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Solving Constrained Flow-Shop Scheduling Problems with Three Machines

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Abstract

A new method is proposed to obtain an optimal scheduling sequence for flow-shop scheduling problems involving transportation time, break down time and weights of jobs (constrained flow-shop scheduling problems) with 3-machines. The proposed method is very simple and easy to understand and also, provides an important tool for decision makers when they design a scheduling for constrained flow-shop scheduling problems with 3 machines. The proposed method is illustrated with help of numerical examples.

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Keywords: Flow-shop scheduling problem; Transportation time; Weights of jobs; Optimal sequence.

1 Introduction

Now-a-days, the decision makers for the manufacturing plant must find a way to successfully manage resources in order to produce products in the most efficient way in the complex manufacturing setting, with multiple lines of products, each requiring many different steps and machines for completion. Also, they need to design a production schedule that promotes on-time delivery as well as minimizes the flow time of a product. Out of these concerns grew an area of studies known as the scheduling problems. In the scheduling problem, one of the central tasks in high-level synthesis is the problem of determining the order in which the operations in the behavioural description will execute. It involves solving for the optimal schedule under various objectives, different machine environments and characteristics of the jobs. The number of possible schedules of the flow-shop scheduling problem involving n jobs and m machines is $(n!)^m$. The optimal solution to the problem is to find the sequence of jobs on each machine in order to complete all the jobs on all the machines in the minimum total time provided each job is processed on machines 1, 2, 3, …, m in that order. The general flow-shop scheduling problem is NP-hard.

The scheduling problem practically depends upon three important factors namely, job transportation time which includes loading time, moving time and unloading time etc., relative importance of a job over another job and breakdown machine time (due to the failure of electric current, the non-supply of raw material or other technical interruptions) . These three factors were separately studied by many researchers [1-3, 5-8]. Chandramouli [4] proposed a heuristic algorithm for flow-shop scheduling problem with 3-machines involving transportation time, break down time and weights of jobs to find an optimal or near optimal sequence.

In this paper, we propose a new method for flow-shop scheduling problems involving transportation time, break down time and weights of jobs (constrained flow-shop scheduling problems) with 3-machines to obtain an optimal sequence. The proposed method is very simple and easy to understand and also, provides an important tool for decision makers when they design a schedule for constrained flow-shop scheduling problems with 3-macines. With the help of the numerical examples, the proposed method is illustrated.

2 Machine flow-shop problem

Consider the following constrained flow-shop problem with 3-machines which can be stated as follows:

(a) Let n - job be processed through three machines A, B and C in the order ABC.
(b) Let ‘i’ denote the job in S where S is an arbitrary sequence.
(c) All jobs are available for processing at time zero.
(d) Let each job be completed through the same production stage, that is, ABC, in other words, passing is not allowed in the flow shop.
(e) Let A_i, B_i and C_i denote the processing time of job ‘i’ on the machine A, B and C respectively.
(f) Let t_i and g_i denote the transportation time of job ‘i’ from A to B and from B to C respectively.
(g) Let job ‘i’ be assigned with a weight w_i according to its relative importance for performance in the given sequence.
(h) The performance measure studied in weighted mean flow time defined by

$$F = \frac{\sum_{i=1}^{n} w_i f_i}{\sum_{i=1}^{n} f_i},$$ where f_i is flow time of ith job.

(i) Let the break down interval (a,b) is already known to us , that is, deterministic nature. The break down interval length $b-a$ which is known.

Then, our aim is to find out the optimal sequence of jobs so as to minimize the total elapsed time.

The above stated problem (P) in the tabular form may be stated as follows:

Job	Machines with Processing times and transporting times					Weight of Jobs
i	A_i	t_i	B_i	g_i	C_i	W_i
1	A_1	t_1	B_1	g_1	C_1	W_1
2	A_2	t_2	B_2	g_2	C_2	W_2
⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	A_n	t_n	B_n	g_n	C_n	W_n

Let us assume that the problem (P) satisfies any one of the following structural conditions involving the processing time and transportation time of jobs hold.

Structural conditions:

1. $Minimum\{A_i\} \geq Maximum\{t_i + B_i\}$

or $Minimum\{C_i + g_i\} \geq Maximum\{t_i + B_i\}$.

2. $Minimum\{A_i + t_i\} \geq Maximum\{B_i\}$

or $Minimum\{C_i + g_i\} \geq Maximum\{B_i\}$.

