On the Solution of Fuzzy Multiobjective Integer Linear Fractional Programming Problem

Omar M. Saad 1, Mohamed Sh. Biltagy 2 and Tamer B. Farag 3

1 Department of Mathematics, Faculty of Science, Helwan University, Cairo, Egypt
omarsd@gmail.com

2 Department of Computer Science, Modern Academy, Maadi, Cairo, Egypt

3 Department of Basic Sciences, Modern Academy, Maadi, Cairo, Egypt

Abstract

In this paper a suggested algorithm to solve fuzzy multiobjective integer linear fractional programming problem (FMOILFP) is described. The basic idea of the computational phase of the algorithm is based mainly upon a modified Isbell-Marlow method together with the branch and bound technique. The fuzzy coefficients are in the right-hand side of the constraint functions and can be characterized by fuzzy numbers via a trapezoidal membership function. In addition; an illustrative example is included to demonstrate the correctness of the proposed solution algorithm.

Keywords: Multiobjective Fractional programming; Fuzzy programming; Integer programming

1. Introduction

Fractional programming has attracted the attention of many researchers in the past. The main reason for interest in fractional programming stems from the fact that programming models could better fit the real problems if we consider optimization of ratio between the physical and / or economic quantities. Literature survey reveals wide applications of fractional programming in different areas ranging from engineering to economics. For comprehensive review of the work in this filed, we refer to [9].
The mathematical optimization problems with a goal function that is a ratio of two linear functions have many applications: in finance (corporate planning, bank balance sheet management), marine transportation, water resources, health care, and so forth. Indeed, in such situations, it is often a question of optimize a ratio debt / equity, output / employee, actual cost / standard cost, profit / cost, inventory / sales, risk-assets / capital, student / cost, doctor / patient, and so on subject to some constraints [12]. In addition, if the constraints are linear, we obtain the linear fractional programming problem (LFP) problem.

Different approaches have been proposed in the literature to solve both continuous (LFP) and integer linear fractional programming (ILFP) problems. These can be divided in studies that have developed solution methods (e.g., [4,10]) and those which concentrated on applications (e.g., [11,3]).

Integer linear fractional programming problem with multiple objectives (MOILFP) is an important field of research and has not received as much attention as did multiple objective linear fractional programming. In [2], an exact method for discrete multiobjective linear fractional optimization has been developed using a branch and cut algorithm to generate the whole integer efficient solutions of the MOILFP problem.

The objective function and the constraints in an optimization problem involve many parameters with values assigned by decision makers. However, the precise values of the parameters are often difficult to identify due to ambiguous information supplied by decision makers. In this case, it is more appropriate to interpret the ambiguous coefficients and the vague aspiration parameters as fuzzy numerical data that can be represented by fuzzy numbers.

A suggested program with fuzzy linear fractional objective and integer decision variables (FILFP) has been considered in [7]. The fuzzy coefficients were involved in the numerator of the linear objective function and have been characterized by trapezoidal fuzzy numbers, where an algorithm has been outlined to solve the FILFP.

In this paper, an attempt is made to extend the study in [7] to cover the investigation of fuzzy multiobjective integer linear fractional programming problem with fuzzy parameters in the constraints (FMOILFP). After an overview of some preliminary concepts regarding fuzzy numbers, we introduce the concept of $\alpha$-efficient solution of the problem under consideration. The original fuzzy programming problem (FMOILFP) is transformed into nonfuzzy version based on the specified $\alpha$ - level sets of the fuzzy numbers. An algorithm is developed to obtain the $\alpha$ -efficient solution to the problem (FMOILFP), followed by an example to illustrate the methodology. Finally, we conclude the paper with some discussion.
2. Problem Formulation

