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# A Mathematical Model for Designing a Multiple MTI Filter Using MIMO Radar Signals

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## Abstract

A ground clutter cancellation in multi-channel noise radar is presented. The radar system under consideration consists of *m* independent noise transmitters working in the same frequency band and *n* receivers. Independent cancellation of clutter echoes originating from successive transmitters is not fully effective therefore joint cancellation of all clutter echoes is proposed.

MIMO (Multiple-input multiple-output) extensions radar systems are enable to number of advantages compared to traditional approaches. These advantages include improved angle estimation and target detection.

**Keywords:** MIMO Radar, Linear Frequency Modulation, waveform design, Mathematical modeling

## 1 Introduction

The term “noise radar” refers to a group of radars using a random waveform for target illumination [1, 2]. This type of radar can be used in a relatively wide range of applications. It is possible to construct surveillance, imaging [3], tracking, guidance, collision warning, subsurface and other types of radar using a noise waveform. Noise radars have several advantages over classical pulse, pulse-Doppler and FMCW radars. Literature survey of a noise waveform ensures the absence of range and Doppler ambiguity and low peak power. One of the crucial problems of the noise radar is the masking effect [4, 5, 8], which consists in hiding weak target echoes in the side lobes of strong returns from clutter and near targets. This problem can be solved by using adaptive methods for strong echoes cancellation [6, 7].

Current research on the noise radar is focused on two main configurations: multistatic (netted) configuration and MIMO (multiple output multiple inputs) configuration. The concept of netted noise radars forming a "fence" along a border has been proposed in [9]. The noise MIMO radar has been described in [10]. In the case when all netted or MIMO transmitters are emitting signals in separated bands there is no interference between them and the whole signal processing can be performed as in a mono-static noise radar. However, frequency resources are very valuable and it is difficult to allocate separate frequency bands for each transmitter, especially when using wide-band signals. An alternative solution is to transmit independent noise waveforms from each transmitter using the same frequency band. For infinite transmitting time, the transmitted noise signals are orthogonal to each other. However, for limited time intervals some correlations between the signals have been presented. It degrades the performance of the whole system. It is possible to orthogonally the finite-length signals before transmission; however, time-delayed and Doppler shifted versions of those signals will be correlated. The lack of orthogonally between received components originating form different transmitters decreases the sensitivity of the system, especially in the case of strong ground clutter, which can completely mask weak echoes of moving targets [11, 12].

This paper presents an effective method of ground clutter removal in multistatic or MIMO noise radars. The clutter removal procedure utilizes a multi-dimensional lattice predictor for signal orthogonalisation. The received signal is then projected onto orthogonalised clutter subspace.

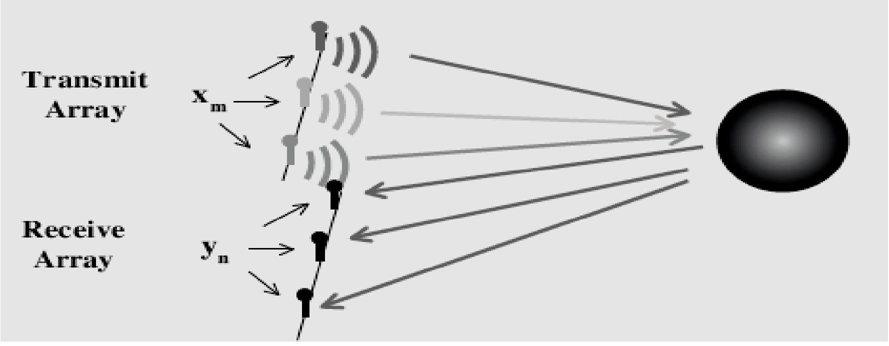

Fig. 1. System concept for m = 3 transmitters and n = 3 receivers.

## 2 System concepts

It is assumed that the signal emitted by the radar is band-pass continuous noise. There are K transmitters and L receivers in the system, located in different positions. The transmitters send independent noise signals $x_m$ (t) in the same frequency band. The receivers have access to the signals $y_1$(t), $y_2$(t) ... n sent by all transmitters. This can be achieved by the use of a separate channel with an antenna directed towards the transmitter or by transmission of the signal samples by a computer network. The concept of the system for m= 3 and n = 3 is presented in Fig.(1).

