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Distance Compatible Set-Labeling Index of Graphs

Bindhu K. Thomas and K. A. Germina

Research Center & PG Department of Mathematics
Mary Matha Arts & Science College
Vemom P.O., Mananthavady - 670645, India
bindhukthomas@gmail.com
srgerminaka@gmail.com

Abstract

Distance compatible set-labeling of a graph G is an injective set-assignment $f : V(G) \rightarrow 2^X$, X a nonempty ground set, such that the corresponding induced function $f^{\oplus} : V(G) \times V(G) \rightarrow 2^X - \{\emptyset\}$, defined by $f^{\oplus}(u, v) = f(u) \oplus f(v)$ satisfies $\mid f^{\oplus}(u, v) \mid = k^f_{(u,v)}\ d(u, v)$ for all distinct $u, v \in V(G)$, where $d(u, v)$ is the distance between u and v, and $k^f_{(u,v)}$ is a constant, not necessarily an integer; G is *distance compatible set-labeled* (or, 'dcsl') *graph* if it admits a dcsl. A dcsl f of a graph G is *k-uniform dcsl* if the constants of proportionality $k^f_{(x,y)}$, $(x, y) \in V(G) \times V(G)$ are all equal to k; G itself is a *k-uniform dcsl graph* if it admits a *k-uniform dcsl*. We define the dcsl index δ_d of graph G as the minimum cardinality of the ground set X such that G admits a dcsl. In this paper we calculate the 1-uniform dcsl index of some classes of graphs.

Keywords: k-uniform distance compatible set-labeling of graphs, 1-uniform distance compatible set-labeling, 1-uniform dcsl index

1 Introduction

For all terminology and notation which are not defined in this paper, we refer the reader to F. Harary. Unless mentioned otherwise, all the graphs considered in this paper are finite, simple and without self-loops.

B. D. Acharya et.al [2] introduced the notion of distance-compatible set-labeling (dcsl) of a graph G as an injective set-assignment $f : V \rightarrow 2^X$, X being a nonempty 'ground set', such that the corresponding induced function $f^{\oplus} : V(G) \times V(G) \rightarrow 2^X - \emptyset$, defined by $f^{\oplus}(uv) = (f(u) - f(v)) \cup (f(v) - f(u))$

satisfies $| f^{\oplus}(u,v) |= k^f_{(u,v)} d(u,v)$ for all distinct $u, v \in V(G)$, where $d(u,v)$ is the distance between u and v and $k_{(u,v)}$ is a constant, not necessarily an integer.
Graham and Pollak [4] proposed assigning addresses in such a way that the distance (number of edges in a shortest path) between two vertices can quickly be determined by comparing their addresses. Then, when a packet reaches a vertex v, en route to a vertex v, it 'knows' the distance $d(u,v)$ to its destination. If then proceeds to the first vertex w that it finds, among those adjacent to u, for which $d(w,v) = d(u,v) - 1$, and thereby guaranteed a minimal-length path to its destination. Assigning all edges of the network to be capable of carrying packets in either direction, and addresses are strings $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_t)$ of 0's and 1's and $*$'s with distance defined as $d(a,b) =| \{k : \{a_k, b_k\} = \{0,1\} |$. If $M = \{m_{ij}\}$ is any $n \times n$ matrix then an addressing of M of length t is an $n \times t$ matrix A with entries from {0, 1, *} such that
$m_{ij} =| \{k : a_{ik} = 0 \textit{ and } a_{jk} = 1,\ 1 \leq k \leq t\} |$ for all i, j between 1 and n.

In this paper, we are calculating the 1-uniform dcsl index of some classes of graphs and establish an addressing of the distance matrix of those graphs are the $0, 1)-$matrices of the hypergraphs corresponding to the 1-uniform dcsl labeling of that graphs with minimal dcsl set such that $\emptyset$ is not assigned to any vertex of any graph in this class.

