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Strong Forms of τ^*-Generalized Continuous Map in Topological Spaces

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Abstract

In this paper, we introduce a new class of maps called τ^*-gc-irresolute map, strongly τ^*-generalized continuous map and perfectly τ^*-generalized continuous maps in topological space and study some of their properties and relations among them.

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1. Introduction

Levine [4] introduced the concept of generalized closed sets in a topological space. Dunham [3] introduced generalized closure operator cl* and defined a topology called τ^* topology. Balachandran, Sundaram and Maki [1] introduced and studied g- continuous maps. Pushpalatha, Eswaran and Rajarubi [5] introduced and investigated τ^*-generalized closed sets. Eswaran and Pushpalatha [2] introduced and studied τ^*-generalized continuous maps in a topological spaces.

In this paper, we introduce new class of maps namely τ^*-gc-irresolute maps, strongly τ^*-g-continuous maps and perfectly τ^*-g-continuous maps

Throughout this paper (X, τ^*) and (Y, σ^*) (or simply X and Y) represent non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. For a subset A of a space (X, τ^*), cl(A), cl*(A) and A^c represent closure of A, closure* of A and complement of A respectively.

2. Preliminaries

Since we need the following definitions and some results, we recall them.

Definition 2.1. [4] A subset A of a topological space (X, τ) is called generalized closed (briefly g-closed) in X if cl(A) $\subseteq$ G whenever A $\subseteq$ G and G is open in X. A subset A is called generalized open (briefly g-open) in X if its complement A^c is g-closed.

Definition2.2. [5] A subset A of a topological space (X, τ*) is called τ*-generalized closed (briefly τ*-g-closed) cl*(A) $\subseteq$ G whenever A $\subseteq$ G and G is τ*-open in X. A subset A is called τ*-generalized open (briefly τ*-g-open) in X if its complement A^c is τ*-g-closed.

Definition 2.3. [3] For the subset A of a topological X, the topology τ^* is defined by $\tau^* = \{G : cl^*(G^C) = G^C\}$

Definition 2.4. [3] For the subset A of a topological X, the generalized closure operator cl* is defined as the intersection of all g-closed sets containing A.

Definition 2.5. [1] A collection $\{A_i : i \in I\}$of g-open sets in a topological space (X, τ) is called a g-open cover of a subset B if $B \subset \cup\{A_i : i \in I\}$.

Definition 2.6. A map $f : X \rightarrow Y$ from a topological space (X, τ) into a topological space (Y, σ) is called

(i) Continuous if the inverse image of every closed set (or open set) in Y is closed (or open) in X.
(ii) generalized continuous[1](g-continuous) if the inverse image of every closed set in Y is g-closed in X.
(iii) gc-irresolute[1]if the inverse image of every g-closed set in Y is g-closed in X.
(iv)strongly g-continuous[1] if the inverse image of every g-open set in Y open in X.
(v) perfectly g-continuous[1] if the inverse image of every g-open set in Y is both open and closed in X.

Definition 2.7. [5] A map $f : X \rightarrow Y$ from a topological space (X, τ*) into a topological space (Y, σ*) is called τ^*-generalized continuous (τ^*-g-continuous) if the inverse image of every g-closed set in Y is τ^*-g-closed in X.

Remark 2.8. In [5], it has been proved that (i) every closed set is τ*-g-closed and (ii) every g-closed set is τ*-g-closed.

3. τ*-gc-irresolute maps in topological spaces

In this section, we introduce a new class of maps called τ*-gc-irresolute maps which is included in the class of τ*-g-continuous maps. We investigate some basic properties also.

Definition 3.1. A map $f : X \to Y$ from a topological space (X, τ*) into a topological space (Y, σ*) is called τ*-gc-irresolute if the inverse image of every τ*-g-closed set in Y is τ*-g-closed in X.

Theorem 3.2. A map $f : X \to Y$ is τ*-gc-irresolute if and only if the inverse image of every τ*-g-open set in Y is τ*-g-open in X.

Proof: Assume that f is τ*-gc-irresolute. Let A be any τ*-g-open set in Y. Then A^C is τ*-g-closed in Y. Since f is τ*-gc-irresolute, $f^{-1}(A^C)$ is τ*-g-closed in X. But $f^{-1}(A^C) = X - f^{-1}(A)$ and so $f^{-1}(A)$ is τ*-g-open in X. Hence the inverse image of every τ*-g-open set in Y is τ*-g-open in X.