The purpose of this paper is to develop a method for solving the following fuzzy multiobjective integer linear fractional programming problem (FMOILFP):

\[
\begin{align*}
\max z_1(x) &= \frac{c^1 x + \alpha^1}{d^1 x + \beta^1} \\
\max z_2(x) &= \frac{c^2 x + \alpha^2}{d^2 x + \beta^2} \\
&\vdots \\
\max z_k(x) &= \frac{c^k x + \alpha^k}{d^k x + \beta^k}
\end{align*}
\]

subject to

\[x \in M(\tilde{b})\]

where \(k \geq 2, c^r, d^r\) are \(1 \times n\) vectors; \(\alpha^r, \beta^r\) are scalars for each \(r \in \{1, 2, \ldots, k\}\). In addition,

\[M(\tilde{b}) = \{x \in \mathbb{R}^n \mid Ax \leq \tilde{b}, x \geq 0 \text{ and integer}\},\]

and moreover \(A\) is an \(m \times n\) real matrix, \(\tilde{b} \in \mathbb{R}^m\) is vector of fuzzy parameters and we suppose that they are given by fuzzy numbers; estimated from the information provided by the decision maker. Throughout this paper we assume that \(M(\tilde{b})\) is nonempty, compact polyhedron set in \(\mathbb{R}^n\) and \(d^r x + \beta^r > 0\) over \(M(\tilde{b})\) for all \(r \in \{1, 2, \ldots, k\}\).

A fuzzy number is defined differently by many authors. The most frequently used definition belongs to a trapezoidal fuzzy type as follows:

**Definition 1 [8]**

It is appropriate to recall that a real fuzzy number \(\tilde{b}\) is a continuous fuzzy subset from the real line \(\mathbb{R}\) whose membership function \(\mu_{\tilde{b}}(b)\) is defined by:

1. A continuous mapping from \(\mathbb{R}\) to the closed interval \([0,1]\),
2. \(\mu_{\tilde{b}}(b) = 0\) for all \(b \in (-\infty, b_1]\),
3. \(\mu_{\tilde{b}}(b)\) is strictly increasing on \([b_1, b_2]\),
4. \(\mu_{\tilde{b}}(b) = 1\) for all \(b \in [b_2, b_3]\),
5. \(\mu_{\tilde{b}}(b)\) is strictly decreasing on \([b_3, b_4]\),
6. \( \mu_b^+(b) = 0 \) for all \( [b_3, +\infty) \),

Figure 1. illustrates the graph of a possible shape of a membership function of a fuzzy number \( \tilde{b} \).

Here, the vector of fuzzy parameters \( \tilde{b} \) involved in problem (FMOILFP) is a vector of fuzzy numbers whose membership function is denoted by \( \mu_b^+(b) \).

In the following we give the definition of the \( \alpha \)–level set or \( \alpha \)-cut of the fuzzy vector \( \tilde{b} = [\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_m] \).

**Definition 2 [8]**

The \( \alpha \)–level set of the vector of fuzzy parameters \( \tilde{b} \) in the problem (FMOILFP) is defined as the ordinary set \( L_\alpha(\tilde{b}) \) for which the degree of its membership function exceeds the level \( \alpha \in [0,1] \), where

\[
L_\alpha(\tilde{b}) = \left\{ b \in R^m / \mu_{\tilde{b}}(b) \geq \alpha \right\}.
\]

For a certain degree \( \alpha = \alpha^* = [0,1] \), estimated by the decision maker. The problem (FMOILFP) can be understand as the following nonfuzzy \( \alpha \)–multiobjective integer linear fractional programming problem (\( \alpha \)–MOILFP).
Multiobjective fractional programming

\[
(\alpha - \text{MOILFP}) : \begin{cases} 
\max z_1(x) = \frac{c^1 x + \alpha^1}{d^1 x + \beta^1} \\
\max z_2(x) = \frac{c^2 x + \alpha^2}{d^2 x + \beta^2} \\
\quad \vdots \\
\max z_k(x) = \frac{c^k x + \alpha^k}{d^k x + \beta^k}
\end{cases}
\]

Subject to

\[x \in M(b)\]

where

\[M(b) = \{x \in \mathbb{R}^n / Ax \leq b, x \geq 0 \text{ and integer, } b \in L_\alpha(\tilde{b})\}\]

If should be emphasized here in the \((\alpha - \text{MOILFP})\) above that the vector of parameters \(b\) is treated as a vector of decision variables rather than constants.

Depending on the basic definition of the \(\alpha\)-level set of the fuzzy numbers, we introduce the concept of the \(\alpha\)-efficient solution to the \((\alpha - \text{MOILFP})\) in the following definition.

**Definition 3 [8]**

A point \(x^* \in M(b^*)\) is said to be an \(\alpha\)-efficient solution to problem \((\alpha - \text{MOILFP})\), if and only if there does not exist another \(x \in M(b), b \in L_\alpha(\tilde{b})\) such that \(z_r(x) \geq z_r(x^*)\) \((r = 1, 2, \ldots, k)\) with strict inequality holding for at least one \(r\), where the corresponding values of parameters \(b^*\) are called \(\alpha\)-level optimal parameters.

