

Almost δ^* - Pre-I-Continuous Multifunctions

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Abstract

In this paper we introduce and investigate a new type of weaker class namely δ^* -pre-I-continuous and almost δ^* -pre-I-continuous multifunctions in ideal topological space. Some characterizations of almost δ^* - pre-I-continuous multifunctions are obtained.

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1.Introduction

Mashhour et al. [9] introduced the concept of preopen sets and pre-continuous functions in topological spaces. In 1968, Velicko [18] introduced the notion of δ -open sets which are stronger than open sets. Many authors like [1], [5], [12] have researched and studied several forms of continuous functions and multi functions. Let X be a topological space. Recall that a subset A of X is said to be preclosed if $\text{cl}(\text{int}(A)) \subset A$. The preclosure of A is denoted by $\text{pcl}A$, is the smallest preclosed set in X containing A . Complement of preclosed sets are called preopen sets. S.Raychaudhuri and M.N. Mukherjee[17] introduced the notion of δ -continuity and δ -preopen sets. Erdal Ekici[5] introduced the concept of almost δ -precontinuous multifunction. Some of the other continuities are semi-continuity[11], α -continuity [10,16], precontinuity [15] and β -continuity [16].

Here we introduce a new type of continuous multifunctions namely almost δ^* - pre-I-continuity in ideal topological space. Some properties and several characterizations are studied and the relationships between other continuous

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multifunctions are also determined.

2. Preliminaries

In this paper, spaces (X, τ) and (Y, ν) always mean topological spaces on with no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X . For a subset A of (X, τ) , $\text{cl}(A)$ and $\text{int}(A)$ represent the closure of A with respect to τ and interior of A with respect to τ , respectively. An ideal is defined as a nonempty collection I of subsets of X satisfying the following two conditions: (1) If $A \in I$ and $B \subset A$, then $B \in I$. (2) If $A \in I$ and $B \in I$, then $A \cap B \in I$. Let (X, τ) be any topological space and I an ideal of subsets of X . An ideal topological space denoted by (X, τ, I) is a topological space (X, τ) with an ideal I on X . For a subset A of X , $A^*(I) = \{x \in X : U \cap A \in I \text{ for each neighbourhood } U \text{ of } x\}$ is called the local function [6] of A with respect to I and τ . Additionally, $\text{cl}^*(A) = A \cup A^*$ defines a Kuratowski closure operator for $\tau^*(I)$. $\delta\text{-cl}(A) = \{x \in X : A \cap \text{int}(\text{cl}(A)) \neq \phi, U \in \tau \text{ and } x \in U\}$, if $A = \delta\text{-cl}(A)$ then A is said to be δ -closed set. The δ -interior of a subset A of X is denoted by $\delta\text{-int}(A)$. A subset A is called δ -open if $A = \delta\text{-int}(A)$. The complement of δ -closed is δ -open.

Definition.2.1. A subset A of a space X is said to be α -open [17] if $A \subset \text{int}(\text{cl}(\text{int}(A)))$, pre-open [9] if $A \subset \text{int}(\text{cl}(A))$, δ -preopen [17] if $A \subset \text{int}(\delta\text{-cl}(A))$, semi-open [8] if $A \subset \text{cl}(\text{int}(A))$, β -open [2] if $A \subset \text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A))$, β -open [1] if $A \subset \text{cl}(\text{int}(\text{cl}(A)))$.

The family of all α -open sets, β -open sets, preopen sets, δ -preopen sets, δ -open sets are denoted by $\alpha O(X, \tau)$, $\beta O(X, \tau)$, $PO(X, \tau)$, $\delta PO(X, \tau)$, $\delta O(X, \tau)$ respectively.

By a multifunction $F : X \rightarrow Y$, we mean a point-to-set correspondence from X into Y and always assume that $F(x) \neq \phi$ for all $x \in X$. For a multifunction $F : X \rightarrow Y$, following [3,4] we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$ respectively, that is, $F^+(B) = \{x \in X : F(x) \subset B\}$ and $F^-(B) = \{x \in X : F(x) \cap B \neq \phi\}$. In particular, $F^-(y) = \{x \in X : y \in F(x)\}$ for each point $y \in Y$. For each $A \subset X$, $F(A) = \cup_{x \in X} F(x)$, then F is said to be a surjection if $F(X) = Y$, or equivalently if for each $y \in Y$ there exists an $x \in X$ such that $y \in F(x)$. Moreover, $F : X \rightarrow Y$ is called upper semi continuous (resp. lower semi continuous) if $F^+(V)$ (resp. $F^-(V)$) is semi open in X for every open set V of Y [11].