Definition 1.1. *[2] Let $G = (V, E)$ be any connected (p, q) graph. A distance compatible set-labeling of a graph G is an injective set-assignment $f : V(G) \to 2^X$, X a nonempty ground set, such that the corresponding induced function $f^{\oplus} : V(G) \times V(G) \to 2^X - \emptyset$, defined by $f^{\oplus}(uv) = f(u) \oplus f(v)$ satisfies $| f^{\oplus}(uv) |= k^f_{(u,v)} d(u,v)$ for all distinct $u, v \in V(G)$, where $d(u,v)$ is the distance between u and v and $k^f_{(u,v)}$ is a constant, not necessarily an integer. G is distance compatible set-labeled (dcsl) graph if it admits a dcsl. We denote a dcsl-graph G with a dcsl, f by the ordered pair (G, f). The corresponding ground set is called a dcsl-set. A dcsl f of a graph G is k-uniform if the constants of proportionality are all equal to k.*

Definition 1.2. *dcsl index of a dcsl-graph (G, f) is the minimum cardinality of the dcsl-set and it is denoted by $\delta_d(G)$.*

Let (G, f) be a 1-uniform dcsl-graph, such that $f(u) \neq \emptyset$ for all $u \in V(G)$. We can construct a hypergraph, H^f_G using the labels of the dcsl-graph, which are subsets of the corresponding dcsl-set. Optimal hypergraphs H^f_G are those hypergraphs, for which f is minimal, in the sense that the dcsl-set with respect to the 1-uniform dcsl f is minimal.The hypergraph H^f_G is defined on the dcsl-set $X = \{x_1, x_2, x_3, \ldots, x_n\}$ as a family $H = (E_1, E_2, E_3, \ldots, E_n)$ of subsets of X such that $E_i = f(v_i)$ for exactly one vertex $v_i \in V(G)$, $E_i \neq \emptyset$, $(1 \leq i \leq n)$, and $\bigcup_{i=1}^{m} E_i = X$

2 1-uniform dcsl index of graphs

As defined already, a dcsl f of a graph $G = (V, E)$ is 1-*uniform* if all the constants of proportionality with respect to f are equal to 1, and if G admits such a dcsl then G is a 1-*uniform dcsl-graph*. Following are some important classes of 1-uniform dcsl graphs, reported in [5].

Theorem 2.1 (5). *All paths are* 1*-uniform dcsl-graphs.*

Theorem 2.2 (5). *All finite stars are* 1*-uniform dcsl-graphs.*

Theorem 2.3 (5). *A graph G with a vertex of full degree is* 1*-uniform dcsl if and only if $G \cong K_{1,n}$.*

Theorem 2.4 (5). *Complete bipartite graph $K_{m,n}$ is* 1*-uniform dcsl if and only if it is isomorphic to $K_{1,n}$ or $K_{2,2}$.*

Theorem 2.5 (5). *Every tree admits a* 1*-uniform dcsl.*

Recall that the minimum cardinality of the underlying set X such that G admits a 1-uniform dcsl is called the 1-uniform dcsl index δ_d of G. Now, we calculate the dcsl index of certain 1-uniform dcsl-graphs

Proposition 2.6. *For any* 1*-uniform dcsl-graph G, $\delta_d(G) \geq diam(G)$.*

Proof. $diam(G) = max\{d(v_i, v_j), v_i, v_j \in V(G)\}$. Now, if X_i, X_j are subsets of X such that $f(v_i) = X_i$, $f(v_j) = X_j$, then $\mid X_i \oplus X_j \mid = d(v_i, v_j)$, only if $\mid X \mid \geq d(v_i, v_j)$, $\forall \ i, j$. Thus, $\mid X \mid \geq diam(G)$. Therefore, $\delta_d(G) \geq diam(G)$. □

The natural question under investigation is whether the bound for $\delta_d(G)$ obtained in Proposition 2.6 is attainable. The following Theorem answers this question affirmatively.

Theorem 2.7. $\delta_d(P_n) = n - 1$, $n > 2$

Proof. Suppose P_n is a 1-uniform dcsl with a set X of cardinality $n-2$. Then, $\mid X_1 \oplus X_n \mid \leq n - 2$, where $f(v_1) = X_1$ and $f(v_n) = X_n$. But, $d(v_1, v_n) = n - 1$, a contradiction. Hence, $\delta_d(P_n) \geq n - 1$. Let $X = \{1, 2, \ldots, n - 1\}$. Consider the labeling $f : V(G) \to 2^X$ defined by $f(v_1) = \{1\}$, $f(v_2) = \{1, 2\}$, $f(v_3) = \{2\}$ and $f(v_i) = \{2, 3, \ldots, i-1\}$, $4 \leq i \leq n$. Then, $\mid f^{\oplus}(v_1v_2) \mid = 1$, $\mid f^{\oplus}(v_1v_3) \mid = 2$ and $\mid f^{\oplus}(v_2v_3) \mid = 1$. Now, $f^{\oplus}(v_iv_j) = f(v_i) \oplus f(v_j) = \{2, 3, \ldots, i-1\} \oplus \{2, 3, \ldots, j-1\}$, $(i \leq j) = \{i, i+1, i+2, \ldots, j-1\}$, which implies $\mid f^{\oplus}(v_iv_j) \mid = j - i = d(v_i, v_j)$. Also, $d(v_j, v_i)(i \geq 4) = i - j = \mid f^{\oplus}(v_jv_i) \mid (j = 1, 2, 3)$. Thus, f is a 1-uniform dcsl of P_n, which is unique up to the cardinalities of the sets in the set-labeling f. □