3. $Minimum\{A_i + t_i\} \geq Maximum\{B_i + g_i\}$

or $Minimum\{C_i\} \geq Maximum\{B_i + g_i\}$.

4. $Minimum\{A_i\} \geq Maximum\{t_i + B_i + g_i\}$

or $Minimum\{C_i\} \geq Maximum\{t_i + B_i + g_i\}$.

3 New proposed method

We, now introduce a new method for finding an optimal sequence to the problem (P).

The new proposed method proceeds as follows.

Step 1 : Reduce the given problem (P) into two machines flow-shop problem by introducing two fictions machines, G and H whose machine processing times G_i and H_i, $i = 1,2,...,n$ are given below:

(a) If the structural condition (1) is satisfied,

then $G_i = A_i + t_i + B_i$ and $H_i = t_i + B_i + g_i + C_i$.

(b) If the structural condition (2) is satisfied,

then $G_i = A_i + t_i + B_i$ and $H_i = B_i + g_i + C_i$.

(c) If the structural condition (3) is satisfied,

then $G_i = A_i + t_i + B_i + g_i$ and $H_i = B_i + g_i + C_i$.

and

(d) If the structural condition (4) is satisfied,

then $G_i = A_i + t_i + B_i + g_i$ and $H_i = t_i + B_i + g_i + C_i$.

The tabular form of the reduced problem is given below:

Job	Machine with processing times		Weight of Jobs
i	G_i	H_i	W_i
1	G_1	H_1	W_1
2	G_2	H_2	W_2
⋮	⋮	⋮	⋮
n	G_n	H_n	W_n

Step 2. Compute $Minimum(G_i, H_i)$.

a. If $Minimum(G_i, H_i) = G_i$, then define $G_i' = G_i - w_i$ and $H_i' = H_i$.

b. If $Minimum(G_i, H_i) = H_i$, then define $G_i' = G_i$ and $H_i' = H_i + w_i$

Step 3. Formulate a new reduced scheduling problem involving two machines as follows:

Job	Machines with processing times	
i	G_i''	H_i''
1	$\frac{G_1'}{w_1}$	$\frac{H_1'}{w_1}$
2	$\frac{G_2'}{w_2}$	$\frac{H_2'}{w_2}$
⋮	⋮	⋮
n	$\frac{G_n'}{w_n}$	$\frac{H_n'}{w_n}$

where G_i' and H_i' are obtained from the Step 2.

Step 4. Determine the optimal sequence to the new reduced scheduling problem obtained in the Step 3. and also, the total elapsed time for the given problem (P) by using Johnson's algorithm. (If the machine processing times of a job are equal, put the job at last in the optimal sequence)
Step 5. Identify the effect of break-down interval (a,b) on different jobs.
Step 6. Modify the given problem using the new machine processing times A_i', B_i' and C_i' which are obtained from one of the following cases.
(i) If the break-down interval (a,b) has no effect on job i, at the time of processing the machines A , B and C, then $A_i' = A_i$, $B_i' = B_i$ and $C_i' = C_i$.
(ii) If the break-down interval (a,b) has affected on job i, at the time of processing the machines A, B and C , then $A_i' = A_i + (b-a)$, $B_i' = B_i + (b-a)$ and $C_i' = C_i + (b-a)$

Step 7. Using the modified scheduling problem and the optimal sequence of the given problem obtained in Step 4., determine the total elapsed time and weighted mean-flow time.

4 Numerical Examples

The proposed method is illustrated by the following examples.
Example 1: Consider the following constrained flow-shop scheduling problem of 4-jobs on 3-machines with processing times, transportation times and the weights of jobs:

Job	Machines with processing times and transporting times					Weight of Jobs
i	A_i	t_i	B_i	g_i	C_i	W_i
1	13	1	7	2	5	3
2	8	3	6	5	9	5
3	7	2	3	4	5	4
4	5	5	2	1	6	2

given that the break-down interval (a, b) = (18, 25).
Now, since $Minimum\{A_i + t_i\}$ = 9 $\geq$ 7 = $Maximum\{B_i\}$, the structural condition (2) is satisfied.
Then, using the Step 1. to the Step 4. of the proposed method, we obtain that (2,1,4,3) is an optimal sequence for the given problem.
Now, the total elapsed time for the optimal sequence (2,1,4,3) is calculated as follows:

Job	Machines with processing times and transporting times					Weight of Jobs
i	A_i	t_i	B_i	g_i	C_i	W_i
2	0-8	3	11-17	5	***22-31***	5
1	***8-21***	1	***22-29***	2	31-36	3
4	***21-26***	5	31-33	1	36-42	2
3	26-33	2	35-38	5	42-47	4