The signal received by the n-th receiver originating form the m-th transmitter can be expressed by the following formula:

$$Y_{n,m}(t) = \sum_{q=1}^{Q} a_q X_m \left[ t - \frac{R_q}{C} \right] + \sum_{P-1}^{P} b_p X_m \left[ t - \frac{R_p}{C} \right] \exp\left( j2\lambda \frac{V_p}{\lambda} t \right) \qquad (1)$$

where $x_m$ (t) is the signal transmitted by the m-th transmitter, $a_q$, $b_p$ are the complex signal amplitudes, $R_q$ , $R_p$ are the biostatic ranges, $V_p$ is the target biostatic velocity, λ is the wavelength, Q is the number of stationary targets, P is the number of moving targets. The first term on the right-hand side of equation (1) corresponds to the reflections from stationary targets. The second term represents contributions from moving targets. The signal received by the nth receiver comprises contributions form each transmitter and it can be expressed as follows:

$$Y_n(t) = \sum_{m=1}^{M} Y_{n,m}(t) + W(t) \qquad (2)$$

where *w(t)* is an additive white Gaussian noise modeling environment and receiver noise.

The detection procedure is based on the matched filtering concept. In order to take into account different Doppler shifts of target echoes, a bank of filters is used, in which each filter is matched to a different frequency shift:

$$\psi_{m,n}(R,V) = \int_n Y_n(t) X_m^{*} \left( t - \frac{R}{C} \right) \exp\left( -j2\lambda \frac{V}{\lambda} t \right) dt \qquad (3)$$

The signal $x_m$ (t) correlates only with the corresponding term $y_m$ (t), *n* of the signal $y_n$ (t). However, signals originating from other transmitters act as additional noise sources. This causes the raise of the noise floor on the range-Doppler surface calculated by (3), which leads to the reduction of system sensitivity. In addition, the returns from stationary targets, i.e. clutter, can mask weaker moving targets.

## 3 Algorithm Description

A single channel ground clutter removal can be achieved using a standard interference canceller based on the Wiener filtering theory, well described in [3, 7]. According to the general structure of the canceller, the clutter estimate is subtracted from the received signal in order to obtain an estimate of the desired non-zero Doppler echo. The clutter estimate, in turn, is obtained at the output of a linear filter excited by the reference signal. This filter can be realized either as a transversal filter or as a well known joint process estimator. The joint process estimator consists of two parts [3, 7]: a lattice predictor and a linear combiner (regression filter). An important feature of the lattice predictor is that it may be viewed as an orthogonalisation transform. Feeding the input of a multistage predictor with the sequence of samples of the reference signal, a corresponding sequence of orthogonal backward prediction errors at the output of each stage is obtained. Prediction errors are then used by a linear combiner to remove the interference from the received signal. The fact that they are orthogonal to each other simplifies the solution to the problem of finding optimal coefficients of the linear combiner.

This idea of clutter removal was presented in [5, 8] in context of the noise radar and FM radio-based radar, respectively. In both publications the interference canceller was implemented as a single channel (scalar) joint process estimation filter. In the case of multiple sources considered in this paper, the usefulness of such solution is restricted since the single channel version of the joint process estimator can be applied only to each single channel independently instead of joint removal from all channels. For this reason, effective clutter suppression in radar system with multiple sources requires the use of the multi-channel version of the joint process estimation filter.

The multi-channel lattice prediction algorithm has the following form [13]:

$$f_{m+1}(n) = f_m(n) - \Gamma^{f}_{m+1} b_m(n-1) \tag{4}$$

$$b_{m+1}(n) = b_m(n-1) - \Gamma^{b}_{m+1} f_m(n) \tag{5}$$

where $f_m(n)$, $b_m(n)$ denote forward and backward prediction errors (M×1 vectors) at the output of the m-th stage. The (M×M) matrices of PARCOR (Partial Correlations), coefficients $\Gamma^{f}_{m+1}, \Gamma^{b}_{m+1}$ and the (M×M) covariance matrices $P^{f}_{m}$, $P^{b}_{m}$ of the prediction errors are defined as follows:

$$\Gamma^{f}_{m+1} = E\left[f_m(n) b^{H}_{m}(n-1)\right]\left[P^{b}_{m}\right]^{-1} \tag{6}$$

$$\Gamma^{b}_{m+1} = E\left[b_m(n-1) f^{H}_{m}(n)\right]\left[P^{f}_{m}\right]^{-1} \tag{7}$$

$$P^{b}_{m+1} = \left\lfloor I - \Gamma^{f}_{m+1}\Gamma^{b}_{m+1} \right\rfloor P^{b}_{m} \tag{8}$$

$$P^f_{m+1} = \lfloor I - \Gamma^f_{m+1}\Gamma^b_{m+1} \rfloor P^f_m \qquad (9)$$

The recursions (4) – (9) are repeated in a loop for m = 0 to M −1 with initial conditions:

$$f_o(n) = b_o(n) = X(n) = [X_1(n), X_2(n)......X_m(n)]^T \qquad (10)$$

$$\Gamma^f_1 = \Gamma^b_1 = R^{-1}_{xx}(0)R_{xx}(1) \qquad (11)$$

$$P^f_1 = P^b_1 = (I - \Gamma^f_1\Gamma^b_1)R_{xx}(0) \qquad (12)$$

where $X_m(n)$ are sampled versions of the transmitted signals and,

$$R_{xx}(m) = E[X(n)X^H(n-m)] \qquad (13)$$

is the (M×M) autocorrelation matrix of the vector reference signal.

To complete the joint process estimation, the coefficients of the linear combiner are determined and calculated for m = 0 to M:

$$h_m(n) = [P^b_m]^{-1}E[b_m(n)e^H_m(n)] \qquad (14)$$

$$e_{m+1}(n) = e_m(n) - h^H_m b_m(n) \qquad (15)$$

For m=0, the $e_o(n) = Y_1(n)$ and $h_o(n) = [R^{-1}_{xx}(n)]^{-1}.E[X(n).Y^*_l(n)]$

where $Y_l(n)$ is the sampled version of the signal received by the n-th receiver. The final estimate of the non-zero Doppler echo from (1) is given by e$_{m+!}$ (n), i.e. the output of the linear combiner. The expectation E[·] in the above formulas is replaced in calculations by a sample mean. Fig.(2), depicts a block diagram of the lattice joint process estimator used for ground clutter removal.