Invoking the definition of an addressing of a matrix M by Graham and Pollak [4] we define,

Definition 2.8. *If $M = (m_{ij})$ is any $n \times n$ matrix, then an addressing of M of length t is an $n \times t$ matrix A with entries from $\{0, 1\}$ such that, $m_{ij} = \mid \{k : a_{ik} = 0$ and $a_{jk} = 1$ or $a_{ik} = 1$ and $a_{jk} = 0,\ 1 \le k \le t\} \mid$ for all i, j between 1 and n.*

Now, we prove that addressing of distance matrix of P_n is the $(0, 1)-$matrix of the hypergraph corresponding to the labeling given in Theorem 2.7.

Distance matrix of P_n is given by

$$D_{n\times n} = \begin{pmatrix} 0 & 1 & 2 & 3 & . & . & . & n-1 \\ 1 & 0 & 1 & 2 & . & . & . & n-2 \\ 2 & 1 & 0 & 1 & . & . & . & n-3 \\ 3 & 2 & 1 & 0 & . & . & . & n-4 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ n-1 & n-2 & n-3 & n-4 & . & . & . & 0 \end{pmatrix}$$

The hypergraph representation of this labeling consists of vertices $1, 2, 3, \ldots, n-1$ and hyperedges $(1), (1, 2), (2), (2, 3), (2, 3, 4), \ldots, (2, 3, 4, , \ldots, n-1)$. The $(0, 1)-$matrix of this hypergraph is

$$A_{n\times n-1} = \begin{pmatrix} 1 & 0 & 0 & 0 & . & . & . & 0 \\ 1 & 1 & 0 & 0 & . & . & . & 0 \\ 0 & 1 & 0 & 0 & . & . & . & 0 \\ 0 & 1 & 1 & 0 & . & . & . & 0 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 0 & 1 & 1 & 1 & . & . & . & 1 \end{pmatrix}$$

Now $d_{ij} = \mid \{k : a_{ik} = 0$ *and* $a_{jk} = 1$ *or* $a_{ik} = 1$ *and* $a_{jk} = 0,\ 1 \le k \le t\} \mid$ for all i, j between 1 and n. Thus the $(0, 1)$ matrix of the hypergraph corresponding to the 1-uniform dcsl labeling of path P_n is an addressing of length $n-1$ of the distance matrix $D_{n\times n}$ of P_n.

Theorem 2.9. $\delta_d(K_{1,n}) = n$.

Proof. Suppose $K_{1,n}$ has a 1-uniform dcsl with respect to the ground set $X = \{1, 2, \ldots, n-1\}$. Let $v_1, v_2, v_3, \ldots, v_{n+1}$ be the vertices of $K_{1,n}$ with v_1 as the central vertex.

Case i: $\mid f(v_1) \mid = 1$. Then, it is necessary that $\mid f(v_i) \mid = 2$ for all i, $2 \leq i \leq n+1$ and $f(v_1) \subset f(v_2)$. But, the number of subsets of X with cardinality 2 containing $f(v_1)$ is $n-2$, which in turn implies one vertex remains unlabeled, a contradiction.
Case ii: $\mid f(v_1) \mid = k, k > 1$. Then, $\mid f(v_i) \mid = k+1$ or $k-1$, $2 \leq i \leq n+1$.
(a) If $\mid f(v_i) \mid = k+1$ $\forall i$ $2 \leq i \leq n$, then $f(v_1) \subset f(v_i)$. But, the number of such subsets is less than n, again a contradiction as in Case(i).
(b) If $\mid f(v_i) \mid = k-1$, $k > 2$ $\forall i$, then, for every i, $2 \leq i \leq n$ $f(v_i) \subset f(v_1)$ and the number of such subsets is less than n, again a contradiction.
If $\mid f(v_i) \mid = k+1$, for some i and $k-1$ for some i then, some vertices would remain unlabeled, again a contradiction. Hence, we conclude that $\delta_d(K_{1,n}) = n$. □