Definition.2.2. A multifunction $F : X \rightarrow Y$, is said to be: 1) upper almost continuous [13,14] or upper precontinuous [14] (upper α continuous, upper β -continuous [15], upper δ -precontinuous [11]) at $x \in X$ if for each open set V of Y containing $F(x)$, there exists $U \in PO(X, \tau)$, ($U \in \alpha O(X, \tau)$, $U \in \beta O(X, \tau)$, $U \in \delta PO(X, \tau)$) such that $F(U) \subset V$.

2) lower almost continuous [13,14] or lower precontinuous [14] (lower α continuous, lower β -continuous[15] , lower δ -precontinuous [11]) at $x \in X$ if for each open set V of Y such that $F(x) \cap V \neq \phi$, there exists $U \in PO(X,\tau)$, ($U \in \alpha O(X,\tau), U \in \beta O(X,\tau), U \in \delta PO(X,\tau)$) such that $F(u) \cap V \neq \phi$ for every $u \in U$.

3) upper (lower) almost continuous or upper (lower) precontinuous, (upper (lower) α -continuous, (upper (lower) precontinuous, upper(lower) δ -precontinuous) if it has this property at each point of X .

3. Almost δ^* - pre-I-continuous multifunctions

The δ^* -interior of a subset A of X is denoted by $\delta^*\text{-int}(A)$. A subset A is called δ^* -open if $A = \delta^*\text{-int}(A)$. A point x of X is called a δ^* -cluster point of A if , $\delta^*\text{-cl}(A) = \{x \in X: A \cap \text{int}(\text{cl}^*(U)) \neq \phi, \text{ whenever } x \in U, U \in \tau\}$ equivalently if $A \cap V \neq \phi$ for every IR open set V containing x , where V is said to be IR open[7] if $\text{int}(\text{cl}^*(V)) = V$. If $\delta^*\text{-cl}(A) = A$, then A is said to be δ^* -closed. The complement of δ^* -closed is δ^* -open.

Definition.3.1. A subset A of a space X is said to be δ^* -pre-I-open if $A \subset \text{int}(\delta^*\text{-cl}(A))$.

The complement of a δ^* -pre-I-open is said to be δ^* -pre-I-closed. The intersection of all δ^* -pre-I-closed sets of X containing A is called the δ^* -pre-I-closure of A and is denoted by $\delta^*\text{-pIcl}(A)$.The union of all δ^* -pre-I-open sets of X contained in A is called the δ^* -pre-I-interior of A and is denoted by $\delta^*\text{-pIint}(A)$. The subset U of X is called a δ^* -pre-I-neighbourhood of a point $x \in X$ if there exists a δ^* -pre-I-open set V such that $x \in V \subset U$. The family of all δ^* -pre-I-open sets, δ^* -pre-I-closed sets of X is denoted by $\delta^*\text{PIO}(X)$, $\delta^*\text{PIC}(X)$.

Theorem.3.2. For a subset A of a topological space (X,τ,I) , the following properties hold: 1) Every δ -open is δ^* -open. 2) Every δ^* pre-I-open is δ preopen. But the converses is not true for both in general.

Proof. It is clear from the definitions.

Example.3.3. Let $X = \{a, b, c\}, \tau = \{\phi, X, \{a\}\}, A = \{a\}$ and $I = \{\phi, \{a\}\}$ then we get A is δ^* open set but it is not a δ open set.

Example.3.4. Let $X = \{a, b, c\}, \tau = \{\phi, X, \{a\}\}, A = \{b, c\}$ and $I = \{\phi, \{a\}\}$ then we get $\{b, c\}$ is δ preopen set but it is not a δ^* pre-I-open set.