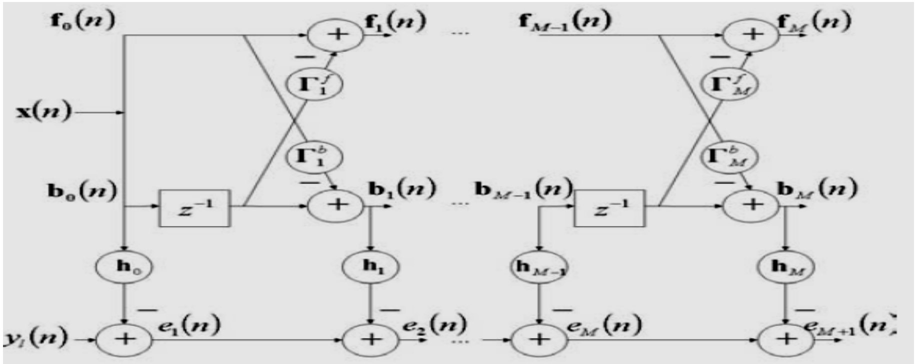

Fig.2. Joint process estimator

# 4 Simulation Results

The algorithms presented in this paper were tested by means of computer simulations. The transmitted signal was a band limited Gaussian noise. The targets were simulated by adding time-delayed and Doppler shifted versions of the transmitted signal to the received signal. The sampling frequency was equal to 200 kHz and the length of the signal blocks was equal to 32768. The range-Doppler surfaces were obtained by correlating appropriate signals using (3). Before correlation a Hamming window was applied to the signals in order to reduce side lobes. In the first simulation scenario, it was assumed that there are three transmitters and one receiver. Some stationary targets were present to simulate ground clutter. In addition, three moving targets were simulated. The environment and receiver noise were neglected. The results presented below were obtained for the first channel but the results of processing of other channels are comparable. The values of the correlation were clipped at the mean noise floor level. In the figure, only ground clutter at zero Doppler frequency is visible. The noise floor level is approximately at 30 dB (arbitrary scale). To remove zero Doppler components, the single-channel lattice filter was used. The input of this filter was the signal from the first transmitter $x_1$ (t). The echoes originating from ground clutter were removed and the noise floor level was decreased to 28 dB. However, the moving targets still cannot be observed, because the signals from other transmitters present in the measurement signal $y_1$ (t) keep the noise floor at a high level.

In the next step, a single-channel lattice filter was used to remove signals originating from all transmitters separately. This led to further reduction of the noise floor to the level of (− 5 dB). The simulated moving targets start to be visible. However, the lattice filters did not remove zero Doppler components completely because were used independently for each channel. The use of the multi-channel lattice filter described in this paper yielded the results. The ground clutter was removed completely and the noise floor level was reduced to -24 dB. The simulated moving targets are clearly visible above the noise floor. In the second simulation scenario, the influence of the number of transmitters on the radar detection performance was investigated. The received signal consisted of the signals originating from all transmitters and additive noise. The signals generated by the transmitters contained zero-Doppler components representing clutter. The power of each of those signals was the same. Additional noise simulating disturbance at the receiver was 40 dB below the level of the useful signals. Different methods for clutter cancellation were used in presented situation. Figure (3) below, showed the noise floor level versus number of transmitters for different versions of the cancellation algorithm. It can be observed that joint cancellation method using multi-dimensional lattice filter yields constant noise floor level independently of the number of transmitters. The fixed noise floor level results from the noise added to the independent signal cancellation of each channel causes the noise floor to rise by several dBs in comparison with the multi-dimensional lattice method. When clutter removal method is not used, the noise floor level increases steadily as a result of larger number of transmitters.

For multiple transmitters, the noise floor level is similar to the case without with out clutter removal procedure.

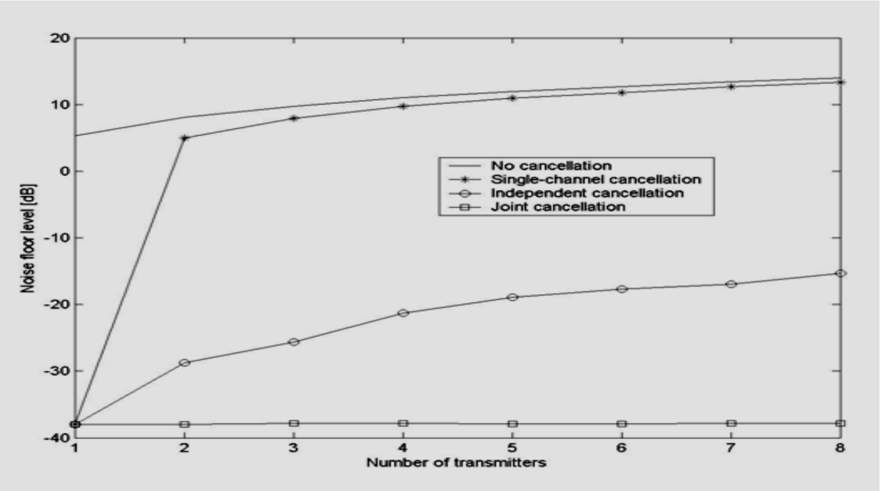

Fig.3. Noise floor level for different clutter cancellation methods versus number of transmitters

## 5. Conclusions

The presented algorithm has been intensively tested on simulated data for different multistatic and MIMO configurations of the noise radar. The random Gaussian band limited noise, frequency shifted to the carrier frequency band, has been used as the transmitted signal. One-dimensional filters were not able to remove ground clutter completely. The application of the multidimensional lattice filter allows us for better clutter cancellation and detection of weaker targets. The presences of strong targets echoes have also a great impact on the radar sensitivity. This problem has been pointed out in [10]. Our work shows that methods described in [10, 11] can be extended to multistatic and MIMO noise radars.

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