Now consider the distance matrix of $K_{1,n}$

$$D_{n+1\times n+1} = \begin{pmatrix} 0 & 1 & 1 & 1 & . & . & . & 1 \\ 1 & 0 & 2 & 2 & . & . & . & 2 \\ 1 & 2 & 0 & 2 & . & . & . & 2 \\ 1 & 2 & 2 & 0 & . & . & . & 2 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 1 & 2 & 2 & 2 & . & . & . & 0 \end{pmatrix}$$

The $(0,1)-$matrix $A_{n+1\times n}$, given by the hypergraph representation of 1-uniform dcsl star $K_{1,n-1}$ is given by

$$A_{n+1\times n} = \begin{pmatrix} 1 & 1 & 0 & 0 & . & . & . & 0 \\ 1 & 0 & 0 & 0 & . & . & . & 0 \\ 0 & 1 & 0 & 0 & . & . & . & 0 \\ 1 & 1 & 1 & 0 & . & . & . & 0 \\ 1 & 1 & 0 & 1 & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 1 & 1 & 0 & 0 & . & . & . & 1 \end{pmatrix}$$

The distance matrix $D(K_{1,n})$ has an addressing of length $n-1$, which is nothing but the $(0,1-)$matrix $A_{n+1\times n}$, given by the hypergraph representation of 1-uniform dcsl star $K_{1,n-1}$

Theorem 2.10. 1-*uniform dcsl index of an even cycle* C_n *is* $\frac{n}{2}+1$.

Proof. Let C_n be the even cycle with vertex set $\{v_1, v_2, \ldots, v_n\}$. Let $X = \{1, 2, \ldots, \frac{n}{2}+1\}$. Define $f : V(C_n) \rightarrow 2^X$ defined by

$f(v_1) = \{1\};$
$f(v_2) = \{1,2\};$
$f(v_n) = \{1,3\};$
$f(v_3) = \{1,2,4\};$
$f(v_{n-1}) = \{1,3,5\};$
.
.
.
$f(v_{\frac{n}{2}+1}) = \{1,2,\ldots,\frac{n}{2}+1\}$. Then, clearly f is a 1-uniform dcsl. Also, note that diameter of C_n is $\frac{n}{2}$, so that the minimum cardinality of X should necessarily be $\frac{n}{2}+1$. Thus, 1-uniform dcsl index of an even cycle C_n is $\frac{n}{2}+1$ □

The distance matrix of C_8 is given below.

$$D(C_n) = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 3 & 2 & 1 \\ 1 & 0 & 1 & 2 & 3 & 4 & 3 & 2 \\ 2 & 1 & 0 & 1 & 2 & 3 & 4 & 3 \\ 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 \\ 3 & 4 & 3 & 2 & 1 & 0 & 1 & 2 \\ 2 & 3 & 4 & 3 & 2 & 1 & 0 & 1 \\ 1 & 2 & 3 & 4 & 3 & 2 & 1 & 0 \end{pmatrix}$$

In this case also we can see that A is an addressing of length $\frac{n}{2}+1$ of the distance matrix $D(C_n)$ as shown in the following matrix.

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

It is a tedious task to find the dcsl index of an arbitrary tree. It may even an NP- complete problem. However, we find the 1-uniform dcsl index δ_d of the classes of trees with less than 6 vertices and of diameters less than or equal to three. We denote a tree on n vertices with diameter d by T_n^d.

Theorem 2.11. *1-uniform dcsl index* $\delta_d(T_n^d) \leq n-1$ *for* $n \leq 6$ *and* $d \leq 3$.

We consider each classes seperately as follows.

1. $\delta_d(T_2^1) = 1$.
   Let v_1 and v_2 are the two vertices of T_2^1. Let $X = \{1\}$. Define $f(v_1) = \emptyset$ and $f(v_2) = \{1\}$. Then, this is a 1-uniform dcsl with dcsl set X. Therefore $\delta_d(T_2^1) \leq 1$. Also by Theorem 2.6 $\delta_d(T_2^1) \geq 1$. Hence $\delta_d(T_2^1) = 1$.

2. $\delta_d(T_3^2) = 2$.
   Let v_1, v_2 and v_3 are the three vertices of T_3^2. Define $f(v_1) = \{1\}$, $f(v_2) = \{1, 2\}$ and $f(v_3) = \{2\}$. Then, f is a 1-uniform dcsl with dcsl set $X = \{1, 2\}$. Therefore $\delta_d(T_3^2) \leq 2$. Also by Theorem 2.6 $\delta_d(T_3^2) \geq 2$. Hence, $\delta_d(T_3^2) = 2$.