Definition.3.5. A multifunction $F: X \rightarrow Y$ is said to be: 1) upper δ^* -pre-I-continuous at a point $x \in X$ if for each open set V of Y containing $F(x)$, there exists a $U \in \delta^*\text{PIO}(X,\tau)$, such that $F(U) \subset V$,

2)lower δ^* -pre-I-continuous at a point $x \in X$ if for each open set V of Y such that $F(x) \cap V \neq \phi$, there exists a $U \in \delta^*\text{PIO}(X,\tau)$, such that $F(U) \cap V \neq \phi$,

3) lower almost δ^* -pre-I-continuous at a point $x \in X$ if for each open set V of Y such that $x \in F^-(V)$, there exists a $U \in \delta^*\text{PIO}(X,\tau)$ such that $U \subset F^-(\text{int}(\text{cl}^*(V)))$,

4) upper almost δ^* -pre-I-continuous at a point $x \in X$ if for each open set V of Y such that $x \in F^+(V)$, there exists a $U \in \delta^*\text{PIO}(X, \tau)$ such that $U \subset F^+(\text{int}(\text{cl}^*(V)))$,

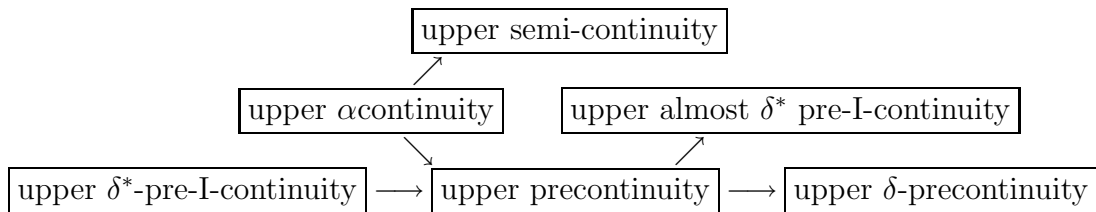
5) lower (upper) almost δ^* -pre-I-continuous if F has this property at each point of X .

Example.3.6. Let $F: (X, \tau, I) \rightarrow (Y, \nu, J)$, where $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a, c\}, \{a, c, d\}\}$, $I = \{\phi, \{a\}\}$, $Y = \{a, b\}$ and $J = \{\phi, Y, \{a\}\}$. The function is defined as follows: $F(a) = F(b) = F(c) = a$ and $F(d) = b$. Let $a \in X$, then $V = \{a\}$ be the open set containing $F(a)$ in Y , then there exists a δ^* -pre-I-open set $U = \{a, c\}$ in X such that $U \subset F^+(\text{int}(\text{cl}^*(V)))$.

Theorem.3.7. For a multifunction $F : X \rightarrow Y$ from a topological space (X, τ) to a topological space (Y, ν) , the following implications hold: 1) Every upper α -continuous is upper semicontinuous. 2) Every upper α -continuous is upper precontinuous. 3) Every upper δ^* -pre-I-continuous is upper precontinuous. 4) Every upper precontinuous is upper δ -precontinuous. 5) Every upper δ^* -pre-I-continuous is upper almost δ^* -pre-I-continuous.

Proof. It is clear from the definitions.

Remark.3.8. The above theorem.3.7. is shown diagrammatically.



Note that none of these implications is reversible. We give an example for the almost upper δ^* -pre-I-continuous but not upper δ^* -pre-I-continuous.

Example.3.9. Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a, c\}, \{a, c, d\}\}$, $I = \{\phi, \{a\}, \{c\}, \{a, c\}\}$, $Y = \{a, b\}$, $\nu = \{\phi, X, \{a\}\}$ and $J = \{\phi, \{b\}\}$. Let $F: (X, \tau, I) \rightarrow (Y, \nu, J)$ be a function defined as $F(a) = F(b) = F(d) = a$ and $F(c) = b$. Let $b \in X$ then $V = \{a\}$ be the open set containing $F(b)$, then there exists a δ^* -pre-I-open set $U = \{b, c\}$ such that $U \subset F^+(\text{int}(\text{cl}^*(V)))$ but not $F(U) \subset V$. (i.e) F is almost upper δ^* -pre-I-continuous but not upper δ^* -pre-I-continuous.

