In this paper we have extended the fuzzy multi objective optimization problem considered by Gupta et al [8] by incorporating another attribute of minimization of number of vehicles. Reducing the number of vehicles produces substantial savings in the global transportation costs though some times this goal is achieved with a compromise of other goal. The problem is solved by enhancing the genetic algorithm (FVRPTW) developed by Gupta et al [8] in MATLAB. Also the time taken to serve a customer is considered to be dynamic since practically it is highly impossible to predict the exact time taken to serve a customer.

**Mathematics Subject Classification:** 90B36, 90C29

**Keywords:** Fuzzy logic, Genetic Algorithm, Vehicle routing problems, Dynamic service time
1. Introduction:

The Vehicle routing problem calls for the determination of the optimal set of routes to be performed by a fleet of vehicles to serve a given set of customers, and it is one of the most important, and studied, combinatorial optimization problems. Dantzig and Ramser [3] introduced this problem in 1959 where they described a real-world application concerning the delivery of gasoline to service station and proposed the first mathematical programming formulation and algorithmic approach. In 1964, Clarke and Wright [2] proposed an effective greedy heuristic that improved on the Dantzig-Ramser approach. Following these two seminal papers, hundreds of models and algorithms were proposed for the optimal and approximate solutions of the different versions of VRP. The literature on the development of genetic algorithms for solving the VRP is rather scant. There is an opportunity for the GA to provide competitive results, given its relative robustness in the presence of complex constraints. Some very effective implementations have been reported in the literature for VRPTW (Potvin and Bengio [5], Thangiah[9]). The work done on the CVRP, including its distance or time-constrained variant, was mostly aimed at evaluating the impact of different parameters of a GA on the efficiency of the search. Breedam [1] compares a GA with previously developed simulated annealing and tabu search heuristics on different types of VRPs. A detailed literature survey on genetic algorithm based VRP has been given by Gupta et al [7].

Recently application of fuzzy set to VRP to help find optimal (or near optimal) solution is widely spreading. A fuzzy vehicle routing and scheduling problem with five attributes was formulated by Lin [4] and they proposed to solve it with a pure genetic algorithm method. They used the concept given by Gen and Cheng [6] to replace the time windows by the concept of fuzzy due time since it can describe customers preference better than fixed time window. We improved this model (refer Gupta et al [8]) by also incorporating the time taken to serve each customer since it affects the time window of the next customer. Here we developed a model with three attributes; maximization of grade satisfaction, distance minimization, waiting time minimization and the service time considered was deterministic (i.e. the time taken to serve the customer was assumed based on experience). However, practically it is difficult to assess the correct service time. Also it was felt that reducing the number of vehicles produces substantial savings in the global transportation costs even though some times this goal is achieved with a compromise of other goal. The decision can be left to the decision maker’s priority. Therefore, in this paper we have enhanced our model FVRPTW with four attributes, including fourth attribute to be minimization of number of vehicles, and making the service time to be dynamic.
2. Mathematical model of Fuzzy VRP

We briefly describe the model taken by us in the previous work [6] and then discuss the improvement done in the model. The concept of time windows does not model the customer’s preference very well. Even though customers are asked to provide a fixed time window for service, they really hope to be served at a desired time if possible. Such a desired time is called a fuzzy due time. Fuzzy vehicle routing problem is formulated based on the concept of fuzzy due time, where the membership function of fuzzy due time corresponds to the grade of satisfaction of a service time. It is improved by adding the time taken to serve each customer dynamically thereby affecting waiting time of the other customers. The objectives considered here are to maximize the average grade of satisfaction over customers, minimize the number of vehicles, the total travel distance and total waiting time for vehicles.