Therefore, the total elapsed time is 47 hrs.
Now, Job 2, Job1 and Job3 have been affected by the break down interval (18, 25) on the optimal sequence (2, 1, 4, 3).
Now, using the Step 6. of the proposed method, we modify the processing times for affected jobs and we obtain the following new scheduling problem in the tabular form:

Job	Machines with processing times and transporting times					Weight of Jobs
i	A_i	t_i	B_i	g_i	C_i	W_i
1	20	1	14	2	5	3
2	8	3	6	5	16	5
3	7	2	3	4	5	4
4	12	5	2	1	6	2

Now, the elapsed time for the optimal sequence (2,1,4,3) after applying the breakdown time is calculated as follows:

Job	Machines with processing times and transporting times					Weight of Jobs
i	A_i	t_i	B_i	g_i	C_i	W_i
2	0-8	3	11-17	5	22-38	5
1	8-28	1	29- 43	2	45-50	3
4	28-40	5	45-47	1	50-56	2
3	40-47	2	49-52	4	56-61	4

The mean weighted flow time $= \dfrac{38x5+(50-8)x3+(56-28)x2+(61-40)x4}{5+3+2+4} =$ 30.4.

Hence, the total elapsed time is 61 hrs and the mean weighted flow time is 30.4 hrs.

Example 2: Consider the following constrained flow-shop scheduling problem of 4-jobs on 3-machines with processing times, transportation times and the weights of jobs:

Job	Machines with processing times and transporting times					Weight of Jobs
i	A_i	t_i	B_i	g_i	C_i	W_i
1	10	2	7	2	5	3
2	8	3	6	5	9	5
3	7	1	3	4	5	4
4	5	4	2	1	6	2

given that the break-down interval (a, b) = (18, 22).
Now, since $Minimum\{A_i + t_i\} = 8 \geq 7 = Maximum\{B_i\}$, the structural condition (2) is satisfied.
Then, using the Step 1. to the Step 4. of the proposed method, we obtain that (3,2,1,4) is an optimal sequence for the given problem.
Now, the total elapsed time for the optimal sequence (3,2,1,4) is calculated as follows:

Job	Machines with processing times and transporting times					Weight of Jobs
i	A_i	t_i	B_i	g_i	C_i	W_i
3	0-7	1	8-11	4	**15-20**	4
2	7-15	3	**18-24**	5	29-38	5
1	**15-25**	2	27-34	2	38-43	3
4	25-30	4	34-36	1	43-49	2

Therefore, the total elapsed time is 49 hrs.
Now, Job 3, Job 2 and Job 1 have been affected by the break down interval (18, 22) on the optimal sequence (3,2, 1, 4).
Now, using the Step 6. of the proposed method, we modify the processing time for affected jobs and we obtain the following new scheduling problem in the tabular form:

Job	Machines with processing times and transporting times					Weight of Jobs
i	A_i	t_i	B_i	g_i	C_i	W_i
1	14	2	7	2	5	3
2	8	3	10	5	9	5
3	7	1	3	4	9	4
4	5	4	2	1	6	2

Now, the elapsed time for the optimal sequence (3,2,1,4) after applying the breakdown time is calculated as follows:

Job	Machines with processing times and transporting times					Weight of Jobs
i	A_i	t_i	B_i	g_i	C_i	W_i
3	0-7	1	8-11	4	15-24	4
2	7-15	3	18-25	5	30-39	5
1	15-29	2	31-38	2	40-45	3
4	29-34	4	38-40	1	45-51	2

The Mean weighted flow time = $\frac{24x4 + (39-9)x5 + (45-15)x3 + (51-29)x2}{5+3+2+4}$ = 27.86 hrs.

Hence the total elapsed time is 51 hrs and the mean weighted flow time is 27.86 hrs.

Note: The Example 2. can not be solved using the algorithm given in [4] since both structural conditions given in [4] are not satisfied.

5 Conclusion

The new method provides an optimal scheduling sequence for constrained flow-shop scheduling problems of 4-jobs on 3-machines. This method is very easy to understand and apply and also, will help managers in the scheduling related issues by aiding them in the decision making process and providing an optimal scheduling sequence in a simple and effective manner. Determining a best schedule for given sets of jobs can help decision makers effectively to control job flows and to provide a solution for job sequencing. We have a plan to extend the proposed method to constrained flow-shop problems with m-machines.

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