Conversely assume that the inverse image of every τ*-g-open set in Y is τ*-g-open in X. Let A be any τ*-g-closed in Y. Then A^C is τ*-g-open in Y. By assumption, $f^{-1}(A^C)$ is τ*-g-open in X. But $f^{-1}(A^C) = X - f^{-1}(A)$ and so $f^{-1}(A)$ is τ*-g-closed in X. Therefore f is τ*-gc-irresolute

Theorem 3.3. A map $f : X \to Y$ is τ*-gc-irresolute if and only if it is τ*-g-continuous.

Proof: Assume that f is τ*-gc-irresolut. Let F be any g-closed set in Y. By Remark 2.8, F is τ*-g-closed in Y. Since f is τ*-gc-irresolut, $f^{-1}(F)$ is τ*-g-closed in X. Therefore f is τ*-g-continuous.

Conversely, assume that f is τ*-g-continuous. Let F be any g-closed set in Y. By Remark 2.8, F is τ*-g-closed in Y. Since f is τ*-g-continuous, $f^{-1}(F)$ is τ*-g-closed in X. Therefore f is τ*-gc-irresolute.

Theorem 3.4. Let X, Y and Z be any topological spaces. For any τ*-gc-irresolute map $f : X \to Y$ and any τ*-g-continuous map $g : Y \to Z$, the composition $g \circ f : X \to Z$ is τ*-g-continuous.

Proof: Let F be any g-closed set in Z. Since g is τ*-g-continuous, $g^{-1}(F)$ is τ*-g-closed in Y. Since f is τ*-gc-irresolut, $f^{-1}(g^{-1}(F))$ is τ*-g-closed in X. But $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$. Therefore $g \circ f$ is τ*-g-continuous.

Theorem 3.5. If $f : X \to Y$ from a topological space (X, τ*) into a topological space (Y, σ*) is bijective, g-open and τ*-g-continuous then f is τ*-gc-irresolute.

Proof: Let A be a τ*-g-closed set in Y. Let $f^{-1}(A) \subseteq O$ where O is g-open in X. Therefore, $A \subseteq f(O)$ holds. Since f(O) is g-open and A is τ*-g-closed in Y, $cl(A) \subseteq f(O)$ holds and hence $f^{-1}(cl(A)) \subseteq O$. Since f is τ*-g-continuous and cl(A) is g-closed in Y, $cl(f^{-1}(cl(A))) \subseteq O$ and so $cl(f^{-1}(A)) \subseteq O$. Therefore, $f^{-1}(A)$ is τ*-g-closed in X. Hence f is τ*-gc-irresolute.

4. Strongly τ*-g-continuous maps and Perfectly τ*-g-continuous maps

Levine [4] and K.Balachandran, P.Sundaram and H.Maki [1] introduced and investigated strongly continuity and strongly g-continuity in topological spaces respectively. In this section we introduce two forms of continuous maps in topological spaces namely strongly τ*-g-continuous and perfectly τ*-g-continuous maps.

Definition 4.1. A map $f : X \to Y$ from a topological space (X, τ*) into a topological space (Y, σ*) is said to be strongly τ*-g-continuous if the inverse image of every τ*-g-open set (or τ*-g-closed set) in Y is g-open (or g-closed) in X.

Theorem 4.2. If a map $f : X \to Y$ from a topological space (X, τ*) into a topological space (Y, σ*) is strongly τ*-g-continuous then it is τ*-g-continuous but not conversely.

Proof: Assume that f is strongly τ*-g-continuous. Let G be any g-closed set in Y. By Remark 2.8, G is τ*-g-closed in Y. Since f is strongly τ*-g-continuous, $f^{-1}(G)$ is g-closed in X. Therefore f is τ*-g-continuous.

Converse of the above theorem need not be true as seen from the following example.

Example 4.3. Let τ = {X, ϕ, {c}}, σ = {Y, ϕ, {c}, {a, b}}. Let $f : X \to Y$ be an identity map. Then f is τ*-continuous. But f is not strongly τ*-g-continuous, since for the τ*-g-closed set G = {c} in Y, $f^{-1}(G)$ = {c} is not g-closed in X.

Theorem 4.4. A map $f : X \to Y$ from a topological space (X, τ*) into a topological space (Y, σ*) is strongly τ*-g-continuous if and only if the inverse image of every τ*-g-closed set in Y is g-closed in X.

Proof: Assume that f is strongly τ*-g-continuous. Let F be any τ*-g-closed set in Y. Then F^C is τ*-g-open set in Y. Since f is strongly τ*-g-continuous, $f^{-1}(F^C)$ is g-open in X. But $f^{-1}(F^C) = X - f^{-1}(F)$ and so $f^{-1}(F)$ is g-closed in X.

