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The Barrier of Decomposition Method

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Abstract

By studying Adomian decomposition method as well as its adaptabilities and capabilities, and by providing experiments, this article attempts to examine the barriers that arise in the course of calculating the above algorithm and that deflect us from the true solution.

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1. Introduction

Unfortunately, in providing or developing a method, we only foucs on its strong points and ignore the barriers that lead us away from the true solution. As numerical methods are meant for problems whose true solutions are hard to obtain, we should have a logical analysis of the method for a variety of problems so that there is greater agreement between the facts and the solutions.

The extensive capabilities of Adomian decomposition method for various problems have been dealth with in a plethora of articles. Its advantage over power series in solving ordinary differential equations [1], solving linear and nonlinear differential equations of various orders [2-4], solving fuzzy equations [5], special cases of the method in solving special equations [6] as well as its advantage over other numerical methods in solving diverse integral equations [7] are only a few of its capabilities.

2. Adomian decomposition method

This method has been thoroughly analyzed in ordinary cases, in double decomposition, and in Adomian polynomials for solving linear and nonlinear derivative equations with initial and boundary conditions [8-10].
It should be noted that in solving the equation in the form of Lu+Nu=g, where L is a linear operator and N is a nonlinear operator, we will obtain, through Adomian decomposition method, a solution [10] in the form

$$u = \sum_{n=0}^{\infty} u_n \tag{1}$$

In the above relation, if we take the first m terms, we will have

$$\Phi_m = \sum_{n=0}^{m-1} u_n \tag{2}$$

In which, as $m\to\infty$, Φ_m approaches u; this means that Φ_m is a suitable approximation of u.
Considering the process of Adomian decomposition method, series terms (1) rapidly approximate zero, because the quotient $\frac{1}{(mn)!}$, has been the coefficient of calculation derived from integration, where m is the number of terms and n is the order of the operator derivation, approaches zero. Therefore, our solution should have a rapid convergence.

Now, let us consider some experiments that illustrate the inefficiency of the method for some equations.

3. Numerical experiments

Problem 1: Consider the nonlinear boundary- value of

$$u'' = 4 + x^3/4 - uu'/8 \qquad 1 \le x \le 3 \tag{3}$$

$$u(1)=17 \ , \ u(3)=43/3$$

where the true solution is $u(x) = x^2 + 16/x$.

Solution:

By transforming equation (3) and by assuming $L=d^2/dx^2$ and the effect of L^{-1} on the above equation, we have

$$L^{-1}Lu = L^{-1}(4 + x^3/4) - L^{-1}(uu')/8 \tag{4}$$

By solving the left side of the equation (4), which is an indefinite integral, and by substituting the right values in the equation, we will have

$$u = A + Bx + L^{-1}(4 + x^3/4) - L^{-1}(uu')/8 \qquad (5)$$

where A and B are integral constants. We obtain the following assumptions from equation (5)

$$u_0 = A + Bx + L^{-1}(4 + x^3/4)$$
$$u_{n+1} = -L^{-1}(u_n u'_n)/8 \qquad n \geq 0 \qquad (6)$$

From the decomposition of relation (6), we will have

$$u_0 = A + Bx + L^{-1}(4 + x^3/4) = A + Bx + 2x^2 + (1/80)x^5$$

$$u_1 = -L^{-1}(u_0 u'_0)/8 = -(4/7)x^7 - (7/400)x^{10} - (69/332800)x^{13} + A - ABx^4 - (1/498073600)x^{19} - (7/6553600)x^{16} + Bx - (1/2048000)Bx^{15} - Bx^6 - (4/5)Ax^5 - (3/160)Ax^8 - (1/192)A^2x^6 - (1/2293760)Ax^{14} - (3/16)B^3x^4 - (11/480)Bx^9 - (19/102400)Bx^{12} - (7/102400)ABx^{10} - (1/16)AB^3x^2 - (11/3840)AB^2x^6 - (1/640)A^2Bx^5 - (1/4)A^2Bx^2 - (5/12)AB^2x^3 - (1/36864)A^2x^9 - (9/56320)Ax^{11} - (13/20)B^2x^5 - (1/3)A^2x^3 - (3/2240)B^3x^7 - (1/48)B^4x^3 - (5/512)B^2x^8 - (23/563200)B^2x^{11} - (17/1120)ABx^7 \qquad (7)$$

By adding up relations (7) and by applying the boundary conditions, we will obtain constants A and B. Now, Table I below presents the comparison of true and approximate solutions.