Throughout this paper, a membership function of the fuzzy vector \(\sim b\) in the following form will be elicited:

\[
\mu_{\sim b}(b) = \begin{cases} 
0, & b \leq b_1, \\
1 - \left(\frac{b - b_2}{b_1 - b_2}\right)^2, & b_1 \leq b \leq b_2, \\
1, & b_2 \leq b \leq b_3, \\
1 - \left(\frac{b - b_3}{b_4 - b_3}\right)^2, & b_3 \leq b \leq b_4, \\
0, & \text{otherwise.}
\end{cases}
\]
Now, before we go any further, problem \((\alpha - \text{MOILFP})\) can be rewritten as follows:

\[
\begin{align*}
\text{(\(\alpha - \text{MOILFP}\))} : \\
&\max z_1(x) = \frac{c^1 x + \alpha^1}{d^1 x + \beta^1} \\
&\max z_2(x) = \frac{c^2 x + \alpha^2}{d^2 x + \beta^2} \\
&\vdots \\
&\max z_k(x) = \frac{c^k x + \alpha^k}{d^k x + \beta^k}
\end{align*}
\]

Subject to
\[
\begin{align*}
x \in M(b) &= \left\{ x \in \mathbb{R}^n / Ax \leq b, l_i^{(0)} \leq b_i \leq L_i^{(0)}, (i = 1,2,...,m) \right\} \\
x \geq 0, \text{ and integer,}
\end{align*}
\]

Note that the constraint \(b \in L_\alpha \left( \frac{b}{\alpha} \right)\) in problem \((\alpha - \text{MOILFP})\) stated above has been replaced by the equivalent one \(l_i^{(0)} \leq b_i \leq L_i^{(0)}, (i = 1,2,...,m)\) where \(l_i^{(0)}\) and \(L_i^{(0)}\) are the lower and the upper bounds on the variables \(b_i, (i = 1,2,...,m)\).

Now it can be observed from the nature of problem \((\alpha - \text{MOILFP})\) that a suitable scalarization technique for treating such problem is to use the \(\varepsilon -\)constraint method [1]. For this purpose, we consider the following integer linear fractional programming problem with a single-objective function as:

\[
P_s(\varepsilon) : \max z_s(x) = \frac{c^s x + \alpha^s}{d^s x + \beta^s},
\]

subject to
\[
z_r(x) = \frac{c^r x + \alpha^r}{d^r x + \beta^r} \geq \varepsilon_r, \quad r = 1,2,3,...,k; r \neq s,
\]
\[
Ax \leq b,
\]
\[
l_i^{(0)} \leq b_i \leq L_i^{(0)}, (i = 1,2,...,m),
\]
\[
x \geq 0 \text{ and integer.}
\]
where \( s \in \{1,2,3,...,k\} \) and can be taken arbitrary and it should be stated here that an \( \alpha \)-efficient solution \( x^* \) for problem \((\alpha-MOILFP)\) can be found by solving single-objective problem \( P_s(\varepsilon) \) and this can be done when the maximum allowable levels \((\varepsilon_1, \varepsilon_2,...,\varepsilon_{s-1},\varepsilon_{s+1},...,\varepsilon_k)\) for the \((k-1)\)-objectives \((z_1, z_2,...,z_{s-1},z_{s+1},...,z_k)\) are determined in the feasible region of solutions of problem \( P_s(\varepsilon) \).

It is clear from [1] that a systematic variation of \( \varepsilon_i \)'s will yield a set of \( \alpha \)-efficient solutions. On the other hand, the resulting problem \( P_s(\varepsilon) \) can be solved using a certain vector of parameters \( \varepsilon = \varepsilon^* \) using a modification of Isbell-Marlow method described in [6] together with the branch-and-bound technique [5].

If \( x^* \) is a unique optimal integer solution of problem \( P_s(\varepsilon^*) \), then \( x^* \) becomes an \( \alpha \)-efficient solution to problem \((\alpha-MOILFP)\) with \( b^* \in R^m \) the \( \alpha \)-level optimal parameters.