2.1 Grade of Satisfaction

The grade of satisfaction of service \( \mu_i(t) \) can be defined for any service time \( t>0 \) as

\[
\mu_i(t) = \begin{cases} 
1, & t \in [e_i, l_i] \\
0, & \text{otherwise}
\end{cases}
\]

(1)

where \((e_i, l_i)\) represents the earliest and the latest start time of customer \(i\) and \(t\) is the time at which the customer is served. In fuzzy set theory, a triangular fuzzy number (TFN) with respect to the grade of satisfaction for service time can be defined by the triplet \((e_i, u_i, l_i)\). The membership function of the fuzzy due time of the customer \(i\) by \(\mu_i(t_i)\) can be shown as equation (2). If a customer is served at his/her desired due time, the grade of satisfaction for him/her is 1 (full satisfaction); otherwise, the grade of satisfaction gradually decreases with the increase of difference between the service time and desired due time. The grade of satisfaction will be 0 (no satisfaction) if the service time falls outside the time interval.

\[
\mu_i(t_i) = \begin{cases} 
0, & t_i < e_i \\
\frac{t_i - e_i}{u_i - e_i}, & e_i \leq t_i \leq u_i \\
\frac{l_i - t_i}{l_i - u_i}, & u_i \leq t_i \leq l_i \\
0, & t_i > l_i
\end{cases}
\]

(2)

The overall degree of service satisfaction of vehicle \(k\) to all its service customers \((SC)_k\) is calculated as

\[
\text{Maximize } (SC)_k = \frac{1}{n} \sum_{i=1}^{n} \mu_i(t_i).
\]

(3)
Where $\mu_i(t_i)$ is the degree of satisfaction for customer $i$
\[ t_i \text{ is the service time for customer } i \]
\[ n_k \text{ is the number of customers served by the vehicle } k \]

We can find out the maximal service satisfaction of vehicle $k$ to all its service customers through maximizing the value of $(SC)_k$.

2.2 Waiting time
With the service satisfaction improved, the number of vehicles, total waiting time occurring under only certain amount of vehicles available for service probably increases at the same time. The relationship of the service time between two different customers of a vehicle is described as follows. After customer $i$ is served at the service time $t_i$ the vehicle arrives and starts to serve customer $j$ at $t_j$. The time $r_{ij}$ denotes the required transport time from customer $i$ to customer $j$ and $s_i$ denotes the time taken to serve the customer $i$ and is generated randomly. A waiting time exists while the service time of customer $j$ is earlier than the earliest time wanted. The waiting time (WT) can be given by the equation
\[ w_j = s_j - (t_j + r_{ij} + s_i), \text{ if } t_j < s_j \]
Where $t_j = t_i + r_{ij} + s_i$

Once the waiting time is minimized, the most effective transportation solution can be reached. With respect to a customer $j$, which is served by vehicle $k$ and needs waiting time, the minimum waiting time can be calculated by
\[
\text{Minimize } (WT)_j = \sum w_j(t_j)
\]
Where $w_j(t_j)$ is the waiting time for a vehicle at customer $i$
\[ t_i \text{ is the service time for customer } i \]

2.3 Total distance traveled
Another effective way to reduce the cost of transportation is shortening of distance. Through the ratio of each distance to the maximum calculated (DT)$_j$, the average ratio for all transportation distance to the maximum can be obtained as
\[
\text{Minimize } (DT)_j = \sum \frac{d_{ij}x_{ijk}}{\sum\sum d_{ij}}, \text{ for } i, j, k
\]
Where $d_{ij}$ is the distance of direct travel from customer $i$ to $j$
\[ x_{ijk} = \begin{cases} 1, & \text{if vehicle } k \text{ travels directly from customer } i \text{ to } j \\ 0, & \text{otherwise} \end{cases} \]

2.4 Fleet Size
The dispatching strategy always mainly focuses on minimizing the number of vehicles in use. A sacrifice of increasing the total transportation distance is always made to achieve this goal but the tradeoff between fleet size and travel cost must be considered. We apply genetic algorithm to search for the most appropriate dispatching strategy to transport the commodities with minimum fleet size.
Minimize \( FS = \sum_{j=1}^{n} \sum_{k=1}^{m} x_{0jk} \)  (7)

Where

\( x_{tjk} = \begin{cases} 1, & \text{if vehicle } k \text{ travels directly from customer } i \text{ to } j \\ 0, & \text{otherwise} \end{cases} \)

### 2.5 Objective Function

Fitness is calculated based on the original objective functions. The weighted sum of objectives for a chromosome is given by:

\[
F = \rho_1 \left[ \frac{(DT)_j}{(DT)_{j_{max}}} \right] + \rho_2 \left[ 1 - \frac{(SC)_{kj}}{(SC)_{kj_{max}}} \right] + \rho_3 \left[ \frac{(WT)_j}{(WT)_{j_{max}}} \right] + \rho_4 \left[ \frac{FS}{(FS)_{max}} \right] \tag{8}
\]