The following theorem gives some characterizations of an upper almost δ^* -pre-I-continuous multifunction.

Theorem.3.10. Let $F: X \rightarrow Y$ be a multifunction from a topological space (X, τ) to a topological space (Y, σ) . Then the following statements are equivalent: 1) F is a upper almost δ^* -pre-I-continuous multifunction, 2) for each $x \in X$ and for each open set V such that $F(x) \subset V$, there exists a $U \in \delta^*\text{PIO}(X, \tau)$ such that if $y \in U$, then $F(y) \in \text{int}(\text{cl}^*(V))$, 3) for each $x \in X$ and for each closed set K such that $x \in F^+(Y \setminus K)$, there exists a δ^* -pre-I-open set H such

that $x \in X \setminus H$ and $F^-(\text{cl}(\text{int}^*(K))) \subset H$, 4) $F^+(\text{int}(\text{cl}^*(V))) \in \delta^*\text{PIO}(X, \tau)$ for any open set $V \subset Y$, 5) $F^-(\text{cl}(\text{int}^*(K))) \in \delta^*\text{PIC}(X, \tau)$ for any closed set $K \subset Y$, 6) for each point $x \in X$ and each neighbourhood V of $F(x)$, $F^+(\text{int}(\text{cl}^*(V)))$ is a δ^* - pre-I- neighbourhood of x , 7) for each point $x \in X$ and each neighbourhood V of $F(x)$, there exists a δ^* - pre-I-neighbourhood U of x such that $F(U) \subset \text{int}(\text{cl}^*(V))$, 8) $\delta^*\text{-pIcl}(F^-(\text{cl}(\text{int}^*(H)))) \subset F^-(\text{cl}(\text{int}^*(\text{cl}(H))))$ for every subset H of Y , 9) $F^+(\text{int}(\text{cl}^*(\text{int}(N)))) \subset \delta^*\text{-pIint}(F^+(\text{int}(\text{cl}^*(N))))$ for every subset N of Y .

Proof. (1) \leftrightarrow (2): obvious.

(2) \rightarrow (3): Let $x \in X$ and K be a closed set of Y such that $x \in F^+(Y \setminus K)$. By (2), there exists a $U \in \delta^*\text{PIO}(X, \tau)$ such that $F(U) \subset (\text{int}(\text{cl}^*(Y \setminus K)))$. We have $(\text{int}(\text{cl}^*(Y \setminus K))) = Y \setminus (\text{cl}(\text{int}^*(K)))$ and $U \subset F^+(Y \setminus (\text{cl}(\text{int}^*(K)))) = X \setminus F^-(\text{cl}(\text{int}^*(K)))$. Thus we obtain $F^-(\text{cl}(\text{int}^*(K))) \subset X \setminus U$. Take $H = X \setminus U$, then $x \in X \setminus H$ and H is a δ^* pre-I-closed set.

(3) \rightarrow (2): It can be obtained similarly as (2) \rightarrow (3).

(1) \rightarrow (4): Let V be any open set of Y and $x \in F^+(V)$. By (1), there exists $U_X \in \delta^*\text{PIO}(X, \tau)$ such that $U_X \subset F^+(\text{int}(\text{cl}^*(V)))$. Therefore, we obtain $F^+(\text{int}(\text{cl}^*(V))) = \cup U_X$. Hence, $F^+(\text{int}(\text{cl}^*(V))) \in \delta^*\text{PIO}(X, \tau)$.

(4) \rightarrow (1): Let V be any open set of Y and $x \in F^+(V)$. By (4), $F^+(\text{int}(\text{cl}^*(V))) \in \delta^*\text{PIO}(X)$. Take $U = F^+(\text{int}(\text{cl}^*(V)))$. Then, $F(U) \subset \text{int}(\text{cl}^*(V))$. Hence, F is upper almost δ^* -pre-I-continous.