## 3. Modification of Isbell-Marlow Method

In this section, a modification is carried out on Isbell-Marlow method [6] to solve the following single-objective integer linear fractional programming problem (ILFP):

\[
\text{ILFP: } \max_{x \in S} \left\{ f(x) = \frac{c^T x + c_0}{d^T x + d_0} \right\}
\]

subject to

\[
s = \{ x \in R^n : Ax \leq b, x \geq 0 \text{ and integer} \}
\]

where \( c, d \in R^n, c_0, d_0 \in R \), \( A \) is \( m \times n \) real matrix, \( b \in R^m \), \( d \neq 0 \), \( d_0 \neq 0 \), \( d^T x + d_0 > 0 \) for all \( x \in s \), and rank \( A = m < n \).

The first sequential method for solving the linear fractional problem, ignoring the integer requirement in problem ILFP above, was suggested by Isbell and Marlow in [6]. The basic idea of this method, which works on a compact feasible region, is to generate a finite sequence of vertices \( x^1,...,x^h \), starting from an initial basic feasible solution \( x^0 \) such that

\[
\max_{x \in S_i} f(x) = f(x^0) < f(x^1) < ... < f(x^h) = \max_{x \in S_f} f(x),
\]

where \( S_f = \{ x \in R^n : Ax \leq b, x \geq 0 \} \). Each vertex \( x^{i+1}, i = 0,...,h - 1 \) is an optimal solution for the linear problem:

\[
\max_{x \in S_f} \phi'(x) = (c^T x + c_0) - f(x)\left( d^T x + d_0 \right)
\]
Isbell-Marlow method is based on the theoretical property stated in the following theorem and the proof can be found in [6]:

**Theorem [6]:**

\[ x^* \] is optimal for problem LFP iff it is optimal for the linear problem:

\[
\max_{x \in \mathcal{S}_I} \varphi^*(x) = (c'x + c_0) - f(x^*) (d'x + d_0)
\]

The modified algorithm to solve the integer linear fractional programming problem ILFP formulated above can be summarized in the following steps:

**Algorithm I:**

**Step 0:** Initialize \( i = 0 \). Find a basic feasible integer solution \( x^i = x^0 \) for the problem ILFP.

**Step 1:** Solve the integer linear problem:

\[
\max_{x \in \mathcal{S}} \varphi'(x) = (c'x + c_0) - f(x^i) (d'x + d_0)
\]

to obtain the optimal solution \( x^{i+1} \).

**Step 2:** If \( x^{i+1} \) is an integer point then go to step 3. Otherwise use the Branch and Bound method [5] to satisfy the integrality condition on \( x^{i+1} \).

**Step 3:** If \( \varphi'(x^{i+1}) = 0 \) then, stop and \( x^{i+1} \) is the optimal integer solution for the problem ILFP. Otherwise, set \( i = i + 1 \) and go to step 1.

4. **A Proposed Solution Algorithm for solving problem \((\alpha - \text{MOILFP})\)**

The algorithm generating an \( \alpha \)-efficient solution \( x^* \) to problem \((\alpha - \text{MOILFP})\) is presented in the following steps:

**Algorithm II:**

**Step 1:** Start with an initial \( \alpha \)-level set with \( \alpha = \alpha^* = 0 \).

**Step 2:** Determine points \( (b_1, b_2, \ldots, b_m) \) for the vector of fuzzy parameters \( \tilde{b} \) in problem \((\alpha - \text{MOILFP})\) to elicit a membership function \( \mu_{\tilde{b}}(b) \) satisfying the assumptions (1)-(6) in Definition 1.

**Step 3:** Convert problem \((\alpha - \text{MOILFP})\) into its non fuzzy version.

**Step 4:** Formulate the integer linear fractional problem with a single-objective function \( P_{\varepsilon}(\varepsilon) \).

**Step 5:** Solve \( k \)-individual integer linear fractional problems \( P_r (r = 1, 2, \ldots, k) \) using the modified Isbell-Marlow Method (Alg. I).

**Step 6:** Construct the payoff table and determine \( n_r, M_r \) (the smallest and the largest numbers in \( r \)th column in the payoff table).
Step 7: Determine the $\varepsilon_r$’s from the formula:

$$\varepsilon_r = n_r + \frac{t}{N - 1} (M_r - n_r), r = 1, 2, ..., k; r \neq s.$$ 

where $t$ is the number of all partitions of the interval $[n_r, M_r]$.