Where \((SC)_{kj} = \sum_{k=1}^{n} \mu_k(t_k), (SC)_{kj_{max}} \) is the maximum satisfaction grade

\((WT)_j = \sum_{k=1}^{n} w_i(t_i), (WT)_{j_{max}} \) is the maximum waiting time

\((DT)_j = \sum_{k=1}^{n} \sum_{j=1}^{m} \sum_{i=1}^{n} d_{ij} x_{ijk}, (DT)_{j_{max}} \) is the maximum distance traveled

\( FS = \sum_{j=1}^{n} \sum_{k=1}^{m} x_{0jk}, (FS)_{max} \) is the maximum number of vehicles used

### 3. Fuzzy Vehicle Routing Problem with Time Windows (FVRPTW) having Uncertainty in Service Time

The fuzzy logic concept was first introduced by L.A. Zadeh in 1965. It was proposed as an extended version of Aristotelian logic considering all values between 0 and 1. The updated mathematical model of fuzzy vehicle routing problems with time windows (FVRPTW) is given below:

**Constants**

- \( n \) is the number of customers
- \( m \) is the number of vehicles
- \( d_{ij} \) is the distance of direct travel from customer \( i \) to \( j \)
- \( w_i(t_i) \) is the waiting time for a vehicle at customer \( i \) when his service time is \( t_i \)
- \( s_i \) is the time taken to service the customer \( i \)
[a_i,b_i] is the time window of customer i
µ_i(t_i) is the degree of satisfaction of customer i

Variables

t_i is the service time for customer i

\( y_{ik} = \begin{cases} 1, & \text{if customer } i \text{ is serviced by vehicle } k \\ 0, & \text{otherwise} \end{cases} \)

\( x_{ijk} = \begin{cases} 1, & \text{if vehicle } k \text{ travels directly from customer } i \text{ to } j \\ 0, & \text{otherwise} \end{cases} \)

Formulation of Mathematical Model

\textbf{Minimize} \quad \sum_{j=1}^{n} \sum_{k=1}^{m} x_{0jk}, \quad (3.1)

\textbf{Maximize} \quad \frac{1}{n} \sum_{k=1}^{m} \mu_i(t_i) \quad (3.2)

\textbf{Minimize} \quad \sum_{i=1}^{n} w_i(t_i) \quad (3.3)

\textbf{Minimize} \quad \sum_{k=1}^{m} \sum_{j=1}^{n} \sum_{i=0}^{n} d_{ij} x_{ijk} \quad (3.4)

\textbf{Subject to,}

\[ \mu_i(t_i) > 0 \forall i \]  

(3.5)

\[ \sum_{k=1}^{m} y_{ik} = 1 \forall i \]  

(3.6)

\[ \sum_{i=0}^{n} x_{ijk} = y_{jk} \forall j, k \]  

(3.7)

\[ \sum_{j=0}^{n} x_{ijk} = y_{ik} \forall i, k \]  

(3.8)

\[ x_{ijk} (t_i + s_i + t_{ij} - t_j) \leq 0 \forall k \]  

(3.9)
where the subscript 0 stands for the central depot of the fleet of the vehicles. Objective (3.1) minimizes fleet size, objective (3.2) maximizes the average grade of satisfaction, objective (3.3) minimizes total travel distance, and objective (3.4) minimizes total waiting time for vehicles. Constraint (3.5) ensures that the service time for each customer is within a tolerable interval of time. Constraint (3.6) ensures that each customer is serviced by one and only one vehicle. Constraint (3.7) and (3.8) ensures that for each customer, there are only two customers directly connected with him, one directly reaches him, and another he directly travels to by a vehicle. Constraint (3.9) and (3.10) guarantee schedule feasibility with respect to time considerations.