(4) \rightarrow (5): Let K be any closed set of Y . Then, $Y \setminus K$ is a open set of Y . By (4), $F^+(\text{int}(\text{cl}^*(Y \setminus K))) \in \delta^*\text{PIO}(X, \tau)$. Since $(\text{int}(\text{cl}^*(Y \setminus K))) = Y \setminus (\text{cl}(\text{int}^*(K)))$, it follows that $F^+(\text{int}(\text{cl}^*(Y \setminus K))) = F^+(Y \setminus (\text{cl}(\text{int}^*(K)))) = X \setminus F^-(\text{cl}(\text{int}^*(K)))$. We obtain that $F^-(\text{cl}(\text{int}^*(K)))$ is δ^* pre-I-closed in X .

(5) \rightarrow (4): It can be obtained similarly as (4) \rightarrow (5).

(4) \rightarrow (6) Let $x \in X$ and V be a neighbourhood of $F(x)$. Then there exists an open set G of Y such that $F(x) \in G \subset V$. Therefore, we obtain $x \in F^+(G) \subset F^+(V)$. Since $F^+(\text{int}(\text{cl}^*(G))) \in \delta^*\text{PIO}(X, \tau)$, $F^+(\text{int}(\text{cl}^*(V)))$ is a δ^* - pre-I-neighbourhood of x .

(6) \rightarrow (7): Let $x \in X$ and V be a neighbourhood of $F(x)$. By (6), $F^+(\text{int}(\text{cl}^*(V)))$ is a δ^* - pre-I- neighbourhood of x . Take $U = F^+(\text{int}(\text{cl}^*(V)))$. Then $F(U) \subset (\text{int}(\text{cl}^*(V)))$.

(7) \rightarrow (1): Let $x \in X$ and V be any open set of Y such that $F(x) \in V$. Then V is a neighbourhood of $F(x)$. By (7), there exists a δ^* - pre-I-neighbourhood of x such that $F(U) \subset (\text{int}(\text{cl}^*(V)))$. Therefore, there exists $G \in \delta^*\text{PIO}(X, \tau)$ such that $x \in G \subset U$ and $F(G) \subset F(U) \subset (\text{int}(\text{cl}^*(V)))$. We obtain upper almost δ^* -pre-I-continuous.

(5) \rightarrow (8) For any subset H of Y , $\text{cl}(H)$ is closed in Y . By (5), $F^-(\text{cl}(\text{int}^*(\text{cl}(H))))$ is δ^* pre-I-closed in X . Therefore, we obtain $\delta^*\text{-pIcl}(F^-(\text{cl}(\text{int}^*(H)))) \subset F^-\text{cl}(\text{int}^*(\text{cl}(H)))$.

(8) \rightarrow (5) Let K be any closed set of Y . Then we have $\delta^*\text{-pIcl}(F^-(\text{cl}(\text{int}^*(K)))) \subset$

$F^-(\text{cl}(\text{int}^*(\text{cl}(K)))) = F^-(\text{cl}(\text{int}^*(K)))$. Thus, $F^-(\text{cl}(\text{int}^*(K)))$ is δ^* -pre-I-closed in X .

(4) \rightarrow (9) For any subset N of Y , $\text{int}(N)$ is open in Y . By (4), $F^+(\text{int}(\text{cl}^*(\text{int}(N))))$ is δ^* -pre-I-open in X . Therefore, we obtained $F^+(\text{int}(\text{cl}^*(\text{int}(H)))) \in \delta^*$ -pint($F^+(\text{int}(\text{cl}^*(H))))$.

(9) \rightarrow (4) Let V be any open set of Y . Then we have $F^+(\text{int}(\text{cl}^*(V))) \in \delta^*$ -pint($F^+(\text{int}(\text{cl}^*(V))))$. Thus, $F^+(\text{int}(\text{cl}^*(V)))$ is δ^* -pre-I-open in X .