Conversely assume that the inverse image of every τ*-g-closed set in Y is g-closed in X. Let G be any is τ*-g-open set in Y. Then G^C is τ*-g-closed set in Y. By assumption, $f^{-1}(G^C)$ is g-closed in X. But $f^{-1}(G^C) = X - f^{-1}(G)$ and so $f^{-1}(G)$ is g-open in X. Therefore, f is strongly τ*-g-continuous.

Theorem 4.5. If a map $f : X \to Y$ is strongly τ*-g-continuous and a map $g : Y \to Z$ is τ*-g-continuous then the composition $g \circ f : X \to Z$ is strongly τ*-g-continuous.

Proof: Let G be any g-closed set in Z. Since g is τ*-g-continuous, $g^{-1}(G)$ is τ*-g-closed in Y. Since f is strongly τ*-g-continuous, $f^{-1}(g^{-1}(G))$ is g-closed in X. But $(g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$. Therefore, $g \circ f$ is strongly τ*-g-continuous.

Theorem 4.6. If a map $f : X \to Y$ is strongly τ*-g-continuous and a map $g : Y \to Z$ is τ*-g-continuous then the composition $g \circ f : X \to Z$ is τ*-g-continuous.

Proof: Let G be any g-closed set in Z. Since g is τ*-g-continuous, $g^{-1}(G)$ is τ*-g-closed in Y. Since f is strongly τ*-g-continuous, $f^{-1}(g^{-1}(G))$ is g-closed in X. By Remark 2.8, $f^{-1}(g^{-1}(G))$ is τ*-g-closed. But $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$. Therefore, $g \circ f$ is τ*-g-continuous.

Theorem 4.7. If a map $f : X \to Y$ from a topological space (X, τ*) into a topological space (Y, σ*) is continuous then it is strongly τ*-g-continuous but not conversely.

Proof: Let $f : X \to Y$ be continuous. Let F be a closed set in Y. Since f is continuous, $f^{-1}(F)$ is closed in X. By Remark2.8, F is τ*-g-closed and since every closed set is g-closed, $f^{-1}(F)$ is g-closed. Hence f is strongly τ*-g-continuous.

Converse of the above theorem need not be true as seen from the following example.

Example 4.8. Let τ = {X, ϕ, {a}, {b, c}}, σ = {Y, ϕ, {a}, {c}, {a, c}}. Let $f : X \to Y$ be an identity map. Then f is strongly τ*-g-continuous. But f is not continuous since for the closed set F = {b} in Y, $f^{-1}(F)$ = {b} is not closed in X.

Theorem 4.9. If a map $f : X \to Y$ from a topological space (X, τ*) into a topological space (Y, σ*) is g-continuous then it is strongly τ*-g-continuous.

Proof: Let $f : X \to Y$ be g-continuous. Let F be a closed set in Y. Since f is g- continuous, $f^{-1}(F)$ is g-closed in X. By Remark2.8, F is τ*-g-closed in Y. Hence f is strongly τ*-g-continuous.

Definition 4.10. A map $f : X \to Y$ from a topological space (X, τ*) into a topological space (Y, σ*) is said to be perfectly τ*-g-continuous if the inverse image of every τ*-g-closed set in Y is both g-open and g-closed in X.

Theorem 4.11. If a map $f : X \to Y$ from a topological space (X, τ*) into a topological space (Y, σ*) is perfectly τ*-g-continuous then it is strongly τ*-g-continuous but not conversely.

Proof: Assume that f is perfectly τ*-g-continuous. Let G be any τ*-g-closed set in Y. Since f is perfectly τ*-g-continuous, $f^{-1}(G)$ is g-closed in X. Therefore, f is strongly τ*-g-continuous.

Converse of the above theorem need not be true as seen from the following example.

Example 4.12. Let τ = {X, ϕ, {a}}, σ = {Y, ϕ, {a}, {b}, {a, b}, {a, c}}. Let $f : X \to Y$ be an identity map. Then f is strongly τ*-g-continuous. But it is not perfectly τ*-g-continuous since for the τ*-g-closed set G = {b, c} in Y, $f^{-1}(G)$ = {b, c} is g-open but not g-closed in X.

Theorem 4.13. A map $f : X \to Y$ from a topological space (X, τ*) into a topological space (Y, σ*) is perfectly τ*-g-continuous if and only if the inverse image of every τ*-g-closed set in Y is both g-open and g-closed in X.