Table I: Comparison of true and approximate solutions to the Problem 1 in Adomian decomposition method.

x	u(exact)	u(approx.)	error
1.0	17.00000000	17.00000000	0.00E+00
1.2	14.77333333	14.71848183	5.49E-02
1.4	13.38857143	13.06315721	3.25E-01
1.6	12.56000000	11.94917101	6.11E-01
1.8	12.12888889	11.31234752	8.17E-01
2.0	12.00000000	11.09710093	9.03E-01
2.2	12.11272727	11.24981400	8.63E-01
2.4	12.42666667	11.71509814	7.12E-01
2.6	12.91384615	12.43285836	4.81E-01
2.8	13.55428571	13.33416315	2.20E-01
3.0	14.33333333	14.33333263	7.00E-07

We obtained the above Table for m=2. Now, if m is of a larger value, we should have a better approximation as is indicated by relation (2). This, however, is not the case in practice since for m=5 we will have:

x	u(exact)	u(approx.)	error
1.0	17.00000000	16.99999391	6.090000E-06
1.2	14.77333333	2601.41863400	2.586645E+03
1.4	13.38857143	5186.46348900	5.173075E+03
1.6	12.56000000	7772.04985600	7.759490E+03
1.8	12.12888889	10358.11321000	1.034598E+04
2.0	12.00000000	12944.52680000	1.293253E+04
2.2	12.11272727	15529.65200000	1.551754E+04
2.4	12.42666667	18090.62900000	1.807820E+04
2.6	12.91384615	20376.14000000	2.036323E+04
2.8	13.55428571	20125.60000000	2.011205E+04
3.0	14.33333333	14.00000000	3.333333E-01

Problem 2: Consider the integral equation below

$$u(x) = e^{-x} - \int_0^x x e^t u(t) dt$$

The true solution to this equation is indeed $u = e^{-x} - \frac{x}{2}$.

Solution: Solving the above equation through Adomian decomposition method in n=4 stages, yields the results listed in the following Table.

Table II: Comparison of true and approximate solutions to the Problem 2 in Adomian decomposition method.

x	u(exact)	u(approx.)	error
0.0	1.00000000	1.00000000	.00E+00
0.1	0.85483742	0.89487328	4.00E-02
0.2	0.71873075	0.77934517	6.06E-02
0.3	0.59081822	0.65413074	6.33E-02
0.4	0.47032005	0.52139491	5.11E-02
0.5	0.35653066	0.38490961	2.84E-02
0.6	0.24881164	0.25002656	1.21E-03
0.7	0.14658530	0.12341582	2.32E-02
0.8	0.04932896	0.01266456	3.67E-02
0.9	-0.04343030	-0.07378350	3.04E-02
1.0	-0.13212060	-0.12499500	7.13E-03

Solving the above equation through Adomian decomposition method in n=10 stages, we have seen that there is no improvement in the results which is listed in the following Table.

x	u(exact)	u(approx.)	error
0.0	1.00000000	1.00000000	0.00E+00
0.1	0.85483742	0.89487328	4.00E-02
0.2	0.71873075	0.77934517	6.06E-02
0.3	0.59081822	0.65413073	6.33E-02
0.4	0.47032005	0.52139456	5.11E-02
0.5	0.35653066	0.38490538	2.84E-02
0.6	0.24881164	0.24999199	1.18E-03
0.7	0.14658530	0.1232035	2.34E-02
0.8	0.04932896	0.01160759	3.77E-02
0.9	-0.04343030	-0.0782580	3.48E-02
1.0	-0.13212060	-0.1416362	9.52E-03

4. Conclusions

Considering the above discussion and experiments, we can say that the Adomian decomposition method seems to suffer an essential barrier in dealing with some derivative and integral equations. Because the solutions to most derivative equations of various orders cannot be obtained easily so as to improve them, so a serious approach is necessary to provide for convergence. We believe that since this method is in some way identical to Taylor series method around u_0 and since Taylor series is convergent for short intervals around zero, so the computation errors increase as we get farther away from the origin, which means that the series becomes more divergent. Therefore, in employing Adomian decomposition method we should consider the necessity for convergence. The convergence of this method was proved in [11] by Cherrault. In addition, our observations [10], indicate that Adomian decomposition method, despite its greater stability in solving equations, suffers certain barriers. It can also be concluded that due to its affinity to Taylor series, Adomian decomposition method has flaws of its own and it is seen that its stability is lower than other numerical methods (for instance: Runge - Kutta method; Finite difference scheme and Collocation method).
At the end, we would like to emphasize that, in a research paper; Golberg [12] showed for linear operator equations, Adomian decomposition method is equivalent to the classical method of successive approximations. Hence, in contrast to previous claims, it is no more rapidly convergent than previous methods and may in fact be divergent. According to the recent aforesaid argument, which manifest the application of Adomian decomposition method to linear equations of the second kind, split equation of the first kind and nonlinear equations [13-16], our testimony of the inefficiency which is illustrated in section 3 of the current paper is not far from the reality.

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