3.1 Fitness Function

Fitness is calculated based on the objective function as shown in equation (8). The four objectives considered are:
(i) minimizing the fleet size
(ii) minimizing the total distance traveled,
(iii) maximizing the average grade of customer satisfaction, and
(iv) minimizing total waiting time over vehicles.
There are tradeoffs between objectives. Thus the solution obtained is a compromised solution (nearly optimal) for considering different objectives at the same time. It is easier for a dispatcher to acquire a best compromise solution by minimizing the overall cost of objectives. He can satisfy the bigger costs of the objectives but sacrifice the smaller to minimize the overall cost.
The application of this improved model in a paper distribution problem where the supplier delivers printing paper shipment to customers has been reconsidered. The necessary data to execute the model is given below:
Table 1: transit distance (km) between distribution center and customers

<table>
<thead>
<tr>
<th>Destinations (Customers)</th>
<th>Distribution Centre</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distribution Centre</td>
<td>0</td>
<td>10</td>
<td>15</td>
<td>8</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>C1</td>
<td>10</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>C2</td>
<td>15</td>
<td>5</td>
<td>0</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>5</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>C3</td>
<td>8</td>
<td>5</td>
<td>10</td>
<td>0</td>
<td>20</td>
<td>5</td>
<td>8</td>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>C4</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>0</td>
<td>8</td>
<td>10</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>C5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>5</td>
<td>8</td>
<td>0</td>
<td>5</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>C6</td>
<td>15</td>
<td>20</td>
<td>5</td>
<td>8</td>
<td>10</td>
<td>5</td>
<td>0</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>C7</td>
<td>20</td>
<td>15</td>
<td>10</td>
<td>5</td>
<td>20</td>
<td>15</td>
<td>0</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>C8</td>
<td>10</td>
<td>15</td>
<td>5</td>
<td>20</td>
<td>15</td>
<td>20</td>
<td>15</td>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: due service time schedules

<table>
<thead>
<tr>
<th>Destinations (Customers)</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Service time</td>
<td>e1</td>
<td>u1</td>
<td>l1</td>
<td>e2</td>
</tr>
<tr>
<td>Required time</td>
<td>8:00</td>
<td>9:00</td>
<td>10:00</td>
<td>9:00</td>
</tr>
<tr>
<td>Destinations (Customers)</td>
<td>C5</td>
<td>C6</td>
<td>C7</td>
<td>C8</td>
</tr>
<tr>
<td>Service time</td>
<td>e5</td>
<td>u5</td>
<td>l5</td>
<td>e6</td>
</tr>
<tr>
<td>Required time</td>
<td>12:00</td>
<td>13:00</td>
<td>14:00</td>
<td>15:00</td>
</tr>
</tbody>
</table>

The paper supplier confronts a vehicle routing problem, which consists of a fleet of 4 vehicles and 8 customers located in the Northern Taiwan. The objective is to design a dispatching strategy for distribution of papers to 8 customers by 4 vehicles in the least time. It is assumed that the load in the vehicle doesn’t exceed its capacity. The service time taken to serve each customer has been generated randomly to make the system robust.

4. Experimental Results

Computational experiments were conducted to test the effectiveness of the proposed algorithm. The basic setting of weights for the evaluation function and the parameters (pop_size, max_gen) for genetic algorithms is given in Table 4.1.
Based on the above settings, the population size was varied from 50 to 150. The average results over 10 runs for each case are reported in Table 4.2. The results show that when the population size is larger than 80, the total fitness is better. We get the best fitness corresponding to population size of 120, in which we achieve the best value for all the attributes with little compromise on slightly more waiting time.

Based on the above settings, the weights of objective functions (8) were varied as shown in Table 4.3 to investigate how they impact on the final decision. From the results we can see that emphasis on the objective of distance minimization will lead to a much better solution (with total fitness of 0.2).

The chromosome and the best route corresponding to each setting have been displayed in Table 4.4. The second column represents the chromosomes as a sequence of customers generated randomly. The third column gives the number of vehicle used to serve the customers. The last column shows the path which each vehicle will follow starting from depot, satisfying the time windows of the customers and coming back to depot.

### Table 4.1: Basic Setting of Function parameters and GA parameters

<table>
<thead>
<tr>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
<th>$\rho_3$</th>
<th>$\rho_4$</th>
<th>pop_size</th>
<th>max_gen</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.3</td>
<td>0.1</td>
<td>0.1</td>
<td>100</td>
<td>500</td>
</tr>
</tbody>
</table>