3. $\delta_d(T_4^3) = 3$.
   Let $V(T_4^3) = \{v_1, v_2, v_3, v_4\}$. Let v_1 and v_4 are the antipodal vertices of T_4^3 and v_2 and v_3 are the internal vertices. Now, $d((T_4^3)) = 3$, implies $\delta_d(T_4^3) \geq 3$. Let $X = \{1, 2, 3\}$. Define $f : V(T_4^3) \to 2^X$ defined by, $f(v_1) = \{1\}$, $f(v_2) = \{1, 3\}$, $f(v_3) = \{3\}$ and $f(v_4) = \{2, 3\}$. Then, f is a 1-uniform dcsl and hence $\delta_d(T_4^3) = 3$.

4. $\delta_d(T_5^3) = 4$.

   Consider T_5^3. Since, T_5^3 contains T_4^3 as an induced subgraph $\delta_d(T_5^3) \geq \delta_d(T_4^3)$. We have $\delta_d(T_4^3) = 3$. Now, consider $x = \{1, 2, 3\}$. There are two different diametral path in T_5^3 and hence we cannot label the antipodal vertices with the subsets of X such that the resulting graph is a 1-uniform dcsl-graph. If $V(T_5^3) = \{v_1, v_2, v_3, v_4, v_5\}$, then the adjacency matrix of T_5^3 is given by

   $$A(T_5^3) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

   It is easy to see that the labeling $f : V(T_5^3) \to 2^X$, $X = \{1, 2, 3, 4\}$, defined by $f(v_1) = \{1\}$, $f(v_2) = \{1, 2\}$, $f(v_3) = \{1, 2, 3\}$, $f(v_4) = \{1, 2, 3, 4\}$ and $f(v_5) = \{2\}$ is a 1-uniform dcsl. Hence, we can conclude that $\delta_d(T_5^3) = 4$.

5. $\delta_d(T_6^3) = 5$.

   Number of trees comes under the class T_6^3 is two upto isomorphism. The corresponding adjacency matrix where $V(T_6^3) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ are given below.

$$A_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Since, T_6^3 contains T_5^3 as an induced subgraph $\delta_d(T_6^3) \geq \delta_d(T_5^3)$. Now, consider the adjacency matrix A_1. In a 1-uniform dcsl-labeling f, if $\mid f(v_1) \mid = 1$, then $\mid f(v_4) \mid = 4 = \mid f(v_6) \mid$. Also, $f(v_1) \subset f(v_4)$, $f(v_1) \subset f(v_6)$ and $f(v_4) \neq f(v_6)$. Therefore cardinality of the dcsl-set should be greater than or equal to five. When we consider the adjacency matrix A_2, if $\mid f(v_1) \mid = 1 = \mid f(v_5) \mid$, then $\mid f(v_4) \mid = 4$, $f(v_1) \subset f(v_4)$ and $f(v_5) \subset f(v_4)$. Then, since f is 1-uniform, $\mid f(v_6) \mid = 3$. Also, $f(v_1) \subset f(v_6)$ and $f(v_5) \subset f(v_6)$. This is possible only if $f(v_6)$ contains an element which does not belong to the set $X = \{1, 2, 3, 4\}$. Which implies that the cardinality of the dcsl-set should be greater than or equal to five. That is, $\delta_d(T_6^3) \geq 5$. Now the labeling $f : V(T_6^3) \to 2^X$, $X = \{1, 2, 3, 4, 5\}$, defined by $f(v_1) = \{1\}$, $f(v_2) = \{1, 2\}$, $f(v_3) = \{1, 2, 3\}$, $f(v_4) = \{1, 2, 3, 4\}$, $f(v_5) = \{2\}$ and $f(v_6) = \{1, 2, 3, 5\}$is a 1-uniform dcsl in the first case and $f(v_1) = \{1\}$, $f(v_2) = \{1, 2\}$, $f(v_3) = \{1, 2, 3\}$, $f(v_4) = \{1, 2, 3, 4\}$, $f(v_5) = \{2\}$ and $f(v_6) = \{1, 2, 5\}$is a 1-uniform dcsl in the second case. Therefore

$$\delta_d(T_6^3) = 5.$$

Remark 2.12. *We strongly believe that* 1*-uniform dcsl index of an arbitrary tree of order* n *is* $n - 1$*; which we pose as a Conjecture.*

Conjecture 2.13. $\delta_d(T) = n - 1$*, where* T *is a tree of order* n*.*

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