Proof: Assume that f is perfectly τ*-g-continuous. Let F be any τ*-g-closed set in Y. Then F^C is τ*-g-open in Y. Since f is perfectly τ*-g-continuous, $f^{-1}(F^C)$ is both g-open and g-closed in X. But $f^{-1}(F^C) = X - f^{-1}(F)$ and so $f^{-1}(F)$ is both g-open and g-closed in X.

Conversely assume that the inverse image of every τ*-g-closed set in Y is both g-open and g-closed in X. Let G be any τ*-g-open set in Y. Then G^C is τ*-g-closed in Y. By assumption, $f^{-1}(G^C) = X - f^{-1}(G)$ and so $f^{-1}(G)$ is both g-open and g-closed in X. Therefore f is perfectly τ*-g-continuous.

Theorem 4.14. If a map $f : X \to Y$ from a topological space (X, τ*) into a topological space (Y, σ*) is strongly τ*-g-continuous then it is τ*-gc-irresolute but not conversely.

Proof: Let $f : X \to Y$ be strongly τ*-g-continuous map. Let F be a τ*-g-closed set in Y. Since f is strongly τ*-g-continuous, $f^{-1}(F)$ is g-closed in X. By Remark 2.8, $f^{-1}(F)$ is τ*-g-closed in X. Hence f is τ*-gc-irresolute.

Converse of the above theorem need not be true as seen from the following example.

Example 4.15. Let τ = {X, ϕ, {c}}, σ = {Y, ϕ, {a}, {b}, {a, b}}. Let $f : X \to Y$ be an identity map. Then f is τ*-gc-irresolute. But f is not strongly τ*-g-continuous since for the τ*-g-closed set F = {c} in Y, $f^{-1}(F)$ = {c} is not g-closed in X.

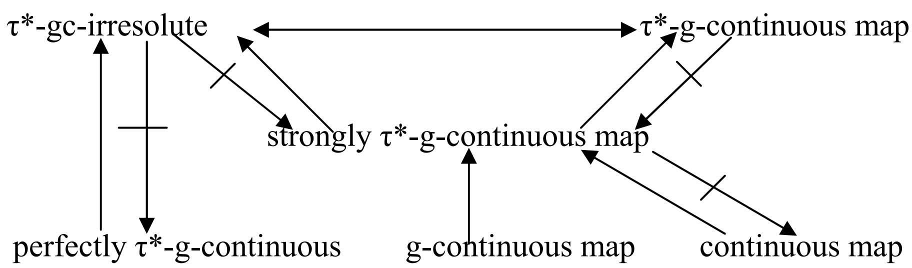
Theorem 4.16 If a map $f : X \to Y$ from a topological space (X, τ*) into a topological space (Y, σ*) is perfectly τ*-g-continuous then it is τ*-gc-irresolute but not conversely.

Proof: Let $f : X \to Y$ be perfectly τ*-g-continuous map. Let F be a τ*-g-closed set in Y. Since f is perfectly τ*-g-continuous, $f^{-1}(F)$ is both g-open and g-closed in X. By Remark 2.8, $f^{-1}(F)$ is τ*-g-closed in X. Hence f is τ*-gc-irresolute.

Converse of the above theorem need not be true as seen from the following example.

Example 4.17 Let τ = {X, ϕ, {a}, {b}, {a, b}, {a, c}}, σ = {Y, ϕ, {b}, {a, b}, {b, c}}. Let $f : X \to Y$ be an defined by f(a) = b, f(b) = c, f(c) = a. Then f is τ*-gc-irresolute. But f is not perfectly τ*-g-continuous since for the τ*-g-closed set F = {c} in Y, $f^{-1}(F)$ = {c} is g-open in X but not g-closed in X.

Remark 4.14. From the above, we obtain the following implications.



References

[1] K.Balachandran, P.Sundaram and J.Maki, On generalized continuous maps in topological spaces, Mem. Fac. Sci. Kochi Univ. (Math.) 12 (1991), 5-13.

[2] S.Eswaran and A.Pushpalatha, τ*-generalized continuous maps in topological spaces, International Journal of Mathematical Sciences and Engineering Applications, Vol. 3, No. IV, December 2009. (will be published in December 2009 issue)

[3] W.Dunham, A new closure operator for non-T topologies, Kyungpook Math. J., 22 (1982), 55 – 60

[4] N.Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo(2), 19 (1970), 89 – 96.

[5] A.Pushpalatha, S.Eswaran and P.Rajarubi, τ^*-generalized closed sets in topological spaces, Proceedings of World Congress on Engineering 2009 Vol II WCE 2009, July 1 – 3, 2009, London, U.K., 1115 – 1117.

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