Step 8: Find $D = \{ \varepsilon \in R^{k-1} | n_r \leq \varepsilon_r \leq M_r, r = 1, 2, ..., k; r \neq s \}.$

Step 9: Choose $\varepsilon_r^* \in D$ and solve the integer linear fractional problem $P_r(\varepsilon^*)$ using Alg. I to find its optimal integer solution $x^*$ with the corresponding $\alpha$-level optimal parameters $b^* \in R^m$.

Step 10: Set $\alpha = (\alpha^* + \text{step}) \in [0,1]$ and go to step 2.

Step 11: Repeat again the above procedure until the interval $[0,1]$ is fully exhausted. Then, stop.

5. An Illustrative Example

In this section we provide a numerical illustration of algorithm II. The example is adapted from one appearing in Chergui and Moulaï [2] and the LINGO software package is used in the computational process.

The problem to be solved here is the following multiobjective integer linear fractional program involving fuzzy vector of parameters $\tilde{b}$ in the right-hand side of the constraints:

$$\begin{align*}
&\text{max } z_1(x) = \frac{x_1 - 4}{-x_2 + 3}, \\
&\text{max } z_2(x) = \frac{-x_1 + 4}{x_2 + 1}, \\
&\text{max } z_3(x) = -x_1 + x_2
\end{align*}$$

Subject to

$$\begin{align*}
-x_1 + 4x_2 &\leq \tilde{b}_1 \\
2x_1 - x_2 &\leq \tilde{b}_2 \\
x_1, x_2 &\geq 0, \text{ and integers}
\end{align*}$$
Set $\alpha = \alpha^* = 0$ with the following membership functions to convert the above fuzzy problem (FMOILFP) into its non fuzzy version.

\[
\mu_{b_1}(b_1) = \begin{cases} 
0, & b_1 \leq a_1, \\
1 - \frac{(b_1 - a_2)^2}{(a_1 - a_2)^2}, & a_1 \leq b_1 \leq a_2, \\
1 - \frac{(b_1 - a_3)^2}{(a_4 - a_3)^2}, & a_3 \leq b_1 \leq a_4, \\
0, & \text{otherwise.}
\end{cases}
\]

\[
\mu_{b_2}(b_2) = \begin{cases} 
0, & b_2 \leq a_1, \\
1 - \frac{(b_2 - a_2)^2}{(a_1 - a_2)^2}, & a_1 \leq b_2 \leq a_2, \\
1 - \frac{(b_2 - a_3)^2}{(a_4 - a_3)^2}, & a_3 \leq b_2 \leq a_4, \\
0, & \text{otherwise.}
\end{cases}
\]

Let also the fuzzy parameters $\bar{b}_1, \bar{b}_2$ are given by the following fuzzy numbers listed in the table below:

<table>
<thead>
<tr>
<th></th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1$</td>
<td>0</td>
<td>0.2</td>
<td>0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>$b_2$</td>
<td>8</td>
<td>8.2</td>
<td>8.3</td>
<td>8.6</td>
</tr>
</tbody>
</table>

It is easy to get: $0 \leq b_1 \leq 0.6, \ 8 \leq b_2 \leq 8.6$

Now, the fuzzy problem (FMOILFP) is converted to the non fuzzy version ($\alpha$–MOILFP) as in the following form:
\[
\begin{align*}
\left( \alpha - \text{MOILFP} \right): & \quad \max z_1(x) = \frac{x_1 - 4}{-x_2 + 3}, \\
& \quad \max z_2(x) = \frac{-x_1 + 4}{x_2 + 1}, \\
& \quad \max z_3(x) = -x_1 + x_2 \\
\text{Subject to} & \\
& -x_1 + 4x_2 \leq b_1 \\
& 2x_1 - x_2 \leq b_2 \\
& 0 \leq b_1 \leq 0.6 \\
& 8 \leq b_2 \leq 8.6 \\
& x_1, x_2 \geq 0, \text{ and integers}
\end{align*}
\]

First, we solve the problem:

\[
p_1 : \quad \max z_1(x) = \frac{x_1 - 4}{-x_2 + 3}
\]