### Table 4.2: Results with varied pop_size

<table>
<thead>
<tr>
<th>Pop_size</th>
<th>Satisfaction grade</th>
<th>Waiting time</th>
<th>Distance traveled</th>
<th>Fleet size</th>
<th>Total fitness</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>0.4835</td>
<td>4.540</td>
<td>116</td>
<td>2</td>
<td>0.579</td>
</tr>
<tr>
<td>80</td>
<td>0.4383</td>
<td>2.426</td>
<td>126</td>
<td>2</td>
<td>0.579</td>
</tr>
<tr>
<td>100</td>
<td>0.3425</td>
<td>2.714</td>
<td>98</td>
<td>2</td>
<td>0.548</td>
</tr>
<tr>
<td>120</td>
<td>0.6001</td>
<td>4.914</td>
<td>86</td>
<td>2</td>
<td>0.467</td>
</tr>
<tr>
<td>140</td>
<td>0.4524</td>
<td>2.2415</td>
<td>99</td>
<td>2</td>
<td>0.5196</td>
</tr>
</tbody>
</table>

### Table 4.3: Results with varied weights

<table>
<thead>
<tr>
<th>Settings</th>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
<th>$\rho_3$</th>
<th>$\rho_4$</th>
<th>SG</th>
<th>WT</th>
<th>DT</th>
<th>FS</th>
<th>TF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.85</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.168</td>
<td>5.768</td>
<td>99</td>
<td>3</td>
<td>0.563</td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
<td>0.85</td>
<td>0.05</td>
<td>0.05</td>
<td>0.481</td>
<td>1.212</td>
<td>133</td>
<td>2</td>
<td>0.500</td>
</tr>
<tr>
<td>3</td>
<td>0.05</td>
<td>0.05</td>
<td>0.85</td>
<td>0.05</td>
<td>0.297</td>
<td>1.756</td>
<td>113</td>
<td>2</td>
<td>0.200</td>
</tr>
<tr>
<td>4</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.85</td>
<td>0.606</td>
<td>2.493</td>
<td>106</td>
<td>2</td>
<td>0.397</td>
</tr>
<tr>
<td>5</td>
<td>0.50</td>
<td>0.30</td>
<td>0.10</td>
<td>0.10</td>
<td>0.476</td>
<td>1.993</td>
<td>124</td>
<td>2</td>
<td>0.580</td>
</tr>
</tbody>
</table>
- SG – Satisfaction grade
- WT - Waiting time (in minutes)
- DT – Distance traveled (in kms)

FS - Fleet size (# of Vehicles)
TF - Total fitness

Table 4.4: Results with chromosomes and corresponding routes

<table>
<thead>
<tr>
<th>Settings</th>
<th>Chromosome [customers]</th>
<th>Fleet size</th>
<th>Vehicle (k)</th>
<th>Routes (0 indicates depot)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[2 3 1 7 4 5 6 8]</td>
<td>2</td>
<td>k=1 k=2</td>
<td>{0 2 3 6 7 0} {0 1 4 5 1 0}</td>
</tr>
<tr>
<td>2</td>
<td>[2 1 3 6 7 4 8 5]</td>
<td>2</td>
<td>k=1 k=2</td>
<td>{0 2 3 6 7 8 0} {0 1 4 5 0}</td>
</tr>
<tr>
<td>3</td>
<td>[2 3 4 6 7 1 5 8]</td>
<td>2</td>
<td>k=1 k=2</td>
<td>{0 2 3 4 6 7 8 0} {0 1 5 0}</td>
</tr>
<tr>
<td>4</td>
<td>[1 5 2 3 6 7 4 8]</td>
<td>2</td>
<td>k=1 k=2</td>
<td>{0 1 5 6 7 0} {0 2 3 4 8 0}</td>
</tr>
<tr>
<td>5</td>
<td>[1 5 2 6 3 8 4 7]</td>
<td>2</td>
<td>k=1 k=2</td>
<td>{0 1 5 6 8 0} {0 2 3 4 7 0}</td>
</tr>
</tbody>
</table>

5. Conclusions

This algorithm provides many compromised strategies to dispatcher which helps him taking a decision in any given situation. Looking at the customer’s preference he can very well plan the priorities of the goals. Reducing the fleet size really helps the dispatcher in saving company’s transportation cost. The solutions given above are the best solutions under given circumstances from dispatcher’s point of view. In case any customer demands an early service or doesn’t want to wait at all, an extra vehicle can be arranged for that customer which will cost more to the dispatcher but customer’s satisfaction grade will be high that will help the dispatcher in future business.

References


Received: August, 2009