Theorem.3.11. Let $F: X \rightarrow Y$ be a multifunction from a ideal topological space (X, τ, I) to a topological space (Y, σ, I) . Then the following statements are equivalent: 1) F is a lower almost δ^* -pre-I-continuous multifunction, 2) for each $x \in X$ and for each open set V such that $F(x) \cap V \neq \phi$, there exists a $U \in \delta^*$ PIO(X, τ) such that if $y \in U$, then $F(y) \cap \text{int}(\text{cl}^*(V)) \neq \phi$, 3) for each $x \in X$ and for each closed set K such that $x \in F^-(Y \setminus K)$, there exists a $U \in \delta^*$ -pre-I-closed set H such that $x \in X \setminus H$ and $F^+(\text{cl}(\text{int}^*(K))) \subset H$, 4) $F^-(\text{int}(\text{cl}^*(V))) \in \delta^*$ PIO(X, τ) for any open set $V \subset Y$, 6) $F^+(\text{cl}(\text{int}^*(K))) \in \delta^*$ PIC(X, τ) for any closed set $K \subset Y$, 7) δ^* -pIcl($F^+(\text{cl}(\text{int}^*(H))) \subset F^+(\text{cl}(\text{int}^*(\text{cl}(H))))$) for every subset H of Y , 8) $F^-(\text{int}(\text{cl}^*(\text{int}(N)))) \in \delta^*$ -pIint($F^-(\text{int}(\text{cl}^*(N))))$) for every subset N of Y .

Proof. It can be obtained similarly as the previous theorem.

Lemma.3.12. Let A and X_0 be subsets of a space (X, τ) . If $A \in \delta^*$ PIO(X, τ) and $X_0 \in \delta^*$ O(X), then $A \cap X_0 \in \delta^*$ PIO(X).

Proof. Let $A \in \delta^*$ PIO(X, τ) and $X_0 \in \delta^*$ O(X), which implies $A \in (\text{int}(\delta^*\text{-cl}(A)))$. $A \cap X_0 \subset (\text{int}(\delta^*\text{-cl}(A))) \cap X_0 \subset (\text{int}(\delta^*\text{-cl}(A)) \cap X_0) \subset (\text{int}(\delta^*\text{-cl}(A) \cap X_0)) \subset (\text{int}(\delta^*\text{-cl}(A \cap X_0)))$. Therefore $A \cap X_0 \in \delta^*$ PIO(X, τ).

Lemma.3.13. Let A and X_0 be subsets of a space (X, τ) . If $A \in \delta^*$ PIO(X, τ) and $X_0 \in \delta^*$ O(X), then $A \cap X_0 \in \delta^*$ PIO(X_0).

Proof. Let $A \in \delta^*$ PIO(X, τ) and $X_0 \in \delta^*$ O(X) by lemma 3.12 $A \cap X_0 \in \delta^*$ PIO(X), Now $A \cap X_0 \cap X_0 \subset X_0 \cap (\text{int}(\delta^*\text{-cl}(A \cap X_0))) \subset (X_0 \cap \text{int}(\delta^*\text{-cl}(A \cap X_0))) \subset (\text{int}_{X_0}(X_0 \cap \delta^*\text{-cl}(A \cap X_0))) \subset (\text{int}_{X_0}(\delta^*\text{-cl}_{X_0}(A \cap X_0)))$. Therefore $A \cap X_0 \in \delta^*$ PIO(X_0).

Lemma.3.14. Let $A \subset X_0 \subset X$. If $X_0 \in \delta^*$ O(X) and $A \in \delta^*$ PIO(X_0), then $A \in \delta^*$ PIO(X).

Proof. Let $A \in \delta^*$ PIO(X_0) and $X_0 \in \delta^*$ O(X). Since $X_0 \subset X$, which implies $A \in \delta^*$ PIO(X).

Theorem.3.15. Let $F: X \rightarrow Y$ be a multifunction and let U be a δ^* -open set in X . If F is a lower(upper) almost δ^* pre-I-continuous, then the restriction multifunction, $F|U : U \rightarrow Y$ is a lower (upper) almost δ^* pre-I-continuous.

Proof. Suppose that V is a open set in Y . Let $x \in U$ and $x \in (F|U)^-(V)$. Since F is a lower almost δ^* pre-I-continuous multifunction, it follows that there exists a δ^* pre-I- open set G such that $x \in G \subset F^-(\text{int}(\text{cl}^*(V)))$. By Lemma 3.13 we obtain that $x \in G \cap U \in \delta^*$ PO(U) and $G \cap U \subset (F|U)^-(\text{int}(\text{cl}^*(V)))$. Thus, we show that the restriction multifunction $F|U$ is a lower

almost δ^* -pre-I-continuous. The proof of the upper almost δ^* -pre-I-continuous of $F | U$ is similar to the above.

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