\text{Subject to}

\[
\begin{align*}
-x_1 + 4x_2 & \leq b_1 \\
2x_1 - x_2 & \leq b_2 \\
0 & \leq b_1 \leq 0.6 \\
8 & \leq b_2 \leq 8.6 \\
x_1, x_2 & \geq 0, \text{ and integers}
\end{align*}
\]

using the modified Isbell-Marlow algorithm (Alg. I) described in section 3. Letting the initial basic integer solution \( x^0 = (0,0) \). Therefore, we have:

\[
\begin{align*}
\max \varphi^0(x) &= [(x_1 - 4) - z_1(x^0)(-x_2 + 3)] \\
&= x_1 - \frac{4}{3}x_2 \\
\text{Subject to} & \\
& -x_1 + 4x_2 \leq b_1 \\
& 2x_1 - x_2 \leq b_2 \\
& 0 \leq b_1 \leq 0.6 \\
& 8 \leq b_2 \leq 8.6 \\
& x_1, x_2 \geq 0, \text{ and integers}
\end{align*}
\]
This non fuzzy problem has been solved using the branch- and- bound method [5] and the optimal integer solution has been found \((x_1 = 4, x_2 = 0)\) with \(\phi_0(x) = 4\). Since \(\phi(x) \neq 0\), then use \(x^1 = (4,0)\) as an initial basic integer solution to solve:

\[
\max \phi^1(x) = [(x_1 - 4) - (x_1(x_1 - 3))]
= x_1 - 4
\]

Subject to
- \(-x_1 + 4x_2 \leq b_1\)
- \(2x_1 - x_2 \leq b_2\)
- \(0 \leq b_1 \leq 0.6\)
- \(8 \leq b_2 \leq 8.6\)
- \(x_1, x_2 \geq 0\), and integers

Again, using the same procedure, the optimal integer solution for problem \(P_1\) is the point \((x_1 = 4, x_2 = 0)\) with \(\phi^1(x) = 0\). Since \(\phi^1(x) = 0\) then, stop.

Now, we proceed to solve problem:

\[
p_2 : \quad \max z_2(x) = \frac{-x_1 + 4}{x_2 + 1}
\]

Subject to
- \(-x_1 + 4x_2 \leq b_1\)
- \(2x_1 - x_2 \leq b_2\)
- \(0 \leq b_1 \leq 0.6\)
- \(8 \leq b_2 \leq 8.6\)
- \(x_1, x_2 \geq 0\), and integers

The initial basic integer solution \(x^0 = (0,0)\) is used to solve:

\[
\max \phi^0(x) = [(x_1 - 4) - z_2(x^0)(x_2 + 1)]
= -x_1 - 4x_2
\]

Subject to
- \(-x_1 + 4x_2 \leq b_1\)
- \(2x_1 - x_2 \leq b_2\)
- \(0 \leq b_1 \leq 0.6\)
- \(8 \leq b_2 \leq 8.6\)
- \(x_1, x_2 \geq 0\), and integers
The optimal integer solution for problem $P_2$ has been found $(x_1 = 0, x_2 = 0)$ with $\varphi^0(x) = 0$ and since $\varphi^0(x) = 0$ then, stop.

Finally, we solve:

$$P_3 : \quad \text{max} \quad z_3(x) = -x_1 + x_2$$

Subject to

$$- x_1 + 4 x_2 \leq b_1$$
$$2 x_1 - x_2 \leq b_2$$
$$0 \leq b_1 \leq 0.6$$
$$8 \leq b_2 \leq 8.6$$
$$x_1, x_2 \geq 0, \text{ and integers}$$

The optimal integer solution for problem $P_3$ is $(x_1 = 0, x_2 = 0)$ and since $x_1, x_2$ are integers, then stop.

Now, the $\varepsilon$–constraint method [1] is used to convert the non fuzzy multiobjective integer linear fractional problem $(\alpha-MOILFP)$ to a single- objective integer linear fractional problem and the payoff table is given below.

