (1,2)*-homeomorphism.

Proposition 5.3 Every (1,2)*-gs-homeomorphism from a (1,2)*-T_b space onto itself is a (1,2)*-homeomorphism.

Definition 5.4 A subset B of a bitopological (X, τ_1, τ_2) is said to be (1,2)*-SGO-compact (resp.(1,2)*-GSO-compact) relative to X if for every cover $\{A_i : i\in\Lambda)$ of B by (1,2)*-sg-open (resp.(1,2)*-gs-open) subsets of (X, τ_1, τ_2), i.e. $B\subset \cup\{A_i : i\in\Lambda)$ where A_i ($i\in\Lambda$) are (1,2)*-sg-open (resp.(1,2)*-gs-open) sets of (X, τ_1, τ_2), there exists a finite subset Λ_0 of Λ such that $B\subset \cup\{A_i : i\in\Lambda_0)$.

If X is (1,2)*-SGO-compact (resp. (1,2)*-GSO-compact) relative to X, (X, τ_1, τ_2) is said to be a (1,2)*-SGO-compact space (resp. (1,2)*-GSO-compact space), shortly. It is evident that the (1,2)*-GSO-compactness implies the (1,2)*-SGO-compactness and the (1,2)*- SGO-compactness implies the (1,2)*-compactness[14].

Proposition 5.5 (i) A (1,2)*-sg-closed subset of a (1,2)*-SGO-compact space (X, τ_1, τ_2) is (1,2)*-SGO-compact relative to X.

(ii) A (1,2)*-gs-closed subset of a (1,2)*-GSO-compact space (X, τ_1, τ_2) is (1,2)*-GSO-compact relative to X.

Since the proof is similar to the case of (1,2)*-compactness, it is omitted.

Proposition 5.6 (i) If $f : X \to Y$ is (1,2)*-sg-continuous (resp.(1,2)*-sg-irresolute) and a subset B of X is (1,2)*-SGO-compact relative to X, then f(B) is (1,2)*-compact in Y (resp. (1,2)*-SGO-compact relative to Y).

(ii) If $f : X \to Y$ is (1,2)*-gs-continuous (resp.(1,2)*-gs-irresolute) and a subset B of X is (1,2)*-GSO-compact relative to X, then f(B) is (1,2)*-compact in Y (resp. (1,2)*-GSO-compact relative to Y).

Proof. (i) Let $\{U_i : i\in\Lambda\}$ be any collection of $\sigma_{1,2}$-open (resp.(1,2)*-sg-open) subsets of Y such that $f(B)\subset \cup\{U_i : i\in\Lambda)$. Then $B\subset\cup\{f^{-1}(U_i): i\in\Lambda\}$ holds and there exists a finite subset Λ_0 of Λ such that $B\subset \cup\{ f^{-1}(U_i): i\in\Lambda_0\}$. Therefore, we have $f(B)\subset \cup\{U_i : i\in\Lambda_0)$ which shows that f(B) is (1,2)*-compact in Y (resp.(1,2)*-SGO-compact relative to Y).

(ii) The proof is similar to that of (i) by using (1,2)*-GSO-compactness and (1,2)*-gs-continuity (resp.(1,2)*-gs-irresolute) in the place of (1,2)*-SGO-compactness and (1,2)*-sg-continuity (resp.(1,2)*-sg-irresolute) respectively.

REFERENCES

[1] Arya, S.P; Nour, T.M. ***Characterizations of s-normal spaces,*** Indian J.Pure Appl. Math.21(1990), no.8, 717-719.

[2] Balachandran, K; Sundaram, P; Maki, H. ***On generalized continuous maps in topological spaces.*** Mem. Fac. Sci Kochi Univ (Math), 12 (1991), 5-13.

[3] Bhattacharyya, P; Lahiri, B.K. **Semi-generalized closed sets in topology.** Indian J. Math, 29 (1897), no.3, 375-382.

[4] Devi, R; Balachandran, K; Maki, H. **Semi-generalized homeomorphism and generalized semi-homeomorphism in topological spaces.** Indian J.Pure Appl. Math, 26 (1995), no.3, 271-284.

[5] Dontchev, J. **On generalized semi-preopen sets.** Mem. Fac. Sci Kochi Univ (Math), 16 (1995), 35-48.

[6] Gene Crossley, S; Hildebrand, S.K. **Semi-topological Properties.** Fund Math, 74 (1972), 233-254.

[7] Lellis Thivagar, M; Ravi, O. **On stronger forms of (1,2)*-quotient mappings in bitopological spaces.** Internat. J. Math. Game theory and Algebra. Vol. 14. No.6 (2004), 481-492.

[8] Lellis Thivagar, M; Ravi, O. **A bitopological (1,2)*-semi-generalized continuous maps.** Bull. Malaysian Math. Sci. Soc. (2) (29) (1) (2006), 79-88.

[9] Lellis Thivagar, M; Ravi, O; Joseph Israel, M; Kayathri, K. **Mildly (1,2)*-Normal spaces and some bitopological functions.** (To appear in Mathematica Bohemica).

[10] Lellis Thivagar, M; Ravi, O; Abd El-Monsef, M.E. **Remarks on bitopological (1,2)*-quotient mappings.** J. Egypt Math. Soc. 16(1) (2008), 17-25.

[11] Lellis Thivagar, M; Salai Parkunan, T; Ravi, O. **Remarks on (1,2)*-semi-generalized homeomorphisms in bitopological spaces.** (Submitted).

[12] Lellis Thivagar, M; Ravi O; Nagarajan, A; Hatir, E. **A Bitopological approach on πg-closed sets and continuity.** (To appear in International Mathematical Forum).

[13] Lellis Thivagar, M; Ravi, O; Hatir, E. **Decomposition of (1,2)*-continuity and (1,2)*-α-continuity.** (To appear in Miskolc Mathematical Notes).

[14] Lellis Thivagar, M; Ravi, O; Ekici, E. **On (1,2)*-sets and decompositions of bitopological (1,2)*-continuous mappings,** Kochi J. Math. 3 (2008), 181-189.

[15] Lellis Thivagar, M; Ravi, O; Joseph Israel, M; Kayathri, K. **Decompositions of (1,2)*-rg-continuous maps in bitopological spaces.** (To appear in Antarctica J. Math.).

[16] Lellis Thivagar, M; Ravi, O; Ekici, E. **Decompositions of bitopological (1,2)*-continuity and complete (1,2)*-continuity.** Analele Universitatii Din Oradea-Fasicola mathematica, Vol.15 (2008), 29-37.

[17] Levine, N. **Generalized closed sets in topology.** Rend. Circ. Mat. Palermo. 19 (1970), no.2, 89-96.

[18] Maki, H; Balachandran, K; Devi, R. **Remarks On semi-generalized closed sets and generalized semi-closed sets.** Kyungpook Math. J. 36 (1996), 155-163.

[19] Maki, H; Sundaram, P; Balachandaran, K. **On generalized homeomorphisms in topological spaces.** Bull. Fukuoka Univ. Ed. Part-III, 40 (1991), 13-21.

[20] Miguel Caldas. **On g-closed sets and g-continuous mappings.** Kyungpook Math. J. 33 (1993), no.2, 205-209.

[21] Sundaram, P; Maki, H; Balachandran, K. **Semi-generalized continuous maps and Semi-$T_{1/2}$ spaces.** Bull. Fukuoka Univ. Ed. Part-III, 40 (1991), 33-40.

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