<table>
<thead>
<tr>
<th></th>
<th>$z_1 = \frac{x_1 - 4}{-x_2 + 3}$</th>
<th>$z_2 = \frac{-x_1 + 4}{x_2 + 1}$</th>
<th>$z_3 = -x_1 + x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1 (4,0)$</td>
<td>0</td>
<td>0</td>
<td>-4</td>
</tr>
<tr>
<td>$x_2 (0,0)$</td>
<td>-4</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>$x_1 (0,0)$</td>
<td>$\frac{4}{3}$</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Max</td>
<td>0</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Min</td>
<td>$\frac{4}{3}$</td>
<td>0</td>
<td>-4</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>$-\frac{4}{3} \leq \varepsilon_1 \leq 0$</td>
<td>$0 \leq \varepsilon_2 \leq 4$</td>
<td>$-4 \leq \varepsilon_3 \leq 0$</td>
</tr>
</tbody>
</table>

Let $\varepsilon_2 = 1$, $\varepsilon_3 = -3$, then we have:
\( P_1(\varepsilon^*) : \quad \max \ z_1(x) = \frac{x_1 - 4}{-x_2 + 3} \)

Subject to

\[
\begin{align*}
-x_1 + 4x_2 & \leq b_1 \\
2x_1 - x_2 & \leq b_2 \\
-x_1 + 4 \quad & \geq 1 \\
x_2 + 1 & \geq 1 \\
x_1 + x_2 & \geq -3 \\
0 & \leq b_1 \leq 0.6 \\
8 & \leq b_2 \leq 8.6 \\
x_1, x_2 & \geq 0, \text{ and integers}
\end{align*}
\]

By using the modified Isbell-Marlow algorithm (Alg. I) and letting the initial basic integer solution \( x^0 = (0,0) \), the problem will be:

\[
\begin{align*}
\max \ \omega^0(x) &= \left[ (x_1 - 4) - z_1(x^0) + x_2 + 3 \right] \\
&= x_1 - \frac{4}{3} x_2
\end{align*}
\]

Subject to

\[
\begin{align*}
-x_1 + 4x_2 & \leq b_1 \\
2x_1 - x_2 & \leq b_2 \\
x_1 + x_2 & \leq 3 \\
x_1 - x_2 & \leq 3 \\
0 & \leq b_1 \leq 0.6 \\
8 & \leq b_2 \leq 8.6 \\
x_1, x_2 & \geq 0, \text{ and integers}
\end{align*}
\]

The optimal solution is found \( (x_i = 3, x_2 = 0) \) with \( \omega^0(x) = 3 \) and again, since \( \omega^0(x) \neq 0 \), then \( x^i = (3,0) \) will be used as an initial integer solution to solve the following problem:
Multiobjective fractional programming

\[ \max \omega^1(x) = \left[ (x_1 - 4) - z_1 (x_1^2 - x_2 + 3) \right] = x_1 - \frac{1}{3}x_2 - 3 \]

Subject to
\[-x_1 + 4x_2 \leq b_1 \]
\[2x_1 - x_2 \leq b_2 \]
\[x_1 + x_2 \leq 3 \]
\[x_1 - x_2 \leq 3 \]
\[0 \leq b_1 \leq 0.6 \]
\[8 \leq b_2 \leq 8.6 \]
\[x_1, x_2 \geq 0, \text{ and integers} \]

Now, the optimal integer solution is found \((x_1 = 3, x_2 = 0)\) with \(\omega^1(x) = 0\). Since \(\omega^1(x) = 0\), Therefore, the optimal integer solution of problem \(P_1(\varepsilon^*)\) is \((3, 0)\) which is an \(\alpha\) - efficient solution to the problem \((\alpha - \text{MOILFP})\).

5. Conclusion

In the presented paper a solution algorithm has been proposed to solve fuzzy multiobjective integer linear fractional programming problem (FMOILFP). Some fuzzy concepts have been given to convert problem (FMOILFP) to a non fuzzy version and a modification of Isbell-Marlow algorithm (Alg. I) has been used to complete the solution process.

Summarizing, many aspects and general questions remain to be studied and explored in the area of fuzzy multiobjective integer linear programming. Despite the limitations, we believe that this paper is an attempt to establish underlying results which hopefully help others to answer some of these questions.

There are however several open points for future research in the area of (FMOILFP), in our opinion, to be studied. Some of these points of interest are stated in the following:

(i) An algorithm is required for solving multiobjective integer linear fractional programming problem with fuzzy parameters (a) in the objective functions and (b) in the left-hand side of the constraints.

(ii) Stability of the \(\alpha\) - efficient solutions to problem \((\alpha - \text{MOILFP})\) should be examined and investigated.

(iii) Computer codes are needed to be constructed to solve the problems recommended above.

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References


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